Surface waves at the interface between a metal and a photovoltaic-photorefractive crystal

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A B S T R A C T
We investigate surface waves at the interface between a metal and a photovoltaic-photorefractive (PP) crystal. These surface waves appear in several forms: delocalized surface waves, shock surface waves, and localized surface waves. Only localized surface waves have limited energy. We demonstrate that the transverse sizes of localized surface waves decrease with an increase in the propagation constant and the amplitudes of localized surface waves increase with the propagation constant. The stability of localized surface waves is investigated numerically and it is found that they are stable.

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1. Introduction
The richness of the nonlinear-optical effects in photorefractive media has given rise to a great deal of new soliton phenomena in these materials. To date, many branches of solitons, such as screening solitons [1,2], photovoltaic solitons [3–5], screening-photovoltaic solitons [6–9], photorefractive polymeric solitons [10,11], and solitons in centrosymmetric photorefractive crystals [12], have already been discovered, of which studies used bulk photorefractive crystals. On the other hand, the presence of interfaces between linear and photorefractive materials strongly affects the propagation of optical beams. Such interfaces can support surface waves localized at the very interface [13–22]. The unique features of such surface waves have no analogues in homogeneous media. Delocalized photorefractive surface waves with the long slowly decaying oscillating tails going away into the volume of the photorefractive crystal have been predicted [17,18] and observed [19]. Of particular interest from a practical point of view are localized photorefractive surface waves, which have no oscillating tails in the volume of the photorefractive crystal. Such localized surface waves at the interface between linear media (dielectric or metal) and biased non-photovoltaic-photorefractive (non-PP) crystals [20,21] or unbiased non-PP crystals [22] have been predicted. However, it is not clear whether localized surface waves between a metal and a PP crystal are possible.

In this paper, we analyze surface waves at the interface between a metal and a PP crystal. We show that these surface waves appear in several forms: delocalized surface waves, shock surface waves, and localized surface waves. Delocalized surface waves and shock surface waves have infinite energy, whereas localized surface waves have limited energy. We demonstrate that the transverse sizes of localized surface waves decreases with an increase in the propagation constant and the amplitudes of localized surface waves increases with the propagation constant and that localized surface waves are stable. Relevant examples are provided where the PP crystal is assumed to be BaTiO3.

2. Theoretical model and results
To start, let us first consider the propagation of an optical beam along the z-axis near the interface between a metal occupying the half-space \( x \geq 0 \) and a PP crystal occupying the half-space \( x < 0 \). For demonstration purposes, let the PP crystal be BaTiO3 with its optical c axis oriented along the x-axis. Moreover, let us assume that the beam is linearly polarized along the x-axis and that the skin layer of the metal, which is of the order of the wavelength, is zero. Under these conditions, the perturbed extraordinary refrac-
tive index $n_c^2$ (along the c-axis) is given by $(n_c^2)^2 = n_e^2 - n_r^2 r_{33} E_{sc}$, where $n_e$ is the unperturbed extraordinary refractive index of the PP crystal, $r_{33}$ is the electro-optic coefficient, and $E_{sc}$ is the induced space-charge field. In typical PP media and for relatively broad beam configurations, the space-charge field can be obtained from the Kukhtarev–Vinetski transport model and it is given by [5]

$$E_{sc} = -E_p \frac{l}{I + l_d} + \frac{K_B T}{e(l/l_d + 1)} \frac{\partial (l/l_d)}{\partial x},$$

(1)

where $l = l(x,z)$ is the intensity of the light beam, $K_B$ is the Boltzmann constant, $T$ is the absolute temperature, $e$ is the electron charge, $l_d$ is the dark irradiance of the crystal, and $E_p$ is the photovoltaic field constant.

In turn, the propagation equation of the optical beam can be obtained by substituting the expression for the perturbed refractive index (induced by the space-charge field) into the paraxial wave equation. After appropriate normalization, the complex amplitude $A$ of the light field is then found to obey the following dynamical evolution equation:

$$i \frac{\partial A}{\partial s} = -\frac{1}{2} \frac{\partial^2 A}{\partial s^2} - \alpha l \frac{1}{I + l_d} A + \frac{\mu}{(l/l_d + 1)} \frac{\partial (l/l_d)}{\partial x} A \quad \text{for} \quad s < 0,$$

$$A = 0 \quad \text{for} \quad s \geq 0.$$  

(2)

Here the parameters $\alpha = \frac{1}{2} r_{33} n_e^2 E_p / l_d$ and $\mu = \frac{K_B T}{e l_d}$ are the relative contribution of the photovoltaic component and the nondiffusion component of the nonlinear response, respectively; $l_d = l(x,z)$ is an arbitrary transverse scale; $k = 2\pi n_e / \lambda$ is the wave number in the area of the PP crystal; $\lambda$ is the free-space wavelength of the lightwave used; and $s = x/k_0$ is the normalized transverse coordinate.

The first term in the right-hand side of Eq. (2) describes the diffraction spreading of the beam; the second term describes the beam self-focusing caused by the photovoltaic effect of the photorefractive nonlinearity; and the last term describes the beam self-bending caused by the diffusion component of the nonlinear response of the PP crystal. The stationary surface waves can be formed at the interface between a metal and a PP crystal owing to compensation of the beam reflection from the interface and its self-bending towards the interface. In this study, the BaTiO$_3$ parameters are taken to be $r_{33} = 80$ pm/V, $n_e = 2.365$, and $E_p = 5$ kV/cm. If we let $\lambda = 500$ nm, $x_0 = 10 \mu$m, and $T = 300$ K, we find that $\alpha = 9.88$ and $\mu = 0.051$.

We look for stationary solutions of the system of Eq. (2) in the form $A = T u(s) \exp(\text{i}kx)$, where the envelope $u(s)$ is the real function and $b$ is the real propagation constant. Furthermore, the initial conditions corresponding to both the continuity of the tangential component of the electric field and the continuity of the normal component of the magnetic induction vector at the interface have that $u(s = 0) = 0$ and $du(s = 0)/ds = m$, where $m$ is the free parameter describing the strength of the nonlinear effects. Direct substitution of this form of $A$ into Eq. (2) leads to the following differential equation:

$$d^2 u \over ds^2 = 2 bu - 2\alpha u + 4 \mu \frac{du}{ds} + \frac{u^2}{1 + u^2} \quad \text{for} \quad s < 0,$$

$$u = 0 \quad \text{for} \quad s \geq 0.$$  

(3)

The above equations cannot be solved analytically and numerical integration is necessary. To find the stationary solutions of Eq. (3), we use a numerical shooting method, which reduces a two-point boundary problem to the Cauchy problem. By varying parameters $b$ and $m$, we obtained various profiles of surface waves at the interface between the metal and BaTiO$_3$ crystal.

Possible classes of solutions of Eq. (3) can be easily obtained from quite general considerations based on the analogy of Eq. (3) for the envelope of the surface wave with the equation describing the motion of a mechanical particle in a potential well with nonlinear dissipation, where the wave envelope $u$ is equivalent to the particle position and the transverse coordinate $s$ is equivalent to time. The potential and kinetic energies of the particle with unit mass are, respectively, defined as

$$U = (\alpha - \beta) u^2 - \alpha \ln(1 + u^2),$$

$$T = \frac{1}{2} \left( \frac{du}{ds} \right)^2,$$

(4)

(5)

In that case, Eq. (3) can be written in the following form:

$$d(T + U) \over ds = 4 \mu \frac{du}{ds} + \frac{u^2}{1 + u^2} \left( \frac{du}{ds} \right)^2,$$

(6)

of which the right-hand side describes the nonlinear friction force that is proportional to the square of the particle velocity $du/ds$. Fig. 1 depicts the typical profiles of the potential well $U$ for different values of the propagation constant $b$ when $\alpha = 9.88$. Note that the potential well $U$ is symmetric with respect to the point $u = 0$. In this figure, we present only the part of the potential well corresponding to $u > 0$.

Let us first consider negative values of the propagation constant $b < 0$, as shown in Fig. 1. In this case, the potential well $U$ has a single stable stationary point $u = 0$, which is a local minimum of potential $U$. A particle with a nonzero initial energy $U + T$ describing the corresponding surface mode performs damped oscillations (as $s$ varies from 0 to $-\infty$), moving periodically from the region of positive $u$ to the region of negative $u$, and consequently losing its energy because of the influence of nonlinear friction. When $s \rightarrow -\infty$, such a particle approaches gradually the stable equilibrium position $u = 0$. This type of particle motion corresponds to delocalized surface waves [21] with long oscillating tails in the volume of the PP crystal. The profiles of such surface waves are shown in Fig. 2. Numerical integration of Eq. (3) shows that delocalized surface waves have infinite energy $W = \int_{-\infty}^{\infty} u^2(s)ds$ because of the very slow decay of the oscillating tail.

Second, consider the case of positive values of the propagation constant $b$ lying in the interval $0 < b < \alpha$. The potential well $U$ has two stable $(u = \pm \sqrt{b/(\alpha - b)})$ and one unstable $(u = 0)$ stationary points (see dash-dot curve in Fig. 1). In this case, a particle with a nonzero initial energy $U + T$ will be periodically transferred from the right side of the potential well (corresponding to $u > 0$) into the left side of the potential well (corresponding to $u < 0$). The particle loses its energy because of the influence of nonlinear friction. When

![Fig. 1. Typical profiles of the potential well $U$ for different values of the propagation constant $b$ when $\alpha = 9.88$.](image-url)
its kinetic energy at the certain moment (at certain $s$) becomes too small for the next transfer through the unstable stationary point $u = 0$, the particle stops either at one of the two stable stationary points $u = \pm \sqrt{b/(a - b)}$ or at the unstable stationary point $u = 0$. The former case corresponds to the shock surface waves with an infinite energy [21] and a nonzero asymptotic form at $s \to -\infty$. Fig. 3 shows the typical profiles of the shock surface waves of the first three orders. In this article, the order of a wave is defined by the number of intersections of its envelope with the $s$ axis, including the point $s = 0$. A close examination of Fig. 3 reveals that at $s \to -\infty$ shock surface waves have the form of decaying oscillations superimposed on the constant background, of which the magnitude is equal to $\pm \sqrt{b/(a - b)}$. On the other hand, shock surface waves are highly unstable in the diffusion medium since they have zero harmonic in the spatial spectrum and are affected by modulation instability. The latter case corresponds to localized surface waves without oscillating tails in the volume of the PP crystal, which have limited energy. Fig. 4 depicts the profiles of localized surface waves of the first three orders for three different values of propagation constant $b$. This figure demonstrates that the transverse sizes of localized surface waves decrease with an increase in the propagation constant and the amplitudes of localized surface waves increases with the propagation constant.

Third, for positive values of $b > a$, the potential well $U$ has a single unstable stationary point $u = 0$, as shown in Fig. 1. A particle with a nonzero energy $U + T$ will go away from $u = 0$, which corresponds to infinitely increasing oscillations of the light field. The finite motion is not possible and it is meaningless to speak about surface waves.

Finally, we will discuss the stability of localized surface waves at the interface between a metal and a PP crystal. Eqs. (2) have been solved numerically using a beam propagation method. Localized surface waves given by Eqs. (3) have been used as the input beam profiles. For examples, Fig. 5(a) shows the evolution of the surface mode of two order for $b = 3$ in Fig. 4(b) when its input amplitude has been perturbed by 20% and moreover, Fig. 5(b) shows the evolution of the surface mode of three order for $b = 5$ in Fig. 4(c) when its input amplitude has been perturbed by 20%. The solitonic behaviour of these surface modes is of course evident in these figures since they do not break up. Similar plots are also obtained for other surface modes in Fig. 4. We conclude that localized surface waves at the interface between a metal and a PP crystal are stable.

3. Conclusion

In conclusion, we have investigated surface waves at the interface between a metal and a PP crystal. Using the simple analogy between surface waves and mechanical particles situated in the certain potential and subjected to the influence of nonlinear friction we have shown that these surface waves appear in several forms: delocalized surface waves, shock surface waves, both with infinite energy, and localized surface waves with limited energy. We have demonstrated that the transverse sizes of localized surface waves decreases with an increase in the propagation constant and the amplitudes of localized surface waves increases with the propagation constant. The stability of localized surface waves has also been investigated using a beam propagation method and we have found that they are stable.

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