Selective reflection of Airy beam at an interface between dielectric and homogeneous atomic medium

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Abstract: We examine selective reflection (SR) spectrum of an Airy beam at an interface between a dielectric and a homogeneous atomic medium. It is shown that both the general reflection (GR) and the SR of Airy beams exhibit accelerating dynamics with a parabolic trajectory, however, the accelerating rate for the SR is slightly greater than that for the GR. Due to interaction of atoms and the Airy beams at the interface between dielectric and resonant atoms, the SR beams can create far-field interference patterns. We also show that the amplitude of the SR can be dramatically modified by the detuning of the incident fields, the reduced $x$-coordinate and the distance from the interface. The SR at the resonant medium interface of Airy beams can probably be a powerful tool in the use of optical power delivering, resonant particle manipulation and spatial spectrum detection of a resonant medium at an interface.

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OCIS codes: (140.3295) Laser beam characterization; (140.3300) Laser beam shaping; (020.1670) Coherent optical effects.

References and links


1. Introduction

An Airy wavepacket was initially predicted within the context of quantum mechanics [1]. It has two typical properties: resistance to diffraction and accelerating dynamics along the transverse direction during the propagation [2,3]. These remarkable features of ideal Airy beams can be implemented in practice by finite power Airy beams maintaining their shape over several diffraction lengths before they are significantly distorted [2,3]. Another feature of the Airy beams is their self-healing when perturbations are imposed on them [4]. This self-healing characteristic of the Airy beam was also observed in the turbulence independence of its centroid position and skewness [5]. It was also shown that an Airy beam can maintain its shape when propagating in a self-defocusing nonlinear medium [6]. These exotic features of Airy beams may have many applications, e.g., controlling of beam trajectory [7], particle manipulation [8], plasma channel generation [9] and near-field imaging of Airy surface plasmons [10]. To describe an Airy beam, a virtual source for generating an Airy wave was identified [11], in which the authors derived a spectral integral expression yielding a freely accelerating, non-diffracting, as well as a finite-energy Airy beam in the paraxial limit.

Self-healing and accelerating dynamics can be observed in reflection and refraction beams from a finite power incident Airy beam at the interface between two dielectric media [12]. In a four-level electromagnetic induced transparency (EIT) atomic vapor system, it is shown that the deflection position and the intensity of the Airy beam can be modulated by the Rabi frequency of the control light. Such a tunable optical behavior may have some potential applications in medicine science [13].

SR spectroscopy is known as an excellent tool to detect the resonant properties of the medium in the vicinity of the interface [14–20]. It is notably beneficial to get high-resolution spectroscopy in order to study atom-surface interaction [14,15,17], and a dense gas atomic medium [16,19,20], sensitively. In this paper, our discussion is devoted to the phenomena associated with the SR at an interface between a dielectric and a homogeneous atomic medium. Both the GR and the SR of Airy beams exhibit accelerating dynamics with a parabolic trajectory. However, the accelerating rate of the SR is slightly greater than that of the GR, and the SR beam can create far-field interference patterns. We also show that the
amplitude of the SR can be dramatically controlled by the detuning of the incident fields, the reduced $x$-coordinate and the distance from the interface. The selective reflected Airy beam can probably be a powerful tool in the use of optical power delivering, resonant particle manipulation and spatial spectrum detection of a resonant medium at an interface.

2. Theoretical model

We consider a two-dimensional system with a homogeneous cold atomic medium 2 sandwiched between two dielectric windows 1 and 1’ [Fig. 1(a)], situated respectively in the regions of $z<0$ and $z>L$. We assume that the inner surface of the window 1’ is antireflection coated, i.e., the Fabry-Perot effect can be neglected in our discussion. The employed lambda-type three-level atomic system in medium 2 is presented in Fig. 1(b), where the transition frequency of $|1\rangle-|3\rangle$ is $\omega_{h_1}$, and that for $|2\rangle-|3\rangle$ is $\omega_{h_2}$. Two laser fields are applied to the system, in which the probe field $E_p$ has frequency $\omega_p$ (with a detuning $\Delta_p = \omega_p - \omega_{h_1}$, impinging on the interface 1-2 at an angle $\theta_p$), and coupling field $E_c$ has frequency $\omega_c$ (with a detuning $\Delta_c = \omega_c - \omega_{h_2}$, impinging on the interface 1-2 along the normal direction). We denote $k_p$ ($k_{p'})$ and $k_c$ ($k_c$) the wave vectors (wavenumbers) of the probe and the coupling fields, respectively.

![Fig. 1.](image)

(a) The geometrical configuration of probe and coupling fields impinging on the interface between a transparent dielectric window and a homogeneous sample of cold atoms. The diverse coordinate systems are also shown. (b) Lambda-type three-level system of atoms.

The field amplitude of a paraxial probe beam with an incident angle $\theta_p$ at the interface can be expressed as

$$E_p(x,z_i = 0) = u_0(x_i) \exp(ik_i \sin \theta_p x_i),$$

where $k_i = n_i k_p$ is the probe wavenumber in medium 1 with dielectric constant $\varepsilon_1$ and refractive index $n_i = \sqrt{\varepsilon_1}$. The coordinate system yields $(x_i,z_i) = (x + s,z + h)$, with $s = h \tan \theta_p$ and $z = -h$ being the depth inside window 1. The transverse input field amplitude for a finite-power Airy beam is $[2,12]$

$$u_0(x_i) = Ai(\pm \frac{x_i \cos \theta_p}{x_0}) \exp(\pm a \frac{x_i \cos \theta_p}{x_0}),$$

where $a$ is the truncation and $x_0$ an arbitrary transverse scale.

Applying a Fourier transformation in coordinates $(x,z)$ to the probe field $E_p$, we can decompose it into many plane wave components and the GR, SR and transmission coefficients of each component are denoted as $R_{g'}(k)$, $R_{s'}(k)$ and $T(k)$. Here, the SR field represents the radiated waves propagating along the reflective direction from the macroscopic
polarization of sandwiched atoms. By applying boundary conditions at the interface, \( R_{p}(k) \), \( R_{p}(k) \), and \( T(k) \) can be expressed by Eqs. (3a), (3b) and (3c) respectively

\[
R_{p}(k) = \frac{(q_{1} - q_{2})}{(q_{1} + q_{2})}, \quad (3a)
\]

\[
R_{p}(k) = i \left[ E_{\text{pin}}^{(k)}(q_{1} + q_{2}) \right] \frac{\partial E_{p}^{(k)}(z)}{\partial z}, \quad (3b)
\]

\[
T(k) = 2q_{1} \left/ (q_{1} + q_{2}) \right., \quad (3c)
\]

where we assume that the probe field is s-polarized, \( q_{1} = \sqrt{k_{1}^{2} - k^{2}} \) is the z wavenumber of the Fourier component in the medium 1, and \( q_{2} = \sqrt{k_{2}^{2} - k^{2}} \) the wavenumber in medium 2 as a refractive index \( n_{z} = 1 \) of a dilute atomic sample is assumed. The special coefficient \( E_{p}^{(k)}(z) \) of each Fourier component of the probe wave in the medium 2 is given by

\[
E_{p}^{(k)}(z) = \int_{-\infty}^{\infty} E_{p}(x, z) \exp(-ikx) dx, \quad (3d)
\]

where \( E_{p}(x, z) \) can be written as

\[
E_{p}(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} E_{p}(x, z = -h) \exp(-ikx) dx \right] T(k) \exp(ikx) \exp(iq_{z}z + iq_{h}h) dk. \quad (3e)
\]

The incident special coefficient \( E_{\text{pin}}^{(k)} \) at the interface in medium 1 can be obtained in a similar way, and we do not show it here.

We assume that each Fourier component of the probe field through atomic medium satisfies the wave equation

\[
\frac{\partial^{2} E_{p}^{(k)}(z)}{\partial z^{2}} + 2iq_{z} \frac{\partial E_{p}^{(k)}(z)}{\partial z} = -4\pi q_{z}^{2} P^{(k)}(z). \quad (4)
\]

The second interface \( 2-1' \) in our system is antireflection coated; hence, it is justified to assume that \( \left[ \frac{\partial E_{p}^{(k)}(z)}{\partial z} \right]_{z=0} = 0 \), resulting in the expression

\[
\left[ \frac{\partial E_{p}^{(k)}(z)}{\partial z} \right]_{z=0} = 4\pi q_{z}^{2} \int_{0}^{L} P^{(k)}(z') \exp(2iq_{z}z') dz'. \quad (5)
\]

In the above equations, the polarization \( P^{(k)}(z) = N \mu_{31} \sigma_{31}^{(k)}(z), \) with \( N \) being the number density of atoms, \( \mu_{31} \) the transition dipole moment corresponding to \( |1\rangle \to |3\rangle \). The density matrix element \( \sigma_{31} \) can be obtained by solving the following coupled equations

\[
\frac{\partial \sigma_{31}}{\partial t} = -\Lambda_{31} \sigma_{31} + iG_{p}^{(k)}(\sigma_{11} - \sigma_{33}) + iG_{c} \sigma_{21}, \quad (6a)
\]

\[
\frac{\partial \sigma_{21}}{\partial t} = -\Lambda_{21} \sigma_{21} - iG_{p}^{(k)} \sigma_{23} + iG_{c} \sigma_{31}, \quad (6b)
\]

\[
\frac{\partial \sigma_{32}}{\partial t} = -\Lambda_{32} \sigma_{32} - iG_{p}^{(k)} \sigma_{23} - iG_{c} (\sigma_{22} - \sigma_{33}), \quad (6c)
\]

where \( \Lambda_{21} = \gamma_{21} - i(\Delta_{p} - \Delta_{c}) \), \( \Lambda_{31} = \gamma_{31} - i\Delta_{p} \) and \( \Lambda_{32} = \gamma_{32} - i\Delta_{c} \), with \( \gamma_{31} \) and \( \gamma_{32} \) being the decay rates from level \( |3\rangle \) to levels \( |1\rangle \) and \( |2\rangle \), respectively; \( G_{p}^{(k)} \) and \( G_{c} \) are the Rabi frequencies defined as \( G_{p}^{(k)} = \mu_{31} E_{p}^{(k)}(z) / h = \mu_{31} E_{\text{pin}}^{(k)} / h \) and \( G_{c} = \mu_{32} E_{c} / h \), respectively, with \( \mu_{32} \) being the dipole moments of the transition \( |2\rangle \to |3\rangle \). It is assumed that the transition...
between levels $|1\rangle$ and $|2\rangle$ is dipole forbidden, as is the case for hyperfine sublevels of an atomic ground state. The nonradiative decay rate (including transient effects and collisional dephasing) between these two lower levels is denoted by $\gamma_{11}$.

We assume that the probe field is much weaker than the coupling field, so $G_p^{(k)}$ in Eq. (6b) can be neglected, and the initial conditions are $\sigma_{11} = 1$, $\sigma_{22} = 0$. Solving Eqs. (6a) and (6b), we get $\sigma_{31} = iG_p^{(k)}(\Lambda_{21}/(\Lambda_{21}\Lambda_{31} + G_c^2))$, and the polarization reads

$$p^{(1)}(z) = iN\mu_0 G_p^{(k)}(\Lambda_{21}/(\Lambda_{21}\Lambda_{31} + G_c^2)). \tag{7}$$

For simplicity the superscript $(k)$ in each density matrix element is omitted, since it does not change the main result in our discussion.

Substituting (7) into (5) and then into (3b), we finally obtain

$$R_{\sigma}(k) = -\frac{4\pi q_2^2}{E_{\text{pin}}(q_1 + q_c)} \int_0^L N\mu_0 G_p^{(k)}(\Lambda_{21}/(\Lambda_{21}\Lambda_{31} + G_c^2)) \exp(2iu_2z')dz'. \tag{8}$$

The envelope function for the SR beam is given by the Fourier integral [12]

$$u(\chi, z) = \frac{1}{2\pi \cos \theta_1} \int d\kappa F_0(k)(\frac{\kappa}{\cos \theta_1})R_{\sigma}(k, \sin \theta_1 + \kappa) \exp \left[ i\kappa \chi - i\frac{\kappa}{2k_i \cos \theta_1} \right]. \tag{9}$$

Here $\chi = x - z \tan \theta_1$ is the reduced $x$ coordinate at the propagation direction of the reflected beam along $x = z \tan \theta_1$ predicted by Snell’s law. The Fourier transformation $F_0(k')$ is given by $F_0(k') = x_0 \exp[i(\pm k'x_0 + i\alpha)/3]$. For the GR beam, the Eq. (9) holds simply with $R_{\sigma}(k)$ replaced by $R_{\text{gr}}(k)$.

It is apparent that the spatial profile of the selective reflected beam can be modulated by the SR coefficient $R_{\sigma}$, i.e., the atomic level structure, the thickness of atomic sample, as well as the detuning and the strengths of the probe and coupling fields. Here we restrict our discussion to the modulation to the SR beam induced by the probe detuning $\Delta_p$, the reduced $x$-coordinate $\chi = x - z \tan \theta_1$, and the distance $z_r$ from the interface, while keeping a fixed value of the sample thickness, the detuning and strength of the coupling field.

3. Numerical results and discussion

In numerical calculation, atomic parameters are chosen to be corresponding to the $D_1$ line of $^{87}\text{Rb}$, the coupling field $E_c$ couples level $|2\rangle$ ($F = 2, S_{1/2}$) to level $|3\rangle$ ($F = 2, P_{1/2}$), and probe beam $E_p$ couples level $|1\rangle$ ($F = 1, S_{1/2}$) to level $|3\rangle$ ($F = 2, P_{1/2}$). Other selected values are $\gamma_{11} = 2\pi \times 4.79 \text{ MHz}$, $\gamma_{22} = 2\pi \times 2.87 \text{ MHz}$, $\gamma_{31} = 2\pi \times 10 \text{ kHz}$, $\omega_s = \omega_1 = 2\pi \times 377.11 \text{ THz}$ and $G_c = \gamma_{31}$. For a cold atomic sample, we assume that $L = 2 \text{ mm}$ and $N = 6 \times 10^{15} \text{ cm}^{-3}$. In all cases, the refractive index of the medium 1 is assumed to be $n_1 = 1.5$; the coupling field is accurately tuned to the transition $|2\rangle - |3\rangle$, i.e., $\Delta_c = 0$. For an s-polarized incident probe Airy beam with incident angle $\theta_1 = 10^\circ$, $x_0 = 20 \mu m$, $a = 0.1$, and lobes developing toward negative $x$, i.e., $F_0(k') = x_0 \exp[i(k'x_0 + ia)/3]$, the diffraction length in medium 1 is $L_{d1} = k_1 x_0 = 4.7 \text{ mm}$. We let $h = L_{d1} \cos \theta_1 = 4.1 \text{ mm}$, and therefore the beam can propagate for one diffraction length before it hits the interface.
First, the SR contribution with symmetrically varying $\Delta_p$ is shown in Fig. 2. It is obvious that a sharp dispersive change emerges in the interval $[-\gamma_{31}, \gamma_{31}]$, at the edges of which the SR is significantly enhanced and a maximum value can be achieved. However, since the transmission is largely enhanced in this interval, the SR is weakened. When the probe beam is far detuned from the resonant frequency, the SR shows a decreasing contribution.

Next, we show the propagation of the GR and the SR beam in Figs. 3(a1) and 3(b1), respectively, up to a distance of 4cm from the interface. The intensity profiles corresponding to Figs. 3(a1) and 3(b1) are respectively plotted in Figs. 3(a2) and 3(b2) at various distances in medium 1 from the interface. Here, the probe detuning is $\Delta_p = \gamma_{31}$ and the amplitude is normalized by the maximum value of the first lobe at $z_r = 0$. 

Fig. 2. The real part contribution of the SR versus the probe detuning $\Delta_p/\gamma_{31}$ for $G_i = 0.3\gamma_{31}$ and $G_c = \gamma_{31}$.
It is obvious that both the GR and the SR are quasi-diffraction-free Airy-like beams, which exhibit accelerating dynamics toward the same direction with the incident beam along the Snell’s law axis. The FWHM of the first lobe for the GR and the SR are \( \sim 34 \mu m \). This feature for the former case can remain almost invariant up to \( \sim 2.8 \) cm, and for the latter case up to \( \sim 1.6 \) cm. This evidently indicates that the quasi-diffraction-free character of the Airy beam can be maintained in the GR and the SR, but the disturbance-resisting distance for the SR beam is smaller than that for the GR beam due to the contribution from resonant atoms. Although the parabolic trajectories of the SR beams are similar to the case of the GR beams, the accelerating rate in the former case is slightly greater than that in the latter case. Due to the spatial periodical modulation of the radiated waves from the resonant atoms along the reflection direction, a quasi-diffraction-free character periodically remains, and a periodic far field interference pattern also emerges in the resonant case.

The evolution of the SR amplitudes versus the position \( \chi_r \) from the Snell’s reflection axis for the Airy beam is plotted in Figs. 4(a)-4(f) for \( z = 0, 1 \) cm, 2cm, 3cm, 4cm and 5cm respectively. The red, blue and green lines correspond to \( \Delta_p = 0, \gamma_{31}, \) and \( 3\gamma_{31} \), respectively.
Fig. 4. SR amplitudes versus the position $\chi$ of the Airy beam for various probe detuning and propagating depth in medium 1. (a), (b), (c), (d), (e) and (f) are for $z_r = 0$, 1cm, 2cm, 3cm, 4cm and 5cm respectively. Red, blue and green lines are for $\Delta_p = 0$, $\gamma$, and $3\gamma$ respectively. Other parameters are $G = \gamma$, $\Delta_c = 0$.

Figure 4 shows that the amplitude of the SR can be dramatically modified by the detuning of the incident fields, the reduced $x$-coordinate and the distance from the interface. As the probe field is exactly tuned to the transition $|1\rangle - |3\rangle$, the EIT effects are manifest due to the ac-Stark splitting and the interference between dressed states created by the coupling laser [21–25]. The absorption of the probe field is largely suppressed, thus the SR from the resonant atoms is very weak (red lines). As the probe detuning is detuned to $\gamma$, the dressed state resonance occurs, the absorption or resonant radiation approaches a maximum value. In this case, the SR approaches maximum value, and the most significant variation of spatial profile also emerges in the propagation (blue lines). When probe field is further detuned to $3\gamma$, the SR turns weak because the dressed state resonance is significantly lost (green lines). It can also be observed that the SR beams can create far-field interference patterns. This is due to the periodic spatial radiation behavior from the resonant atoms resulting from the similar modulation behavior of the incident Airy beam. This type spectrum of the beam can have potential application in the detection of resonant properties of particles, as well as the manipulation of the resonant particles.
4. Conclusion

We have investigated the GR and the SR properties of an Airy beam at an interface between a
dielectric and a homogeneous atomic medium. Both the GR and the SR of Airy beams are
Airy-like and exhibit accelerating dynamics toward the same direction with the incident
beam. Although the parabolic trajectories of the SR beams are similar to the case of the GR
beams, the accelerating rate of the former is slightly greater than that of the latter. Due to the
contribution of resonant atoms, the SR beams can create far-field interference patterns. It is
also observed that the amplitude of the SR can be dramatically modified by the detuning of
the incident fields, the reduced \( x \)-coordinate and the distance from the interface. The SR of
Airy beams can probably be a powerful tool in the use of optical power delivering, resonant
particle manipulation and spatial spectrum detection of resonant medium at an interface.

Acknowledgments

This work is supported by the Science and Technology Project of Xi’an in China (No.
CXY1134WL02, CX12189WL02).