We propose a scheme for giant enhancement of the Kerr nonlinearity in a four-level atomic system in which spontaneously generated coherence is present. The physics mechanism of the enhancement of Kerr nonlinearity is mainly based on the presence of an extra atomic coherence induced by the spontaneously generated coherence. Numerical values obtained by solving the density matrix equations agree well with these exact analytical values.

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1. Introduction

Quantum coherence and interference in an atomic system have shown a number of important phenomena, such as lasing without inversion (LWI) [1–4], cancellation of spontaneous emission [5], subluminal and superluminal light [6–9], the enhancement of the refractive index [10–12], coherent population trapping (CPT) [13], the electromagnetically induced transparency (EIT) [14–17], and so on. There are many ways to generate quantum coherence and interference in an atomic system. Generally, they are produced by a coherent driving field or by initial coherence. It is now well known that another kind of coherence, spontaneously generated coherence (SGC), can be created by the interference of spontaneous emission channels. Such coherence can be created when a single excited state decays to a closely spaced lower doublet (Λ-type atom) [18,19] or a closely spaced excited doublet decays to a single ground state (V-type atom) [20,21], and it can also be created in the equispaced-atomic-level case (ladder-type atom) [22,23]. The existence of such coherence effects depends on the nonorthogonality of the two dipole matrix elements. The study of SGC has attracted considerable attention [24–38]. At present the effects of SGC on gain, dispersion and populations in an atomic system have been intensively investigated [19,25,39,40]. All the interesting features due to SGC could have useful applications in laser physics and other areas of quantum optics.

Little work, however, has been reported on the effect of SGC on the Kerr nonlinearity. The Kerr nonlinearity corresponds to the refractive part of the third-order susceptibility in optical media. Since optical nonlinear susceptibilities play important roles in all-optical switch, all-optical logic operation, polarization phase gates, and so on, it is desirable to have large nonlinear susceptibilities with small absorption under conditions of low light powers. In a four-level double dark resonance system without SGC, Niu et al. proposed that the interacting double-dark resonance can be effective in enhancing the Kerr nonlinearity by proper tuning of the coherent control field [41]. The generic four-level Λ-type system consists of a single excited state decaying to three closely spaced lower levels, which can be treated as two Λ-type system, the com-
mon excited state is coupled by the same vacuum modes to two lower closely spaced levels leading to spontaneously generated coherence effect. So, in this Letter, we explore the main effect of SGC on the Kerr nonlinearity in the same four-level atomic system with double dark resonances. We find that the enhancement of Kerr nonlinearity can be obtained even without the condition of proper detuning of the coherent control field. Moreover, in the case of Kerr nonlinearity can be obtained even without the condition of nonorthogonality of the matrix elements \( \vec{m}_{21} \) and \( \vec{m}_{23} \) and the interference between the spontaneous emission pathways from \( (2) \leftrightarrow (3) \) and \( (2) \leftrightarrow (4) \). The parameter \( \eta \) denotes the angle between the two dipole moments \( \vec{m}_{21} \) and \( \vec{m}_{23} \). If the matrix elements \( \vec{m}_{21} \) and \( \vec{m}_{23} \) are orthogonal to each other, i.e., \( \eta = 0 \), the SGC disappears. So the existence of the SGC depends on the nonorthogonality of the matrix elements \( \vec{m}_{21} \) and \( \vec{m}_{23} \). Here we only consider the case where each field acts only on one transition and the polarization direction of the driving field is parallel to the polarization direction of the coupling field. Then the Rabi frequencies \( \Omega_p, \Omega_c \) and \( \Omega_d \) are connected to the angle \( \theta \) and represented by \( \Omega_i = \Omega_i^{(0)} \sin \theta = \Omega_i^{(0)} \sqrt{1 - \eta^2} \) (\( l = p, c, d \)). To gain the remarkable SGC effect on optical properties of the system we assume that three lower levels \( (1), (3) \) and \( (4) \) are nearly degenerate, i.e., \( \omega_{13} \approx \omega_0 \) and \( \omega_{14} \approx \omega_0 \) or else the terms with \( \eta \) and accompanied by an exponential factor \( \exp(\pm i \omega_{13} t) \) and \( \exp(\pm i \omega_{14} t) \), which are not shown here, have been averaged out to zero and thereby the SGC effect vanishes [20,30,43].

As is known, the response of the atomic medium to the probe field is governed by its polarization \( P = \varepsilon_0 (\varepsilon_p X + \varepsilon_p^x X^x) / 2 \), with \( \chi \) being the susceptibility of the atomic medium. By performing a quantum average of the dipole moment over an ensemble of \( N \) atoms, we find \( P = N \mu (\mu_{24} + \mu_{23} + \mu_{24}) \). In order to derive the linear and nonlinear susceptibilities, we need to obtain the steady-state solution of the density matrix equation. In the present approach, an iterative method is used and the density matrix elements are expressed as \( \rho_{mn} = \rho_{mn}^{(0)} + \rho_{mn}^{(1)} + \rho_{mn}^{(2)} + \rho_{mn}^{(3)} + \ldots \). Assuming that the coupling field and the driving field are much stronger than the probe field, the zeroth order solution will be \( \rho_{11}^{(0)} = 1 \) and other elements are equal to zero. Under the weak-probe approximation, we can get the matrix element \( \rho_{12} \) up to the third order:

\[
\begin{align*}
\hat{\rho}_{12}^{(1)} &= -i(\gamma_1 + \gamma_2 + \gamma_3) (\Delta_1^2 - \Omega_2^2 - \Omega_p^2) \\
\hat{\rho}_{12}^{(2)} &= -i(\gamma_1 + \gamma_2 + \gamma_3) (\Delta_1^2 - \Omega_2^2 - \Omega_p^2) + \frac{1}{2} i (\Delta_1^2 - \Omega_2^2 - \Omega_p^2) - i (\gamma_1 + \gamma_2 + \gamma_3) (\Delta_1^2 - \Omega_2^2 - \Omega_p^2) \\
\hat{\rho}_{12}^{(3)} &= -i (\gamma_1 + \gamma_2 + \gamma_3) (\Delta_1^2 - \Omega_2^2 - \Omega_p^2) + \frac{1}{2} i (\Delta_1^2 - \Omega_2^2 - \Omega_p^2) - i (\gamma_1 + \gamma_2 + \gamma_3) (\Delta_1^2 - \Omega_2^2 - \Omega_p^2). 
\end{align*}
\]

Therefore, the first-and third-order susceptibility \( \chi^{(1)} \) and \( \chi^{(3)} \) should be

\[
\chi^{(1)} = -\frac{2N|\vec{m}_{12}|^2}{\varepsilon_0 \mu_{12}} \rho_{12}^{(1)} ,
\]

\[
\chi^{(3)} = -\frac{2N|\vec{m}_{12}|^4}{3\varepsilon_0 \hbar^2 \Omega_p^2} \rho_{12}^{(2)} ,
\]

and \( \chi \) is be defined as

\[
\chi = \chi^{(1)} + 3|\varepsilon_p|^2 \chi^{(3)}.
\]
3. Discussion and results

In this part, we will focus on the dependence of the third-order susceptibility on the SGC. According to the expressions (3) and (4), we show in Fig. 2 the linear absorption (dotted curve) and the refractive part of the third-order susceptibility (solid curve) as a function of the probe detuning. From Eq. (2a), we can find the linear absorption is independent of the SGC (i.e. \( \eta \)), the nonlinear absorption given by the imaginary part of \( \chi^{(3)} \), which is dependent of \( \eta \) from Eq. (2c), should be considered, so we also plot the imaginary part of the third-order susceptibility (dashed curve) in Fig. 2. For simplicity, we scale all the parameters by the decay rate \( \gamma_1 \), setting \( \Omega_c^{(0)} = \Omega_c \sin \theta = \Omega_c^{(0)} \sqrt{1 - \eta^2} \), \( \Omega_d^{(0)} \sin \theta = \Omega_d^{(0)} \sqrt{1 - \eta^2} \) and the parameters \( \Omega_c^{(0)} = 1.0 \gamma_1 \), \( \Omega_d^{(0)} = 0.2 \gamma_1 \), \( \gamma_3 = \gamma_4 = \gamma_1 \). From this figure, we can see that when \( \eta = 0 \), which means no interference between spontaneous emission channels, a couple of general linear absorption and Kerr nonlinearity curves occur [39,44]. In other words, if the SGC is absent, the maximal Kerr nonlinearity is accompanied by strong linear and nonlinear absorptions, as can be seen from Fig. 2(a), implying that, in this case, it is not desirable for applications of all-optical switch because the accompanying thermal effect of the devices is not negligible. In order to eliminate the thermal effect, Ref. [41] proposed a scheme for giant enhancement of the Kerr nonlinearity by proper tuning of the coherent driving field in the same four-level \( \Lambda \)-type system without SGC. Here, when the SGC is taken into account, as shown in Figs. 2(b) and (c), a giant Kerr nonlinearity with vanishing absorption also can be obtained under the condition of keeping the coherent driving field resonant. Moreover, the Kerr nonlinearity is enhanced gradually as the SGC changes from \( \eta = 0 \) to 0.9. Compared with the case of \( \eta = 0 \), the maximal value of the Kerr nonlinearity is enhanced by about five times at \( \eta = 0.9 \), which is different from the scheme proposed by Ref. [41], in which the
maximal value of $\text{Re}[\chi^{(3)}]$ is invariable when the coherent driving field is detuned. On the other hand, we find, in the case of $\eta = 0.9$, that the enhanced Kerr nonlinearity is located in the nonlinear gain region and the linear absorption is zero near zero probe detuning. So in the case, we achieve giant Kerr nonlinearity accompanied by zero linear absorption and negative nonlinear absorption via SGC.

4. Qualitative explanation of above numerical results

In this part, we will provide a qualitative explanation for the above numerical results. Here, we mainly discuss the effect of SGC on the Kerr nonlinearity. In Eq. (2), since the matrix elements $\rho_{21}^{(1)}$, $\rho_{22}^{(2)}$, $\rho_{23}^{(2)}$, $\rho_{24}^{(2)}$ are independent of $\eta$, we do not present the analytical expressions. It can be seen from Eq. (2b) that the density matrix element $\rho_{22}^{(2)}$ now is an extra term introduced by SGC. This term according makes the third-order susceptibility acquire an additional term associated with SGC. As the analytical expression for $\rho_{22}^{(3)}$ shows, the first term is the product of the coupling field and the additional term. That is to say, in the present generic four-level $\Lambda$-type atomic system, the spontaneous decays from the common excited state to the lower closely spaced levels interfere and give rise to an extra coherence between the lower two levels [36, 45, 46]. Thereby the coupling field interacts with this extra coherence and consequently the Kerr nonlinearity is clearly enhanced. Here, we designate the first term of Eq. (4) as $F_1(\eta)$ and another term as $F_2$, then $\chi^{(3)} = F_1(\eta) + F_2$. We plot $\text{Re}[F_1(\eta)]$ as a function of $\Delta_1$ in Fig. 3 with different SGC. Obviously, the stronger the SGC is, the more remarkable the coupling field interacts with this extra coherence. Hence, we attribute the enhancement of Kerr nonlinearity to the extra coherence between the lower two levels induced by SGC.

Another a question arises, that is why the peak value of the enhanced Kerr nonlinearity enters the electromagnetically induced transparency and whether the position variation of $\text{Re}[\chi^{(3)}]$ is related to SGC. To answer this question, we plot $\text{Re}[F_1(\eta)]$ and $\text{Re}[\chi^{(3)}]$ as a function of $\Delta_1$ in Fig. 4 with different SGC. Obviously, one can see that the peak position of the enhanced $\text{Re}[\chi^{(3)}]$ is approximately coincident with that of $\text{Re}[F_1(\eta)]$. Hence, when the optimal SGC effect is taken into account, a giant Kerr nonlinearity accompanied by vanishing absorption can be realized near zero probe detuning even without the condition of proper detuning of the coherent control field.

5. Conclusion

We have investigated the effect of SGC on the Kerr nonlinearity in a generic four-level $\Lambda$-type atomic system. The results show that the spontaneously generated coherence is capable of enhancing the Kerr nonlinearity even without the condition of Ref. [41]. In the case of optimal SGC, both enhanced Kerr nonlinearity and negligible linear and nonlinear absorption can be realized simultaneously. Compared with the scheme that offered by Ref. [41], the enhancement of Kerr nonlinearity can be enhanced by some times even without the condition of proper detuning of the coherent control field. Close inspection of the analytical expression undoubtedly shows that the spontaneously emission interference induces an extra coherence term that is responsible for the large enhancement of the Kerr nonlinearity, suggesting the application of the above-mentioned results in all-optical switch, frequency conversion, polarization phase gates and relevant fields appear to be promising.

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