Manipulation of a ring-shaped beam via spatial self- and cross-phase modulation at lower intensity

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We report a tunable ring-shaped diffraction pattern via either nonlinear spatial self- or cross-phase modulation caused by the EIT-like effect in rubidium atomic vapor. During the propagation of an input Gaussian-profile beam, its output wavefront exhibits a ring-shaped diffraction pattern. Furthermore, the spot center can be tuned from dark to bright by varying individual experimental parameters, such as power, frequency detuning, the polarization state of the incident beams and atomic temperature, which makes the nonlinear phase shift evolve beyond 2π. In particular, the input intensity can be as low as 500 W m⁻².

1 Introduction

Manipulation of an optical field via a nonlinear effect was first reported in 1967. Till now, various optical phenomena, such as self-focusing, self-defocusing and splitting have been extensively investigated. In particular, the nonlinear phase shift, relating to a light-induced refractive index modulation, can reshape the wavefront of an optical field during its propagation, and has been observed in a variety of nonlinear media such as liquid crystals, semiconductors and carbon nanotubes. As a typical example, ring-shaped diffraction patterns have potential applications in optical switching, optical limiting, trapping and guiding of atoms.

While ring-shaped diffraction patterns modulated via self-phase modulation (SPM) have been extensively studied in atomic vapor, cross-phase modulation (XPM) has received much less attention. Because of the light-intensity-dependent nature of the SPM effect, a high-intensity beam (for example, 753 × 10⁵ W m⁻² or 1973 × 10⁵ W m⁻²), is usually required to generate a SPM effect. From a practical application point of view, it is more preferable to control the diffraction patterns via a low intensity. Therefore, the combination of SPM and XPM in atomic vapor may be a promising mechanism to realize field pattern modulation, as the atomic coherence provides a fertile ground to enhance the spatial self- and cross-Kerr nonlinear phase shift. For example, electromagnetically induced transparency (EIT) in a coherently prepared multi-level atomic medium is a powerful tool to produce large XPM.

In this work, an efficient diffraction pattern manipulation via spatial SPM and XPM effects caused by an EIT-like effect is first experimentally demonstrated under an input intensity of as low as 500 W m⁻². In addition, the switch of the evolution of the beam from a Gaussian beam to a ring profile (dark center) is explained theoretically. The spatial intensity distribution is mainly dependent on the power, detuning and polarization of the incident lights and the temperature of the Rb vapor. The reported effects can be further exploited for studies in quantum information and quantum computation.

2 Experimental scheme and basic theory

The experimental setup is shown in Fig. 1(a). A collimated laser from an external cavity diode laser (ECDL) at a center wavelength of ~780.234 nm is split into two sub-beams by a PBS denoted as E₁ (p-polarization) and E₁' (s-polarization). The rotating HWP₁ and HWP₂ are placed in front of the Rb vapor to modify the polarization orientation, where z₁,₂ is the rotation angle between the axis of HWP₁,₂ and the y axis in a clockwise direction. The purpose of HWP₁ is to change the polarization of E₁' from s- to p-polarization with the angle z₁ = 45°. Then, changing the z₂ of HWP₂ can modify the polarization of both E₁ and E₁' simultaneously. As shown in Fig. 1(b), the ⁸⁷Rb atoms act as a two-level atom system, including a ground state 5S₁/₂, F = 2(1) and an excited state 5P₃/₂ (2) [see Fig. 1(b)].

The parameters of the laser beams are defined as follows: frequency detuning Δ₁ denotes the difference between the resonant transition frequency Ω₁ and the frequency ωᵢ of Eᵢ (i = 1, 1'); Gᵢ = μᵢEᵢℏ is the Rabi frequency between the energy |j⟩ and |k⟩ (j, k = 1, 2), where μᵢ is the transition dipole matrix element.

The two beams have the same polarization and are seeded into

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a 10 cm-long thermal Rb vapor cell wrapped by µ-metal for magnetic shielding. The temperature of the Rb vapor is maintained at 97 °C by a heated tape. Generally, there is no EIT window when the frequencies of two fields are absolutely the same, while a small frequency difference can result in EIT.25

Our system can give rise to an EIT-like effect, which is generated by the frequency detuning shifting to ω1 − kv for hot atoms with a velocity of v along its propagation direction (z axis). As a result, the frequency detuning of E1 and E1′ should be modified as A1′ = A1 + k1ν and A1′′ = A1′ + k2ν cos θ. k1 and k2 (k1 = k2 = 2π/λ) are the wave vectors of E1 and E1′, λ is the wavelength.21 The XPM can be enhanced and modulated by the Kerr nonlinearity of the EIT-like effect.25 A charge-coupled device (CCD) camera is positioned at a distance of 0.04 m away from the output plane of the cell to receive the images of the transmission beams.

Considering that the input E1 and E1′ are collimated Gaussian beams with a beam radius ω2 = 0.8 mm, the radial intensity distribution is expressed as I(laser) = I0 exp(−2r2/ω2) (i = 1, 1′); I0 = P0/πω2 is the on-axis intensity of E0.

Taking E1 as an example, the intensity distribution of transmitted field at the observation plane is given by Kirchhoff’s diffraction integral, yielding

\[ I(r, z) = \left( \frac{2\pi}{Z} \right)^2 I_0 \int_0^\infty J_0 \left( \frac{2\pi r_0}{Z} \right) \exp \left( -\frac{2r^2}{\omega^2} \right) \times \exp \left[ -i \left( \phi_D + (\phi_{NL}^S + \phi_{NL}^X)(r) \right) \right] dr, \]

where \( \phi_D = k[1^2/2Z + r^2/2R] \) is the diffractive phase; \( Z = 0.04 \) m signifies the distance between Rb and CCD; \( J_0 \) is the zero-order Bessel function; \( r \) is the radial coordinate on the projected plane; \( R = \infty \) is the radius of the wavefront curvature of the collimated light; \( \phi_{NL}^S \) and \( \phi_{NL}^X \) are the spatial self- and cross-Kerr nonlinear phase shift caused by the change of the refractive index \( n_2, n_3 \). The propagation equation28 for E1 is \( \nabla E_1 = -i\nabla I_1 + n_2^2 E_1 \). The propagation equation for E1′ is \( \nabla E_1' = -i\nabla I_1' + n_3^2 E_1' \), where \( n_1 \) is the linear refractive index, and \( n_2^2 \) and \( n_3^2 \) are the self- and cross-Kerr nonlinear coefficients, respectively. \( n_2 \) values should be evaluated in Rb vapor. Hence, the Kerr nonlinear coefficient \( n_2 \) can be expressed as

\[ n_2 = \frac{\text{Re} \rho^{(3)}_{21}}{E_0} = n_2^S + n_2^X = \frac{\text{Re} \rho^{(3)}_{21} + \rho^{(3)}_{21} \Gamma_0}{E_0}. \]

Here, \( N \) is atom density; \( \rho^{(3)}_{21} \) is the nonlinear susceptibility. \( \rho^{(3)}_{21} \) denotes the third-order nonlinear density matrix element expressed as

\[ \rho^{(3)}_{21} = -i2G_2^2G_1 \frac{d^2\phi_N}{dz^2}, \]

where \( d_4 = \Gamma_2 + i\Delta_2 \), \( d_5 = \Gamma_4 + i\Delta_4 \). \( \Delta_2 \) denotes the energy detuning shifting to \( (\Delta_2) = \omega_2 \) and \( \Delta_4 \) is the nonlinear phase shift. If the \( \Delta_\phi_D \) is greater than or equal to \( 2\pi \), a ring pattern appears at the observation plane. Moreover, the transverse propagation wave vector is introduced by

\[ \delta k(r) = \frac{\partial \phi_{NL}(r)}{\partial r} = -\frac{8\pi n_2 I_0 \exp(-2r^2/\omega^2)}{n_1}, \]

where \( \delta k(r) \) is the unit vector along the transverse axis. The relationship between \( \delta k \) and \( r \) is shown as the black curve in Fig. 1(c). Here, \( \Delta_\phi_D \) is the maximum phase shift. If the \( \Delta_\phi_D \) is greater than or equal to \( 2\pi \), a ring pattern appears at the observation plane. Moreover, the transverse propagation wave vector is introduced by

\[ K \approx \Delta_\phi_D/2\pi. \]

The value of \( K \) is an largest integer that is less than \( \Delta_\phi_D/2\pi \). According to eqn (4), the nonlinear phase shift is influenced by the intensity of incident beams and Kerr nonlinear coefficient. In order to verify this, detailed experiments are set up to study the influence of the parameters on the diffractive patterns.

### 3 Results and discussion

By changing the power of \( E_1 \) and \( E_1' \) simultaneously, the intensity distribution of the transmitted beams are taken. The image is initially a bright spot [see Fig. 2(a1) and (a2)] and then a dark spot region gradually appears [see Fig. 2(a3)]. After further increasing the power, the intensity of the center portion rises progressively [see Fig. 2(a4)] until the center intensity is...
The spatial variation of the phase shift is directly proportional to the spatial variation of the refractive index \( n_2 I_i \), where \( n_2 \) is a constant and \( I_i \) varies as the input power increases [here, \( I_i = P|\pi w_2^2 \) and \( w_2 = 0.8 \text{ mm} \)]. Hence, the increase of the intensity eventually leads to the increase of the nonlinear phase shift. The corresponding calculated intensities of diffraction patterns using eqn (1) are depicted as Fig. 2(b1)–(b5). A good agreement between the calculations and observations is obtained. By comparing the experimental results with calculated results, the former show weaker signals in the outer ring than those expected from the models. This discrepancy may be attributed to the uniform absorption caused by the inhomogeneous atomic distribution. From the oblique side view of the calculated intensity distribution in eqn (1), as shown in Fig. 2(c1)–(c5), the center and outer intensity profiles can be clearly described. When only one ring (dark center) occurs in Fig. 2(a3), the nonlinear phase shift is \( \pi \). Meanwhile, one ring with a bright center shows that the nonlinear phase shift is eventually greater than \( \pi \) and less than \( 4\pi \) due to the number of the rings that can be obtained by eqn (6). The variation of the phase shift caused by SPM requires an intensity increase from \( 322 \times 10^3 \) to \( 753 \times 10^3 \text{ W m}^{-2} \) or more, whereas the same result using two beams in our work requires an intensity change from 100 to 500 \text{ W m}^{-2} \) (the corresponding power increase is from 0.2 to 1 \text{ mW} \), as shown in Fig. 2(a1)–(a5). When one beam incidents the Rb vapor, the detected signal is the Gaussian beam; when the other beam is added, the Gaussian beam presents a diffraction ring due to the EIT-like effect, which can produce large factor of the refractive index, enabling accurate detection of an extremely large nonlinear phase shift.\(^{20}\) Furthermore, by attenuating one of the beams up to extinction, the diffraction of the other beam is changed from one diffraction ring (\( 4\pi > \phi_{\text{NL}} > 2\pi \)) to a dark spot region in the center of the beam (\( \phi_{\text{NL}} \approx 2\pi \)), and finally the Gaussian beam shape. Here, only the experiment results of two beams with the same power are given.

In addition to the intensity, the measurement of the output patterns is carried out via changing the frequency detuning, which is related to the \( n_2 \) [see Fig. 3(a1)–(a6)]. According to eqn (1), the corresponding theoretical calculation for one obtained beam (marked in white dashed box) is shown in Fig. 3(b1)–(b6). The nonlinear coefficient \( n_2 \) is changed as detuning varies [see Fig. 3(c)]. The calculation of \( n_3 \propto \text{Re} \chi^{(3)} \) from eqn (2) indicates that \( n_3 \) has a positive value at \( A_1 < 0 \) and a negative value at \( A_1 > 0 \). Furthermore, we can obtain the relationship that \( \phi_{\text{NL}} \propto n_2 \) from eqn (4), so \( \phi_{\text{NL}} \) is dominated by \( n_2 \). The cross-Kerr coefficient \( n_2 \) changes with the tuning of \( A_1 \) from \( -120 \) to \( 120 \text{ MHz} \) [see Fig. 3(c)]. The \( |n_2| \) is maximum at \( A_1 \approx -40 \) and \( 40 \text{ MHz} \). Here, the center of the far-field diffraction pattern is still bright, because the absolute value of the nonlinear phase shift is larger than \( 2\pi \) and less than \( 4\pi \). For the images at \( A_1 < 0 \) and \( A_1 > 0 \), the distribution of light intensity is asymmetric, as shown in Fig. 3(a2) and (a5), due to the integral items in eqn (1), including the diffraction phase \( \phi_{\text{NL}} \) and the nonlinear phase \( \phi_{\text{NL}} \). The diffraction phase \( \phi_{\text{NL}} \) is a constant, while the nonlinear phase term has an opposite sign when the sign of probe detuning is altered.

Considering that the temperature of the Rb vapor affects the Kerr nonlinear coefficient, the diffraction patterns should also be manipulated by changing the temperature of the Rb vapor. The experimental results [see Fig. 4(a1)–(a6)] show that the radius of the outer ring first increases and then decreases as the temperature increases. The corresponding nonlinear phase shift first rises and then decreases. The experimental results are related to the variation of atomic density \( N \) and average velocity \( v \) [see in eqn (2)], according to \( \phi_{\text{NL}} \propto n_2 \propto N(1/d_1^2T_{11} + 1/d_2d_3) \) \( d_2 \) and \( d_3 \) are related to \( v \). On one hand, the atomic density \( N \) increases as the temperature goes up, expressed as \( \ln N = 4.312 - 4040/T - \ln(kT) \).\(^{31}\) On the other hand, the average

Fig. 2. Evolution of output images with the input power increasing from 0.2 to \( \sim 1 \text{ mW} \). (a1)–(a5) Experimental results, (b1)–(b5) and (c1)–(c5). The top view and oblique side view of the calculated output patterns, respectively. The frequency detuning of the p-polarized fields are \( -40 \text{ MHz} \).
velocity of hot atoms \( v \) \((v \propto \sqrt{T})\) rises, which reduces the term of \( 1/d^2 \Gamma_{11} + 1/d^2 \Gamma_{13}, \) first increasing to maximum and then decreasing.\(^{32}\) Hence, considering the combined effect of these two factors, the nonlinear phase shift \( \varphi_{\text{NL}} \) first soars and then decreases with rising temperature. As a result, the diffraction patterns can only be observed at appropriate temperatures. The calculated results of one beam by theory, shown as Fig. 4(b1)–(b6), agree well with the experimental results. This cell temperature is \( \sim 25^\circ \) lower than that of previous work.\(^{19}\)

Fig. 5 depicts the polarization dependence of the signals on the rotation angle \( \alpha_2 \) of the HWP. As \( \alpha_2 \) increases from \( -25 \) to \( 0^\circ \) (horizontal polarization), the intensity of the Gaussian beam’s central spot [see Fig. 5(a1)] first becomes dark and the diameter of the dark region increases [see Fig. 5(a2) and (a3)], and then the central spot varies from a small bright spot to a big bright spot [see Fig. 5(a4) and (a5)].

We begin by theoretically exploring the underlying mechanism of the polarization to modify the Kerr nonlinearity. The nonlinear polarization intensity along the \( l (l = x, y) \) direction expressed as \( P_{l}^{(3)} = \varepsilon_0 \sum_{ijkl} \chi_{ijkl} E_{l}^{*} E_{i} E_{j} E_{k} \), where \( \chi_{ijkl} \) is the tensor component of the third-order nonlinear susceptibility. For an isotropic medium like Rb atomic vapor, the tensor elements \( \chi_{xxyy} = \chi_{yyyy} \). The different polarization states of the incident fields induce different nonlinear susceptibilities. Using a HWP, to change the polarization of the \( E_{l} \) there are two components, expressed as \( E_{l} = E_{l} \sin 2\alpha_2 \) and \( E_{l} = E_{l} \cos 2\alpha_2 \). The corresponding \( x \) component is \( P_{x}^{(3)} = \varepsilon_0 \chi_{xxyy} |E_{x}|^2 |E_{y}|^2 \), and the \( y \) component is \( P_{y}^{(3)} = \varepsilon_0 \chi_{yyyy} |E_{y}|^2 |E_{y}|^2 \).\(^{33}\) In experiment, we use a CCD to detect the transmitted signal (horizontal polarization) after PBS. Hence, the nonlinear susceptibility elements \( \chi_{ij}^{\text{NL}} \) in eqn (2) should be modified to \( \chi_{ij}^{\text{NL}} = \chi_{ij}^{(3)} |E_{i}|^2 |E_{j}|^2 \).

Thus, \( n_2 \) increases as the \( \alpha_2 \) increases due to the increasing nonlinear susceptibility by substituting \( \chi_{ij}^{(3)} \) into eqn (2). Hence, it is found that the nonlinear phase shift of transmitted signal gradually increases as the polarized angle \( \alpha_2 \) increases due to \( \varphi_{\text{NL}} \propto n_2 \) in eqn (4). Fig. 5(b1)–(b5) show the calculated normalized intensity profile in the radial direction at different polarization states, which match well with the experimental results.

### 4 Conclusion

In conclusion, we generated a twin ring-shaped far-field diffraction pattern in atomic Rb vapor based on the spatial SPM and XPM given by the EIT-like effect. By controlling the experimental parameters, such as power, frequency detuning, the polarization state of the incident beam and atomic density, the spot center of the output beams can be tuned from dark to bright and the nonlinear phase shift covers a range beyond \( 2\pi \). In particular, the input intensity can be as low as \( 500 \text{ W m}^{-2} \), which is three orders of magnitude smaller than previous works.\(^{14,19}\) Therefore, manipulation of far-field diffraction based on our method holds great promise for the development of practical applications in optical communication systems.

### Conflicts of interest

There are no conflicts to declare.

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### Notes and references