Controllable azimuthons of four-wave mixing and their applications

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Abstract
We report controllable azimuthons of four-wave mixing (FWM), which can be modulated by several parameters in experiment. The spot number, splitting depth, rotation angular velocity and direction of such azimuthons can be controlled by the frequency and intensity of the FWM signal or the dressing field through the cross-phase modulation due to atomic coherence. The intensity gain of the azimuthons can be modulated by frequency detuning through quantum parametric amplification. The quantum correlated FWM vortex is observed in experiment. We also discuss the applications of such controllable azimuthons in all-optical circulators, multiplexers (demultiplexers), routers, cross-connects and optical amplifiers.

Keywords: optical solitons, four-wave mixing, atomic coherence

(Some figures may appear in colour only in the online journal)
controlled by light intensity, it can be used for the construction of demultiplexers. Since the modern integrated optical circuits require the incorporation of a variety of functionalities onto one device [25], the device can significantly simplify the optical circuits and suppress multiple reflections and loss in the couplings among components.

In this paper, for the first time to our knowledge, we report the simultaneous control of spatial splitting, azimuthal rotation and intensity gain of the quantum amplified FWM azimuthons in experiment. We demonstrate the control of the number of splitting spots, the modulation depth, as well as the rotation angular velocity of the vortices by changing the frequency detuning and intensity of the FWM signal or the dressing fields via the atomic coherence enhanced cross-phase modulation. The rotation direction of the vortex is controlled by the phase of three interference waves in the FWM process. By quantum parametric amplification, we demonstrate the modulation of the FWM signal intensity gain by changing the frequency detuning, and experimentally get the quantum correlated FWM vortex. Furthermore, based on experimental results, we propose potential applications of such controllable azimuthons in all-optical communication, including circulators, multiplexers (demultiplexers), routing, cross-connects and amplifiers.

2. Basic theory and experimental scheme

Figure 1(a) shows the three-level V-type atomic system employed in the experiment. Three energy levels (|0⟩ (3S1/2), |1⟩ (3P1/2) and |2⟩ (3P3/2)) in a sodium atom (in a heat pipe oven without buffer gas and with atom number density of approximately 5.6×10^13 cm^-3) are involved. When the energy level |1⟩ (3P1/2) is not used, the atomic system changes into a two-level system (figure 1(b)). The spatial alignment of the beams is displayed in figure 1(c), in which two laser beams Ei (with frequency ωi, wavevector ki, and Rabi frequency Gi) and Ei (ωi, ki, Gi), with a small angle θi≈0.3° between them, propagate in the opposite direction to the weak probe beam E0(ω0, k0, G0).

These three beams are from one near-transform-limited dye laser DL1 (10Hz repetition rate, 5ns pulse-width and 0.04 cm^-1 linewidth) with the frequency detuning Δ1 = Ω - ω0, where Ω is the resonant frequency. The other three laser beams E2 (ω2, k2, G2), E3 (ω3, k3, G3) and E3 (ω2, k3, G3) are from another dye laser DL2 (which has the same characteristics as the DL1) with a frequency detuning Δ2 = Ω - ω2. Among them, E2 co-propagates with E1; E3 co-propagates with E1, while E2 propagates with a small angle θ2 ≈0.3° between E2 and E3.

In three-level V-type atomic systems, because the frequency difference between ω0 and ω2 is large, so two beams with ω0 and ω2 cannot interfere—only the beams from one laser can spatially interfere with another. In this case, we only obtain a vortex soliton formed by the degenerated FWM signal E0 (with phase-matching conditions kF3 = k2 - k1 + k0 and E2 (kF2 = k0 - k2 + k3), which can be modulated by the spatial interfered patterns of Ei, E′i and E0, E′2, E′3 and E′3, respectively. In the two-level atomic system, six beams are all controlled to drive the transition between |0⟩ and |2⟩. In such a case, besides the degenerated vortex beams E0 and E′2, we can also obtain another six non-degenerated FWM signals, which satisfy the phase-matching conditions kF3 = k2 - k1 + k0, kF4 = k2 - k2 + k3, kF5 = k1 - k1 + k0, kF6 = k2 - k1 + k3, kF7 = k0 - k2 + k3, and kF8 = k0 - k1 + k3, respectively.

When three plane waves overlap in the medium, the interference patterns can give rise to phase singularity [6]. In mathematics, complex amplitude vectors of these waves form a spatial closed polygon pattern in a complex plane, so the phase singularity occurs at the position where the summed complex amplitude is zero. At the observation plane, the phase helically changes from 0 to 2π along the azimuthal direction round the singular point, and is always equal along the radial direction. Hence the light has energy flowing along the azimuthal direction on the observation plane and the optical vortex forms. In our experiment, the controllable vortex results from the combination of two factors: (i) the vortex pattern carrying phase singularity, formed by interference among the three waves with nearly degenerate frequency, participating in generating the FWM signal; (ii) the cross-phase modulation (XPM) enhanced by the atomic coherence created by the strong dressing fields E′i, which can induce the spatial splitting and change the rotation angular velocity of the FWM vortex. By the theoretical calculation, we can find that the summed complex amplitude of the interfered lights forms an ellipse in the complex plane and can be expressed as the following:

\[ E = A_c \cos(\varphi + \delta_1) + i A_c \cos(\varphi + \delta_2), \]

(1)

where the initial phases are

\[ \delta_1 = -\arctan \left( \frac{\sum_{i=1}^{n} E_{n i} k_i y \sin \theta_{ni}}{\sum_{i=1}^{n} E_{n i} k_i x \sin \theta_{ni}} \right) \]

and

\[ \delta_2 = -\arctan \left( \frac{\sum_{i=1}^{n} E_{n i} k_i x \cos \theta_{ni}}{\sum_{i=1}^{n} E_{n i} k_i x \cos \theta_{ni}} \right) \]

with

\[ \theta_{ni} = \varphi_{ni} + n(k_i x \xi_{ni} + k_i y \zeta_{ni}), \]

in which (T1, T2) is the coordinate of the singularity point; n1 = n0 + \sum_{n=1}^{m} n^{2\text{min}} |E_n|^2 is the nonlinear refractive index with n^2 = Re[\chi^{(3)}/(\epsilon_0 c n^2)] being the cross-Kerr nonlinear coefficient. The third-order nonlinear density matrix element responsible for the Kerr effect. In equation (1), the coefficient A1(A0) is the amplitude of the real (imaginary) part. Comparing equation (1) with the phase structure εµ = cos(µf) + isin(µf) of a typical vortex soliton, we can deduce that the topological charge carried by the FWM vortex is |n| = 1 or |n| = 0. When 0 < δ1 < δ2 < π (π < δ1 < 2π), n = |m| = 1, which means the phase helically increases clockwise (anticlockwise). The electrical field of the FWM vortex soliton with Gaussian envelope and a phase singularity at the vortex core can be written as [26, 27]

\[ E(r, \varphi, z) = U e^{i \Phi(r, \varphi, z)}, \]

(2)

\[ E_0 \tan \left( \frac{r}{w_0} \right) \exp \left( -\frac{r^2}{w(z)^2} \right) \exp[i \Phi(r, \varphi, z)], \]

(2)
in which the phase is \( \Phi(r, \varphi, z) = kr^2/2R(z) + m \varphi + kz; \)
the vortex core width is related to the nonlinear coefficients as \( w_i = E k^2 (n_0/n_i^2)^{1/2} \), with \( n_i^2 \) being the self-Kerr nonlinear coefficients; \( w(z) \) is the transversal beam size; \( R(z) \) is the radius of curvature of the wave front. The transverse wave vector of vortex soliton is defined as \( \mathbf{k}_v = \mathbf{k} + \mathbf{k}_\varphi = (\partial \Phi/\partial r)\hat{r} + r^{-1}(\partial \Phi/\partial \varphi)\hat{\varphi} \), where \( \hat{r} \) and \( \hat{\varphi} \) are the unit vector along the radical and azimuthal directions, respectively. So the vortex beam has transverse wave vector \( \mathbf{k}_v = (kr/R(z))\hat{r} + (m/r)\hat{\varphi} \). This phase gradient determines the rotation rate and trajectory of the vortex soliton. It is obvious that the azimuthal wave vector is uniform. Therefore, the rotation angular velocity around the vortex core is constant. From a more physical perspective, the angular velocity can be defined as

\[
\Omega = M_s/I,
\]
where \( I = \int |U(r)|^2 r^2 dr \) is analogous to the moment of inertia of a rigid body. The angular momentum of vortex can also be obtained as

\[
M_s = \int \frac{\partial \Phi}{\partial \varphi} U^2(r) dr = m \int U^2(r) dr.
\]

In the experiment, strong dressing fields \( E_{1,2} \) co-propagate with the FWM signal in a nonlinear medium, the cross-phase modulation (XPM) and self-phase modulation (SPM) can strongly affect the rotation rate and spatial patterns of the FWM vortex. Furthermore, when the spatial diffraction is balanced by the XPM or SPM, modulated vortex solitons will be obtained. The nonlinear phase shift induced by the dressing field at a position of vortex beam is

\[
\phi_p = \frac{2kn_s^2 I}{n_0 w_i^2} e^{-\left(r^2+n_0^2-2n_0\cos(\varphi-\varphi_0)/w_i^2\right)\left(r-r_0\cos(\varphi-\varphi_0)\right)} w_i^2,
\]

where \( r_0 \) and \( \varphi_0 \) are the relative distance and angle of the dressing field to the vortex core, respectively; \( I \) and \( I_0 \) are the intensities of the dressing field inducing Kerr nonlinearity and the FWM signal, respectively. The additional transverse wave vector introduced by such cross Kerr nonlinearity is

\[
\delta \mathbf{k}_v = \delta \mathbf{k}_r + \delta \mathbf{k}_\varphi = -\frac{4kn_s^2 I}{n_0 w_i^2} e^{-\left(r^2+n_0^2-2n_0\cos(\varphi-\varphi_0)/w_i^2\right)} \left\{ [r-r_0\cos(\varphi-\varphi_0)] \hat{r} + r_0\sin(\varphi-\varphi_0) \hat{\varphi} \right\}.
\]

Now the total phase gradient in the azimuthal direction is \( \left( \mathbf{k}_\varphi + \delta \mathbf{k}_\varphi \right) \). The angular momentum of vortex is

\[
M_a = \int \left( \mathbf{k}_\varphi + \delta \mathbf{k}_\varphi \right) U^2(r) dr,
\]
so the rotation angular velocity is changed by the cross Kerr effect. When \( \mathbf{k}_\varphi + \delta \mathbf{k}_\varphi = 0 \), a non-rotating vortex will be obtained. Moreover, because the additional transverse wave vectors \( \delta \mathbf{k}_r \) and \( \delta \mathbf{k}_\varphi \) are not uniform in the transverse plane of the vortex, they also cause the vortex beam to split globally. Therefore, the cross-phase modulated azimuthon forms. The electrical field of this azimuthon can be written as

\[
E(r, \varphi, z) = E_0 \left\{ \frac{r}{w_v} \exp \left( -\frac{r^2}{w_0^2} \right) \cos(N\varphi) + ipN\varphi \right\} \times \exp \left[ i \phi_p (r, \varphi, z) + i \phi_p (r, \varphi, z) \right],
\]
in which \( p \) is the modulation parameter (1 – \( p \) is the modulation depth), and \( 2N \) is the number of intensity peaks. As the

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**Figure 1.** (a) Three-level V-type and (b) two-level atomic systems with six laser beams. (c) Spatial beam geometry used in the experiment. (d) Diagram of double-pump conical emission and transverse optical pattern. (e) The self-diffraction process in a two-level atomic system. (f) The signals detected in Stokes and anti-Stokes channels without seeding the FWM beam.
splitting of the vortex arising, the angular momentum of the modulated azimuthon is determined by three different factors: the topological charge, the nonlinear phase shift and the number of intensity peaks.

The propagation equation of vortex solitons in the Kerr nonlinear medium can be further written as

$$\frac{\partial E(r, \varphi, z)}{\partial z} = - \frac{i}{2} \left[ \frac{\partial^2 E(r, \varphi, z)}{\partial r^2} + \frac{1}{r} \frac{\partial E(r, \varphi, z)}{\partial r} \right] + \frac{1}{r^2} \frac{\partial^2 E(r, \varphi, z)}{\partial \varphi^2} = \frac{ik}{2} \chi E(r, \varphi, z),$$  \hspace{1cm} (8)

in which \( \chi \) is the total susceptibility that includes the linear susceptibility, nonlinear self-Kerr and cross-Kerr ones, as

$$\chi = \chi_R + i \chi_i = 2n_0 - 1 + \sum_{\rho=2}^{2k} \pi_\rho \hat{I}^\rho_R + \sum_{\rho=2}^{2k} \pi_\rho \hat{I}^\rho_i + i \left( \chi^{(1)} + \sum_{m=3}^{2k+1} \chi_{m}^{(m)} \hat{I}_R^{m-1} + \sum_{m=3}^{2k+1} \chi_{m}^{(m)} \hat{I}_i^{m-1} \right).$$  \hspace{1cm} (9)

The real area of susceptibility relates to the linear and nonlinear refraction of the FWM forming the vortex soliton. The competition of cubic-quintic nonlinearity can be controlled by the intensity of dressing field \( I_x \), and such saturable nonlinearity can support stable vortex solitons [28, 29]. The imaginary part of susceptibility \( \chi_i^{(m)} \) contributes to the loss. For convenience, the loss is characterized by an absorption coefficient defined as \( \alpha = \alpha_0 + \Delta \alpha(I) \), where \( \alpha_0 = \alpha_0 \chi_i^{(1)} / 2cn_0 \) is the linear absorption coefficient, and \( \Delta \alpha(I) \) the nonlinear one. If we only consider the third-order nonlinear effect, the nonlinear absorption coefficient is specified into \( \Delta \alpha(I) = -6 \omega_0 \chi_i^{(3)} I_3 + \chi_i^{(3)} I_3 \) for \( c = \chi_i^{(3)} \).

Now, we consider the gain of the vortex light propagating in the nonlinear medium. When two strong pumping beams \( E_i \) and \( E_f \) counter-propagate through the sodium vapor, two conical emission (CE) beams emerge both forward and backward (as shown in figure 1(d), respectively [30, 31]). In general, the generation of CE is derived from the nonlinear diffraction or nonlinear wave-mixing processes [32, 33]. Here, the forward CE can be interpreted as the self-diffraction process (as seen in figure 1(e)), in which the FWM effect leads to the annihilation of the two photons from the pumping fields \( E_b \) and generation of two photons participating in weak Stokes field \( E_s \) (with wave-vector \( k_s \)) and anti-Stokes field \( E_{as} \) (with \( k_{as} \)). This process is accompanied by a phase mismatching \( \Delta k = k_s - k_{as} - k_{f} \). At the same time, in a phase-conjugate FWM process, in addition to the \( E_s \) photon, a backward propagating photon of the field \( E_b \) (with \( k_b \)) is also generated, while one photon of \( E_i \) and another of \( E_s \) annihilate with phase mismatching \( \Delta k = k_b - k_s - k_{f} \). The real area of susceptibility relates to the linear and nonlinear refraction of the FWM forming the vortex soliton. The competition of cubic-quintic nonlinearity can be controlled by the intensity of dressing field \( I_x \), and such saturable nonlinearity can support stable vortex solitons [28, 29]. The imaginary part of susceptibility \( \chi_i^{(m)} \) contributes to the loss. For convenience, the loss is characterized by an absorption coefficient defined as \( \alpha = \alpha_0 + \Delta \alpha(I) \), where \( \alpha_0 = \alpha_0 \chi_i^{(1)} / 2cn_0 \) is the linear absorption coefficient, and \( \Delta \alpha(I) \) the nonlinear one. If we only consider the third-order nonlinear effect, the nonlinear absorption coefficient is specified into \( \Delta \alpha(I) = -6 \omega_0 \chi_i^{(3)} I_3 + \chi_i^{(3)} I_3 \) for \( c = \chi_i^{(3)} \).

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\[ \tilde{a}_2 = a_0 \cosh(gz) + e^{2\phi}b_0^\dagger \sinh(gz) = \sqrt{Q} a_0 + e^{2\phi} \sqrt{Q - 1} b_0^\dagger, \quad (11a) \]

\[ \tilde{b}_2 = e^{2\phi} a_0^\dagger \sinh(gz) + b_0 \cosh(gz) = e^{2\phi} a_0^\dagger \sqrt{Q - 1} + b_0 \sqrt{Q}, \quad (11b) \]

where \( Q = \cosh^2(gz) \) is the gain coefficient; \( \phi = \phi_{\text{pump}} - \phi_E - \phi_{\text{AS}} \) is the relative phase. For a coherent injected FWM field presented by coherent state \( |\alpha\rangle \), the total number of photons at the outputs of the channels \( E_{F1} \) and \( E_{AS} \) are

\[ \langle \tilde{N}_E \rangle = Q |\alpha|^2 + (Q - 1), \quad (12a) \]

\[ \langle \tilde{N}_{\text{AS}} \rangle = (Q - 1) |\alpha|^2 + (Q - 1). \quad (12b) \]

It is obvious that such a system acts as a quantum amplifier, in which the injected FWM beam is amplified about \( Q \) times in the quantum amplification, while the anti-Stokes field is amplified by \( Q - 1 \) times. Besides the intensity amplification, the spatial modes of the generated Stokes field imitate the ones of the injected field, and the spatial modes of Stokes and anti-Stokes fields have quantum correlation. Therefore, we can get the quantum amplified FWM azimuth in an appropriate condition.

3. Experimental results and discussions

First, we investigate the dependence of the spatial splitting of the FWM signal on the power of the dressing field. Figure 2(a) shows the images of FWM signal \( E_{F1} \) at different powers \( P_2 \) of the dressing field \( E_2 \) in a V-type three level system in self-focusing region. With \( P_2 \) increases from left to right, we observe that the image of \( E_{F1} \) changes from a strip split in the \( y \)-direction, to vortex pattern and then to a strip split in the \( x \)-direction. At the same time, the intensity of \( E_{F1} \) decreases. Here, the beam splitting of \( E_{F1} \) is due to the XPM process. For a fundamental beam that is not vortical, the nonlinear phase shift is simplified as

\[ \phi_n = 2k_0 n I e^{-\alpha y_{\text{in}}^2} z / (n_0 L), \]

and the additional transverse wave vector as

\[ \delta k_4 = -4\pi k n I I e^{-\alpha y_{\text{in}}^2} z / (n_0 L^2). \]

In the self-focusing (self-defocusing) region, the Kerr coefficient \( n_2^2 > 0 (n_2^2 < 0) \), so the beam will be attracted towards (repulsed from) the dressing field according to the expression of \( \delta k_4 \). In our experiment \( n_2^2 < 0 \), the FWM beam is repulsed by the strong dressing field \( E_2 \). When \( E_2 \) is weak, for the \( E_{F1} \) signal, the repulsion of \( E_2 \) can be ignored and its overlapping with strong field \( E_1 \) in the \( y \)-direction leads to the splitting in the \( y \) direction (as seen in figure 1(c)). With \( P_2 \) increasing, the intensity of \( E_{F1} \) is suppressed by the dressing effect of \( E_2 \) when \( \Delta_1 + \Delta_2 = 0 \) is satisfied. With the further increasing \( P_2 \), the repulsion induced by \( E_2 \) increases. Simultaneously, the fields \( E_n, E_1 \) and \( E_3 \) with same frequency interfere and construct a polarization ellipse, creating a vortex intensity pattern with phase singularity, the electrical field of which in the complex plane can be expressed by equation (1) and the initial phase can be specified as.

\[ \text{Figure 3. Images of (a) } E_{F1} \text{ and (b) } E_{F2} \text{ versus the frequency detuning } \Delta_1 \text{ and } \Delta_2 \text{ in a V-type three level atomic system. (c) } E_{F3} \text{ and (d) the probe beam } E_3 \text{ versus the frequency detuning } \Delta_1 \text{ in a two-level atomic system. Lower images are the simulated results.} \]
\[ \delta_1 = -\arctan \frac{E_i n_i k_i \sin \theta_{\phi} + E_i' n_i k_i \sin \theta_{\phi} + E_i n_i k_i \sin \theta_{\phi}}{E_i n_i k_i \sin \theta_{\phi} + E_i' n_i k_i \sin \theta_{\phi} + E_i n_i k_i \sin \theta_{\phi}} \]
\[ \delta_2 = -\arctan \frac{E_i n_i k_i \cos \theta_{\phi} + E_i' n_i k_i \cos \theta_{\phi} + E_i n_i k_i \cos \theta_{\phi}}{E_i n_i k_i \cos \theta_{\phi} + E_i' n_i k_i \cos \theta_{\phi} + E_i n_i k_i \cos \theta_{\phi}} \]

with \( \theta_{\phi} = \psi_{\phi} + n_i \left( k_i T_i + k_i T_j \right) \).

The equation (14) shows that the nonlinear phase shift reduces as the intensity of \( E_1 \) increases. This indicates that the nonlinear phase shift \( \phi_2 \) decreases as the intensity of \( E_2 \) increases. This is equivalent to the change in the population of the azimuthal component of the vortex beam, which is modulated by the interference pattern of vortex fields. The rotation angular velocity of vortex increases with \( p \) increasing. In a word, we can experimentally control the successive transformation of spiral vortex beams into azimuthal vortex beams.

Figure 4. The rotating of vortex light (a) \( E_{F1} \), (b) \( E_{F2} \) and (c) \( E_{F3} \) versus the temperature. Higher images are the spots measured in experiment. Lower images are the simulated results. (d) Schematic of a 3D multiport circulator.

According to the spatial configuration of laser beams, the field \( E_1 \) mainly induces splitting in the \( x \)-direction, while \( E_2 \) mainly induces splitting in the \( y \)-direction. So the probe field \( E_T \) splits to four spots. Moreover, with the intensity of \( E_T \) increasing, the splitting depth of the vortex decreases. As mentioned above, the splitting of the vortex is attributed to the anisotropic phase gradient. The equation (14) shows that the nonlinear phase shift \( \phi_2 \) decreases as the intensity of \( E_1 \) increases. This indicates that the nonlinear phase shift \( \phi_2 \) decreases as the intensity of \( E_1 \) increases. This is equivalent to the change in the population of the azimuthal component of the vortex beam, which is modulated by the interference pattern of vortex fields. The rotation angular velocity of vortex increases with \( p \) increasing. In a word, we can experimentally control the successive transformation of spiral vortex beams into azimuthal vortex beams.

Next, we investigate the influence of frequency detunings on the pattern formation of the FWM signal and the probe beam. Figures 3(a)–(b) give the degenerated FWM signals and the probe beam. In the experiments, we set laser beams at the proper powers and scan the frequency detuning \( \Delta_1 \) and \( \Delta_2 \), respectively. In the self-defocusing region (\( \Delta_2 > 0 \)), \( E_{F1} \) can form azimuthal vortices because of the interference among three generation fields with the same frequency and the XPM of the strong dressing field \( E_1 \). The pattern development in the \( x \)-direction due to the strong repulsion of \( E_2 \) is attributed to the anisotropic phase gradient. The equation (14) shows that the nonlinear phase shift reduces as the intensity of \( E_1 \) increases. This indicates that the nonlinear phase shift \( \phi_2 \) decreases as the intensity of \( E_1 \) increases. This is equivalent to the change in the population of the azimuthal component of the vortex beam, which is modulated by the interference pattern of vortex fields. The rotation angular velocity of vortex increases with \( p \) increasing. In a word, we can experimentally control the successive transformation of spiral vortex beams into azimuthal vortex beams.

Figure 5. The frequency dependence on the pattern formation of (a) \( E_{F1} \), (b) \( E_{F2} \) and (c) \( E_{F3} \) versus the temperature. Higher images are the spots measured in experiment. Lower images are the simulated results. (d) Schematic of a 3D multiport circulator.
\( \phi \) and the modulation depth. The azimuthon can form when \( n_x^2 \) is relatively large and decays into single point when \( n_x^2 \) approaches zero. In the self-focusing region, similar to \( E_{F_1} \) and \( E_{F_2} \), transforms into a dipole-type pattern. Figure 3(d) shows the frequency dependence of the probe beam \( E_s \). We can see that the four-spot azimuthon of \( E_3 \) appears both in the self-defocusing and self-focusing regions. It is attributed to the co-modulation of two strong dressing fields \( E_1 \) and \( E_2 \) which have the same distance from \( E_p \). The result of co-modulation could counterbalance the self-focusing effect. These phenomena give the chance to practice the frequency modulation and demodulation.

Now we focus on the medium temperature dependence of the cross-phase modulated azimuthon at \( \Delta_1 = 15 \text{ GHz} \) and \( \Delta_2 = -15 \text{ GHz} \). The temperature is changed from 240°C to 280°C, which gives the atomic density \( N_0 \) an increasing multiplicity, and leads to the increase of the effective propagation distance of the beams. The calculated increment in propagation distance is about \( z = aL_h \approx 22 \text{ cm} \), where \( L_h \) is the half-length of the heat pipe oven. Figure 4(a) shows the images of \( E_{F_1} \) at different temperatures. With the temperature (effective propagation distance) increasing, \( E_{F_1} \) circumvolves clockwise (\( m = 1 \)). The propagation distance is 32 times longer than the diffraction length \( L_d = k_iw^2 = 0.67 \text{ cm} \) of \( E_{F_1} \). So we can conclude that \( E_{F_1} \) forms a modulated vortex soliton. However, for non-degenerated FWM signal \( E_{F_1} \) (figure 4(b)), it can only form a vortex in certain temperatures (around 260°C), and presents single spot pattern at both low and high temperatures. Such a phenomenon can be explained by the fact that the frequencies of \( E_1, E_2, \) and \( E_3 \) are different, and therefore the coherence among them is weaker than that of degenerated signal. According to the expression of nonlinear phase

\[
\phi = 2kn_x^2 I e^{-\left(e^{2n_x^2 - 2\pi n_x^2(z/2r)} - e^{z/2r}\right)/n_0} \frac{z}{n_0}
\]

\( \phi \) is determined by the intensities of beams and propagation distance \( z \). Higher temperature, equivalent to longer propagation distance \( z \), leads to larger nonlinear phase \( \phi \). However, as the temperature increases even higher, greater absorption will come, which results in the decrease of beam intensity and therefore the decrease of \( \phi \). At ideal temperature, \( E_{F_0} \) forms a vortex with counterclockwise rotation, as shown in figure 4(b). Figure 4(c) gives the images of \( E_3 \) from 240°C to 280°C, which presents stationary non-rotating (the angular velocity \( \Omega = 0 \)) four-spot azimuthon patterns in this range. In our experiment, the angular velocity of a modulated azimuthon is determined by three factors: the topological charge, the nonlinear phase shift and the number of intensity peaks. At certain frequency detuning and intensity values, the counterbalance of these three factors could be obtained. So the rotation of a modulated azimuthon is restrained.

4. All-optical circulator and cross-connect with the controllable azimuthons

Finally, we discuss the potential applications of the cross-phase modulated azimuthon in all-optical networks. All-optical networks are expected to dominate the communications systems in the future due to their high speed and ability to overcome the ‘electronic bottleneck’ in today’s electronic or electro-optic networks. Optical cross-connect is a key function in most communications systems including spatial optical switching, routing, multiplexers and demultiplexers. The cross-phase modulated azimuthon can be used to obtain ideal optical cross-connects and all-optical circulators.

As discussed above, the rotation angular velocity and orientation of the azimuthon can be controlled by the dressing fields and the interference of three generation waves. Such kinds of phenomena can be used to develop a three-dimensional (3D) multiport circulator. Figure 4(d) indicates the schematic of a 3D circulator. When choosing proper interference phase and powers of dressing fields, we get the clockwise rotating azimuthon \( E_{F_1} \) with three ports. These three ports rotate in a controlled angular velocity \( \Omega \) around the singularity. After a certain path length \( l \), the signal input at port 1 will be output at port 2, port 2 output at port 3, and port 3 output at port 1. The function of loop optical transmission which is useful for circular chromatic dispersion compensation is achieved. If we increase the path length to \( 2l \), the signal input at port 1 will be output at port 3. Therefore, it is possible to route a signal from any input port to any output port by properly setting the path length and angular velocity.

We introduce two parameters to measure the circulator. One is the circulation velocity, which in our experiment is decided by the rotating angular velocity \( \Omega \) of the azimuthon. Larger circulation velocity means higher response speed and short response time of the circulator. When the angular velocity of the azimuthon is set at zero, the signal exchange between two ports stops. This means that the control of the rotation ‘on’ and ‘off’ states of the circulator can be demonstrated. The other parameter is the cycle efficiency, which can be defined as

\[
T = \frac{l_{on}}{l_{off}}
\]

in our experiment. The larger the cycle efficiency is, the smaller the loss of the circulator. Such a device avoids multiple reflections between components, so the loss of energy is dominated by the absorption of the medium, and the cycle...
efficiency is $T = \exp(-al)$. As mentioned above, if we only consider linear and third-order nonlinear cross-Kerr effects, the absorption coefficient is $\alpha = \frac{a_0 \chi^{(1)}}{2nc_0} - \frac{6a_0 \chi^{(3)} I_C}{\varepsilon_0 c^2 n_0^2}$ which can be controlled by the intensity of the dressing field $I_C$. At large $I_C$, the medium will show as nearly transparent because a very low loss of energy can be achieved. Moreover, if the frequency detuning is set at a large value and the pump power is above a critical level [32], the FWM injected into the Stokes channel can also be strongly amplified. So we can observe the quantum amplified modulated vortices under such conditions. Figure 5 gives the images of the outputs in Stokes and anti-Stokes channels when FWM signal $E_{F1}$ is seeded into the Stokes channel. According to equation (12), the output intensity of the Stokes channel is $I_2 = h \omega_1 (\tilde{N}_F) / \pi a^2 \approx Q I_{F1}$. Here, the gain coefficient $Q = \cosh^2(gz)$ can be controlled by the frequency detuning $\Delta_1$ due to the dependence of $g$ on $\Delta_1$. So we can get a quantum amplified vortex in a certain range of $\Delta_1$. At the same time, an intensity of $I_{\omega_1} \approx (Q-1) I_{F1}$ can be measured in the anti-Stokes channel. In figure 5, with $\Delta_1$ at the large detuning region, vortex patterns are observed in the anti-Stokes channel. This is because the generated Stokes signal inherits the spatial vortex pattern of the injected $E_{F1}$, and there exists quantum correlation between the spatial modes of the Stokes and anti-Stokes signals. Such spatial quantum correlation can be used in ghost imaging and logic operations [35]. With $\Delta_1$ near the resonant point, the quantum correlation is destroyed by the strong absorption and spontaneous emission, so the vortex pattern of the anti-Stokes beam disappears. On the other hand, this system also gives a way to achieve the integration of an optical amplifier in a 3D circulator.

Figure 6(a)–(c) is the experimental result of the switched spatial patterns of the FWM signal $E_{F1}$ and probe field $E_2$, when the frequency detuning $\Delta_1$ in the two-level atomic system is repeatedly switched. As shown in figure 6(a), when $\Delta_1$ is repeatedly switched between 0 and 15GHz, the FWM signal $E_{F1}$ repeatedly transforms between the single-spot state to the three-spot state. Therefore, when the beam $E_{F1}$ carries information, the input information can be distributed into three channels. Such a phenomenon can be used as a spatial demultiplexer and router. Figure 6(b)–(c) shows the switched splitting of the probe beam $E_1$ versus switched $\Delta_1$ with and without dressing field, respectively. When the dressing field $E_1$ is on, the beam $E_1$ splits into four spots in two perpendicular directions, as shown in figure 6(b), which means the spatial demultiplexer with $2 \times 2$ channels is obtained. As the dressing field $E_1$ is off, the nonlinear phase shift $\phi_2$ decreases. $E_1$ shows three spots around the ring of vortex, as shown in figure 6(c). Therefore, we can easily control the routing channels by the nonlinear phase shift of the dressing field. Moreover, contrast index is an important parameter for the spatial demultiplexer. It can be defined as $\eta = \frac{(I_{\max} - I_{\min})}{(I_{\max} + I_{\min})}$, where $I_{\max}$ is the intensity of split spots, and $I_{\min}$ is the intensity of gap between split spots. In our experiment, contrast index $\eta$ is related to the modulation parameter $p$ which can be controlled by the power of signal or the dressing field. As seen in figure 6(d), the power of the beam $E_1$ is switched among $25\mu W$, $15\mu W$, and $5\mu W$, and as a result, the contrast index $\eta$ is switched from 0 to 50% and 80%. The larger the contrast index $\eta$, the better the demultiplexing effect, which means that one channel is less affected by the others, so the information can be transmitted with greater accuracy.

Now consider we investigate the co-propagation of two FWM signals $E_{F1}$ (with frequency $\omega_1$) and $E_{F2}$ ($\omega_2$) as seen in figure 6(e). Firstly, two beams are input at port 1 with single spot state (at resonance). When we set the frequency detuning $\Delta_1$ of $E_{F1}$ at a proper value of the self-defocusing region and the frequency detuning of $E_{F2}$ keeps resonance ($\Delta_2 = 0$), the beam $E_{F1}$ splits into three spots while $E_{F2}$ keeps single-spot state. So the signal with frequency $\omega_2$ will be connected straight to output port 2, while the signal $E_{F1}$ is routed to three output ports: 3, 4, and 5. Therefore, the wavelength-selective cross-connect and routing is achieved.

5. Conclusion
In conclusion, we have demonstrated the control of rotation, splitting and output intensity of FWM azimuthons both in theory and experiment. With the frequency or intensity of the signal and dressing field modulated, the modulation of the spot number and splitting depth of FWM azimuthons has been demonstrated. Such a mechanism can act as a frequency or amplitude modulator (demodulator). With introducing an additional phase gradient in the azimuthal direction by the...
dressing field, we have achieved the control of the rotation angular velocity of the azimuthon, in which a stationary non-rotating azimuthon has been observed in experiment. On the other hand, the output intensity of the FWM signal is amplified through quantum parametric amplification, and the gain coefficient can be modulated by frequency detuning. Furthermore, the quantum correlated FWM vortexes are observed in experiment. Such quantum amplified FWM azimuthons can be used for the demonstration of ideal optical amplifiers and all-optical cross-connects, including circulators, multiplexers (demultiplexers) and routing.

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References