Theoretical calculation of annular upward flow in a narrow annuli with bilateral heating

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Abstract

Based on separated flow, a theoretical three-fluids model predicting for annular upward flow in a vertical narrow annuli with bilateral heating has been developed in present paper. The theoretical model is based on fundamental conservation principles: the mass, momentum, and energy conservation equations of liquid films and the momentum conservation equation of vapor core. Through numerically solving the equations, liquid film thickness, radial velocity, and temperature distribution in liquid films, heat transfer coefficient of inner and outer tubes and axial pressure gradient are obtained. The predicted results are compared with the experimental data and good agreements between them are found. With same mass flow rate and heat flux, the thickness of liquid film in the annular narrow channel will decrease with decreasing the annular gap. The two-phase heat transfer coefficient will increase with the increase of heat flux and the decrease of the annular gap. That is, the heat transfer will be enhanced with small annular gap. The effects of outer wall heat flux on velocity and temperature in the outer liquid layer, thickness of outer liquid film and outer wall heat transfer coefficient are clear and obvious. The effects of outer wall heat flux on velocity and temperature in the inner liquid layer, thickness of inner liquid film and the inner wall heat transfer coefficient are very small; the similar effects of the inner wall heat flux are found. As the applications of the present model, the critical heat flux and critical quality are calculated.

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1. Introduction

Two-phase annular flow with heat transfer is prevalent in many processes such as industrial and energy transformation processes. Recently, advances in high performance electronic chips and the miniaturization of electronic circuits in which high heat flux will be created and other compact systems such as the refrigeration/air conditioning, automobile environment control systems have resulted in a great demand for developing efficient heat transfer techniques to accommodate these high heat fluxes (Warrier et al., 2002). One of the simplest heat transfer systems may be using single-phase forced convection or two-phase flow boiling in small tubes or rectangular channels or annular gaps or lunate channels. Experiments have showed that annular flow are main flow pattern in narrow annular gaps (Sun et al., 2001). It is easy to form vapor slug in the narrow channel due to the accumulation of bubbles. Thus, vapor slug is longer than liquid column. And then, annular flow is formed. Annular flow is a particularly important flow pattern since it frequently occurs in a wide range of such parameters as quality, pressure and flow conditions. In this regime,
an inherently unsteady liquid film covers the surface of the heated tube while vapor with entrained droplets flows in the central core. The liquid film structure is influenced by such variables as system geometry, flow orientation, liquid/vapor velocity difference, etc. A lot of literatures of annular flow have been presented up to now. Entrainment and deposition are very important for developing a theoretical model of annular flow. Thus, there exist a lot of reports of entrainment and deposition in annular flow. Jr et al. (2002) studied the liquid entrainment, droplet concentration, and pressure gradient at the onset of annular flow in a vertical pipe. Okawa et al. (2002) presented an entrainment rate correlation in annular flow and declared that their correlation may be applicable to wide range of flow conditions. Pan and Hanratty (2002a,b) developed correlations of entrainment for annular flow in vertical and horizontal pipes. Bertodano et al. (2001) scaled the entrainment rate mechanism by the Kelvin–Helmholtz instability at the liquid film surface and developed a new correlation based on their experimental data obtained in 10 mm tubes for low-viscosity fluids in the ripple annular flow. Kataoka et al. (2000) presented a simple approximate correlation obtained from equilibrium state where entrainment rate and deposition rate become equal. Binder and Hanratty (1991, 1992) modeled the droplet deposition in vertical gas/liquid annular flow by considering dispersion from a ring source on the wall and described droplet using Lagrangian method in horizontal gas–liquid annular flows. A relatively simple model was proposed for predicting deposition rate based on local measurements of droplet fluxes in the core of horizontal annular flow (Paras
and Karabelas, 1991). Experimental data of annular flow have been obtained by many workers and numerous correlations have been developed. Examples include those of Hurlburt and Newell (1999), Holt et al. (1999), Fu and Klausner (1997), Klausner et al. (1991) and Jung and Radermacher (1989) for pressure drop, and Nozu and Honda (2000), Fossa et al. (1995) and Kattan et al. (1998) for heat transfer. However, most of these studies are based on data obtained from relatively large diameter conduits. Due to the nature of the geometries of narrow annular channel, the heat transfer performances and the associated pressure drop can be very different from that in large tubes or channels. Only a few studies in the open literatures report on single- and two-phase flow boiling heat transfer in annular gap. Hasan et al. (1990) reported that experiments on subcooled flow boiling heat transfer were carried out in a vertical annular channel, the inner wall of which is heated and the outer wall insulated. Fron-113 was the working fluid. Flow boiling heat transfer data were reported at three mass velocities (579, 801, and 1102 kg m$^{-2}$s$^{-1}$), three pressure (312, 277, and 243 kPa), and three inlet subcoolings (20.0, 30.0, and 36.5$^\circ$C). A multiple-hysteresis phenomenon is identified. The measured wall heat transfer coefficients were compared with those predicted by various correlations and an improvement of the Shah correlation for annuli was suggested. Bibeau and Salcudean (1994) studied the bubble ebullition in forced convective subcooled nucleate boiling using a vertical circular annulus at atmospheric pressure, for mean flow velocities of 0.08–1.2 m s$^{-1}$ and subcoolings of 10–60$^\circ$C. The maximum bubble diameter and condensation time were shown to be influenced by the location relative to the onset of significant void. Shah (1976, 1983) presented that a general correlation for heat transfer during subcooled and saturated boiling in tubes had been compared with a large amount of data for boiling in annuli. The data included the fluids water, Fron-113, and methanol; mass flow rate from 278 to 7774 kg m$^{-2}$s$^{-1}$; heated fluxes from 0.03 to 4.4 kW m$^{-2}$; liquid subcoolings from 2 to 109$^\circ$C; and reduced pressures from 0.009 to 0.89. The inner tube diameter varied from 4.5 to 42.2 mm and the annular gap from 1 to 6.6 mm. The data included heating of the inner, outer, and both tubes of the annular channel. Yin et al. (2000) experimentally studied the subcooled flow boiling heat transfer and visualized the associated bubble characteristics for refrigeration R-134a flowing in a horizontal annular duct having inside diameter of 6.35 mm and outside diameter of 16.66 mm. The effects of the imposed wall heat flux, mass flow rate, liquid subcooling, and saturated temperature of R-134a on the resulting nucleate boiling heat transfer and bubble characteristics were examined in detail. Sun et al. (2001) numerically calculated the liquid nitrogen film thickness, distribution of velocities and temperatures in the liquid layer for vertical two-phase flow in an annular gap in the range of heat flux from 6000 to 12000 W m$^{-2}$; mass flow rate from 500 to 1100 kg m$^{-2}$s$^{-1}$. Su et al. (2003) presented a theoretical model which describes the model of vertical upward annular flow in narrow annular channel with bilateral heating under low mass flow rate. However, the profiles of velocity in the liquid layers are linear due to that the number of calculation grids in liquid layers is not large enough when the model was numerically solved. It is obvious that the linear solution will departure from the true solution. Thus, there is need to further study the annular flow in a vertical upward annular gap.

2. Analytical model

2.1. Basic equations

The flow in the bilaterally heated annular gap is presented in Fig. 1. In the two-phase annular flow regime, there exist the inner and the outer liquid films due to the bilateral heating. The inner liquid film covers the outer surface of inner tube, and the outer liquid film covers the inner surface of outer tube while the vapor with entrained droplets flows in the central core. The following assumptions are necessary in order to analyze the annular flow under these conditions:

1. The flow is steady and incompressible;
2. The liquid film is uniform around the tube periphery;
3. The pressure is uniform in the radial direction;
4. Vapor is produced by the evaporation at the liquid/vapor interface;
5. Liquid droplets entrained in the vapor core are uniformly distributed;
6. There is no slip between the vapor and the droplets.
In the annular region, there exists the mass transfer between the liquid droplets in the vapor core and the liquid films. On the one hand, the liquid droplets in the vapor core will be deposited on the liquid film surfaces. On the other hand, the liquid droplets will be continuously entrained from the liquid films into the vapor core and thus there will be more droplets in the vapor core. The vapor core is supplied with the liquid evaporated at the liquid/vapor interface and with the vapor bubbles due to the boiling process in the liquid films. Based on these observations, the mass flow rate of the liquid films will be changed as a function of the vapor core. Thus, it can be expressed by the momentum conservation equation of the inner film will be

$$2\pi r \tau \Delta Z = 2\pi (r_1 + h_1) \Delta Z - \rho_f g \left[(r_1 + h_1)^2 - r_2^2\right] \Delta Z$$

$$\frac{dP}{dz} \left[(r_1 + h_1)^2 - r_2^2\right] \Delta Z$$

where, $r_1$ is the outer radius of the inner tube; $h_1$ is the thickness of the inner liquid film; $\tau$ is shear stress at the liquid/vapor interface of inner liquid film; $\rho_f$ is the liquid density; $g$ is the gravitational acceleration.

The shear stress in the inner liquid film can be obtained from Eq. (3), that is

$$\tau = \frac{(r_2 - h_0)}{r} \tau_0 - \frac{1}{2} \left(\rho_f g + \frac{dP}{dz}\right) \left[\left(r_2 - h_0\right)^2 - r_2^2\right]$$

The shear stress $\tau$ at radius $r$ in the outer liquid film can be obtained using the same method. It can be written as:

$$\tau = \frac{(r_2 - h_0)}{r} \tau_0 - \frac{1}{2} \left(\rho_f g + \frac{dP}{dz}\right) \left[\left(r_2 - h_0\right)^2 - r_2^2\right]$$

where, $r_2$ is the inner diameter of the outer tube; $h_0$ is the thickness of the outer liquid film; $\tau_0$ is shear stress at the liquid/vapor interface of the outer liquid film; $dP/\Delta Z$ is axial pressure gradient, and can be calculated by the momentum conservation equation of vapor core. Thus, it can be expressed by

$$\frac{dP}{dz} \left[\tau_0 \rho_f + \tau_0 \rho_f + \frac{1}{A_{b}} \frac{d}{dx} \left(\rho_v \alpha_v u_c^2\right) + \rho_f \right]$$

where, $\rho_v$, $\alpha_v$, and $u_c$ are the density, void fraction, and mean velocity of vapor core, respectively. See the Appendix A for their calculating equations.

Knowing the shear stress in the liquid film, the velocity in the liquid film can be obtained according to
where, $u$ is the local liquid velocity in the liquid film; $T_{sat}$ is the saturated temperature of fluid; $y$ is the distance from the wall; $u$ is the local liquid velocity in the liquid film; $T_{w}$ is the wall temperature.

The present study calculates the heat transfer coefficient of annular gap according to its basic concept. The heat transfer coefficient of two-phase flow is defined by

$$ h = \frac{q}{T_w - T_{sat}} $$

where, $q$ is heat flux; $T_w$ is the wall temperature.

### 2.2. Constitutive equations

The variables $\epsilon$, $\eta_\epsilon$, $\tau_f$, and $E_n$ must be specified in order to simulate the annular flow using the above model because the relations above are not yet closed. The interfacial shear stress is one of the most important variables that effect the solutions of the present model and can be calculated by the correlation proposed by Wallis (1969)

$$ \tau_f = \frac{1}{2} \frac{\rho_i u_*^2}{\mu_f} $$

where, $u_* = (1 + 300\beta)/D_g$; $D_g = D_w - D_i$; $\beta = 0.079Re^2/D_g$; and $Re = D_g/\mu_f u_*$. Detailed turbulence measurements for the liquid films of annular flow in the vertical annular gap are not available. The usual practice in treating the liquid film eddy viscosity is to assume the wall turbulence is similar to that of single-phase flow and employ the single-phase flow formulas for liquid film eddy viscosity. Although such an assumption has yet to be experimentally validated, modified single-phase flow eddy viscosity and turbulent Prandtl number can be used (Fu and Klausnert, 1997). Thus, $\epsilon$ can be calculated by the following equations:

$$ \frac{\epsilon}{\nu} = 0.001 y^{1.3} $$

where, $y^+$ is the wall heat flux; $\nu$ is the molecular viscosity.

$$ T = T_{sat} $$

The boundary conditions of Eq. (7) are as follows:

$$ \begin{cases} y = 0, & u = 0 \\ y = y_0, & \frac{du}{dy} = \frac{\tau_{int}}{\mu_f} \end{cases} $$

where, $y_0$ is the distance from the wall; $u$ is the local liquid velocity in the liquid film; $\tau_{int}$ is the shear stress at the liquid/vapor interface; $\mu_f$ is the effective viscosity of the fluid; $\mu_L = \mu_f$ for laminar flow, and $\mu_L = \epsilon \mu + \tau_f$ for turbulence flow; $\epsilon$ is the eddy viscosity.

The mass flow rate of liquid film can be obtained by integrating the velocity:

$$ \int \rho_i u \, dy $$

The boundary conditions of Eq. (9) are as follows:

$$ \begin{cases} q_n = -k \frac{dT}{dy}, & y = 0 \\ T = T_{sat}, & y = \delta \end{cases} $$

where, $k$ is the thermal conductivity.

There are a lot of boiling heat transfer correlations of two-phase flow that can be found in the open literatures. Such correlations normally attempt to predict heat transfer coefficient both in the convective and nucleate flow boiling regions. However, these correlations are only satisfactory under certain conditions.
The relationship between the liquid film eddy viscosity and eddy thermal diffusivity is as follows (Fu and Klessner, 1997; Celata et al., 2001):

\[ \nu_t = \frac{D_{ff}}{H_{ff}} \]  

(13)

where, \( \nu_t \) is the turbulent Prandtl number and can be calculated by

\[
\begin{align*}
\nu_t &= 1.07, \quad y^+ < 5 \\
\nu_t &= 1.0 + 0.855 \times \tanh(0.2(y^+ - 7.5)), \quad y^+ \geq 5
\end{align*}
\]

(14)

The deposition can be calculated with the equation given by Kataoka, Ishii and Nakayama (2000) based on fully developed annular flow with heat insulation

\[
D_{ep} \frac{D_{ep}}{\mu_t} = 0.222 R_{ff}^{0.74} \left( \frac{\nu_t}{H_{ff}} \right)^{0.26} \psi^{0.74}
\]

(15)

where, \( \psi \) is the fraction of liquid flowing as droplets in the vapor core and is defined as (Ishii and Mishima, 1984; Celata et al., 2001): \( \psi = (j_i - j_o)/j_i \), where, \( j_i \) is the volumetric flux which pertains to the liquid phase, and \( j_o \) is the volumetric flux which pertains to the liquid film. \( R_{ff} \) is the liquid Reynolds number when the liquid singly flows through the channel.

In the annular gap with bilateral heating, on the one hand, the deposition rate strongly depends on the concentration of droplet (number of droplet per unit volume) in the vapor core. The area of the interface between the outer liquid film and the vapor core is larger than that between the inner liquid film and the vapor core. Therefore, the rate of deposition on the outer liquid film is larger than that on the inner liquid film. Thus, the Eq. (15) must be modified in order to be used to calculate the deposition rate in a bilateral heating annuli by a dimensionless factor \( A_K \) (Doerffer et al., 1997)

\[ D_{ep,K} = D_{ep} A_K \]  

(16)

where, \( A_K \) is a modified factor and is defined as: \( A_K = r_k/(r_o + r_i) \), \( K = i, 0 \) (i denotes inner liquid film and o denotes outer liquid film).

On the other hand, the deposition may be blocked by the evaporative effect on the interface between the liquid film and vapor core. Thus, the deposition rate caused by this effect can be given by (Milashenko and Nigmatulin, 1989)

\[ D_{ep,e,K} = -C_2 \delta K \]  

(17)

where, \( \delta K \) is the block coefficient, and is defined by \( \delta K = \left( q_0/(0.165 q_0 V_{ep,K}) \right) \) and \( V_{ep,K} = (q_0(h_{fg},p_0)) \) is the evaporative velocity of liquid film, \( K = i, o \) as above. \( C \) is the concentration of droplet in the vapor core and can be calculated by \( C = W_{in} / (W_{in} + W_{t}) \). \( W_{in} \) and \( W_{t} \) is mass flow rate of liquid droplet and of vapor in the vapor core, respectively. Thus, the deposition rate may be finally calculated by

\[ D_{ep,K} = D_{ep,0,K} + D_{ep,e,K} \]  

(18)

The droplet entrainment rate, \( E_{en} \), can be calculated considering the contribution of two different mechanisms of droplets formation: breakup of disturbance waves and boiling in the liquid film (Mishima et al., 1989). According to the analysis of Kataoka and Ishii (1983), droplet entrainment is mainly attributed to the breakup of large disturbance waves on the liquid/vapor interface. Regarding the second mechanism of droplet formation, according to the analysis of Mishima et al. (1989) and Kay (1994), burst of boiling bubbles in the liquid film may cause droplet entrainment at sufficiently high heat flux. Thus, entrainment rate may be obtained by the following equation:

\[ E_{en} = E_{en,w} + E_{en,b} \]  

(19)

where, \( E_{en,w} \) is wave droplet entrainment rate (entrainment due to disturbance waves): \( E_{en,b} \), is boiling droplet entrainment rate (entrainment due to boiling in the liquid film).

The \( E_{en,w} \) is evaluated by the following equations (Kataoka et al., 2000; Mishima et al., 1989; Ishii and Mishima, 1984):

\[
\begin{align*}
E_{en,w} &= 0.72 \times 10^{-9} R_{ff}^{1.73} [\psi / \varphi_{\infty} \left( \frac{1 - \psi}{\varphi_{\infty}} \right) ]^{0.25} \left( 1 - \frac{\psi}{\varphi_{\infty}} \right)^2, \quad \frac{\psi}{\varphi_{\infty}} \leq 1 \\
E_{en,w} &= 0.222 R_{ff}^{0.74} \left( \frac{\nu_t}{H_{ff}} \right)^{0.26} \psi^{0.74}, \quad \frac{\psi}{\varphi_{\infty}} = 1 \\
E_{en,b} &= 0
\end{align*}
\]

(20)

\( Re_{ff} < Re_{bl} \)
where, $\varphi_\infty$ is the equilibrium entrainment fraction (Ueda et al., 1981), and can be given by $\varphi_\infty = \tanh(7.25 \times 10^{-3} \cdot 20^{0.25})$, and $We$ is Weber number for entrainment: $We = (\rho_l / \rho_g) \cdot 10^{0.347}$, $\sigma$ is the surface tension. $Re_f$ is liquid Reynolds number, and can be given by $Re_f = (G(1-x)(1-\psi)D_e/\mu_f)$, where, $Re_f$ is the liquid film Reynolds number, and can be calculated by $Re_f = G(1-x)(1-\psi)D_e/\mu_f$. $Re_{ff}$ is the minimum liquid film Reynolds number, below which no droplet entrainments exist due to superficial waves occurs (Celata et al., 2001), it can be calculated by $Re_{ff} = (y^{**}/0.347)^{1.5} (\rho_l / \rho_g)^{1.7} (\mu_f / \mu_g)^{1.5}$, where $y^{**}$ is the non-dimensional distance associated with the onset of the entrainment. According to Celata et al. (2001), $y^{**} = 10.0$

The entrainment due to boiling in the liquid film, $E_{ne}$, may be given by the correlation of Ueda et al. (1981):

$$E_{ne} = 4.77 \times 10^5 \left( \frac{2\rho_l}{\rho_l \cdot \rho_g \cdot \sigma} \right)^{0.75} \frac{4 \cdot h_l}{h_l + h_g} \delta_i \delta_o$$  \hspace{1cm} (21)

The above correlations for entrainment are only suitable to circular tubes. They must be modified in order to be used to calculate the entrainment rates in a bilateral heating annuli. That is:

$$E_{sw,i} = E_{sw} A_i$$

### 3. Solution procedure

The relations above are closed now. Knowing the properties of fluid, heat fluxes, mass flow rate, quality, and geometric parameters, the above model can be solved. Fig. 2 is a flowchart describing the solution procedure to the above equations, which is also summarized as:

1. An initial guess is made for the film thickness: $\delta_i$ and $\delta_o$. ($\delta_i$ denotes the film thickness of inner liquid film and $\delta_o$ denotes that of outer liquid film).
2. Eq. (6) is used to compute the pressure gradient $dP/dZ$.
3. Eq. (7) is used in conjunction with Eqs. (4) and (5) to compute the inner and outer liquid film velocity profiles: $u_i(y)$ and $u_o(y)$, and Eq. (8) is employed to calculate the inner and outer liquid film mass flow rates. The inner liquid film mass flow rate is hereby denoted by $W_{li}$, and the outer liquid film mass flow rate is denoted by $W_{lo}$.
4. Eqs. (1) and (2) are also used to compute the inner and outer liquid film mass flow rates. To distinguish between the results obtained by steps (3) and (4), the inner liquid film mass flow rate obtained by present step is hereby denoted by $W_{li}$, and the outer liquid film mass flow rate is denoted by $W_{lo}$.

![Fig. 2. N-S flowchart of solution procedure.](image-url)
(5) Check if $W_{li,3}$ equals to $W_{li,4}$ and $W_{lo,3}$ equals to $W_{lo,4}$. If not, guess the initial values of the film thickness $\delta_i$ and $\delta_o$ again. Steps (2)–(5) are repeated.

(6) The heat transfer coefficient defined in Eq. (10) is evaluated.

It is noted that variable-step integration method should be used in the procedure of solving the velocity Eq. (7) in order to ensure that the step is small enough or the number of the grid is large enough to obtain high accuracy. If the fixed step is employed, the velocity profiles in the liquid film will be linear (Su et al., 2003).

4. Verifications of present model

To validate the correction of proposed model, an attempt has been made to put together a database that covers a wide range of flow and thermal conditions. However, No database with bilateral heating in the annular gap has been found in the open literatures. Therefore, the present model has been verified by our experimental data. Fig. 3 is a sketch of the experimental apparatus used in this study to investigate the flow boiling in a vertical annular gap. It consists of two main loops, namely, the primary and the secondary loops. The secondary loop, which is connected to the primary one through the condenser, is used to condense the primary loop and is not shown in detail in Fig. 3. The primary loop contains a pump, a pressurizer, an electrical pre-heater, a mass flow meter, a test section, a condenser, and connected pipes. The temperature and mass flow rate in the secondary loop must be controlled to have enough cooling capacity for condensing the fluid in the primary loop and subcooling the water to a preset subcooled temperature. Two power supplies were used to heat the inner and outer tubes in the test section. Distilled water is circulated in the primary loop. By adjusting charge of $N_2$ through the nitrogen ($N_2$) charging line, the system pressure can be regulated. By adjusting the gate opening of the by-pass valve, the mass flow rate of the test section can be regulated. Note that the mass flow rate and system pressure should be further adjusted simultaneously by the needle valve at the upstream of the flow meter section and the power of the heater in the pressurizer, respectively, in order to control them at the required levels. The mass flow rate of test section is measured by a precise mass flow meter which consists of

![Fig. 3. Schematic diagram of experimental apparatus.](image-url)
orifice, three valves and differential pressure transmitter. After subcooled, the distilled water flows back to the main pump. The signals from the thermocouples, pressure transducers and other transducers were recorded by a data acquisition system and then analyzed by a computer immediately. The absolute pressure at the entrance of the test section is measured with two absolute pressure transducers. The pressure drop is measured with a differential pressure transducer. The temperatures of fluid are measured by 1.0 mm K-type NiCr-NiSi thermocouples. The inner wall temperatures of inner tube are measured with 15

![Fig. 4. Comparison of heat transfer and axial pressure gradient between the proposed model and experiment.](image)

- (a) One example of comparison of two-phase heat transfer coefficient between the experiment and the present model with \( s = 0.95 \) mm in the axial direction.
- (b) One example of comparison of two-phase heat transfer coefficient between the experiment and the present model with \( s = 2.0 \) mm in the axial direction.
- (c) Comparison of two-phase heat transfer coefficient between the experiment and the present model.
- (d) Comparison of axial pressure gradient between the experiment and the present model.
- (e) Wall temperature and saturated temperature curves obtained by the proposed model.
Fig. 4. (Continued).
Table 1
Accuracy of measurement system

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>0.75</td>
</tr>
<tr>
<td>Mass flow rate</td>
<td>0.251</td>
</tr>
<tr>
<td>Pressure</td>
<td>0.251</td>
</tr>
<tr>
<td>Pressure drop</td>
<td>0.251</td>
</tr>
<tr>
<td>Heated power</td>
<td>1.619</td>
</tr>
</tbody>
</table>

0.5-mm o.d. K-type thermocouples. The outer wall temperatures of the outer tube are measured with 0.1-mm o.d. T-type thermocouples, which are attached at the four locations of the circumference with 90° apart. The accuracy of the measurements is listed in Table 1.

The two-phase heat transfer coefficients and axial pressure gradients obtained by the proposed model are compared with those of experiments, as shown in Fig. 4. The Fig. 4a and b show the comparison of two-phase heat transfer coefficient between proposed model and experiments in different gap (s denotes gap size). A good agreement has been found. Fig. 4c shows the deviation between the experiment and present model is ±25%. Its root mean square (RMS) error is 15.11%. However, about 70% data of heat transfer coefficient were over-predicted (Fig. 4a–c) within above error range. That is to say, there are about 70% two-phase heat transfer coefficients are larger than those of experimental data. The reason may be that the wall temperature obtained by the proposed model is lower than that of experiment. The outer wall temperatures were measured by thermocouples in our experiments. The accuracy of thermocouples is 0.75%. The outer wall temperatures vary very small along the axis. The thermocouples may not measure these small changes along the axis. Thus, the wall temperatures did not change along the axis. However, the pressure decreases slightly along the axis. Thus, the saturated temperature decreases slightly. The thickness of the liquid film also decreases along the axis. Therefore, the wall temperatures obtained by the proposed model also decrease along the axis, as shown in Fig. 4c. Thus, (T_w – T_sat) obtained by proposed model is lower than that obtained by experiment, and the two-phase heat transfer coefficient obtained by proposed model is larger than that obtained by experiment.

Fig. 4d shows the comparison of axial pressure gradient between the present model and experiment. The deviation between them is ±30%, the RMS is 18.86%. The present paper selects quite a number of correlations to close the set of equations for the annular flow model, which are developed for large tube. Although the modified factor is employed to apply them for the case of annular gap, but the errors for the axial pressure gradient seem quite large. Therefore, the further theoretical study must be done to develop a model for narrow annular gap. As the first step, the error (±30%) is acceptable.

5. Results and discussions

Fig. 5 shows the temperature profiles in the liquid films obtained by the proposed model with six different annular gap and for three values of wall heat flux of inner tube or of outer tube under the mass flow rate is 120 kg m⁻² s⁻¹ conditions. Abscissas denote the non-dimensional distance from wall and y⁺ = y/δ in Figs. 5 and 6, where y is the distance from wall and δ is the thickness of liquid films. y⁺ = 0 is at the wall, and y⁺ = 1 is at the liquid/vapor interface. The y⁺-axis denotes non-dimensional temperature T/T_sat or T/T_sat in Fig. 5, where T_i is the temperature in the inner liquid layer, T_o is the temperature in the outer liquid layer, and T_sat is the saturated temperature. Two figures in which the gap size is different are shown under the same conditions in Figs. 5–8 in order to clearly express the general parametric trends of gap size. As observed in Fig. 5, the temperature in the inner or outer liquid layer will almost decrease linearly from the wall temperature to saturated temperature while the distance changes from the wall to the liquid/vapor interface. The liquid film mass flow rate is very low (120 kg m⁻² s⁻¹), thus the flow is laminar one. The energy equation is a conductive one. Therefore, the temperature in the liquid layer changes almost linearly. Fig. 5a shows the effects of outer wall heat flux on the temperature in the inner liquid layer with different annular gap with a fixed heat flux of inner tube, q_w = 50 kW m⁻¹. The effects are very small. With increase of outer wall heat flux, the temperature in the inner liquid layer decreases very slightly. The more liquid will evaporate with increasing outer wall heat flux, the velocity of vapor core will increase slightly. The
Fig. 5. The temperature curves in the liquid films. (a) Effects of outer wall heat flux on the temperature profiles in the inner liquid layer. (b) Effects of outer wall heat flux on the temperature profiles in the outer liquid layer. (c) Effects of inner wall heat flux on the temperature profiles in the inner liquid layer. (d) Effects of inner wall heat flux on the temperature profiles in the outer liquid layer.
Fig. 5. (Continued).

G=120 kg/m²s

T / T₀

0.0 0.2 0.4 0.6 0.8 1.0

0.8, 0.6, 0.4, 0.2

T / T₀

0.0 0.2 0.4 0.6 0.8 1.0

0.8, 0.6, 0.4, 0.2

G=120 kg/m²s

T / T₀

0.0 0.2 0.4 0.6 0.8 1.0

0.8, 0.6, 0.4, 0.2

T / T₀

0.0 0.2 0.4 0.6 0.8 1.0

0.8, 0.6, 0.4, 0.2
Fig. 6. The velocity profiles in the liquid layers. (a) The velocity profiles in the inner liquid layer with different outer wall heat flux. (b) The velocity profiles in the outer liquid layer with different outer wall heat flux. (c) The velocity profiles in the inner liquid layer with different inner wall heat flux. (d) The velocity profiles in the outer liquid layer with different inner wall heat flux.
Fig. 6. (Continued).
Fig. 7. The liquid layer thickness. (a) Effects of outer wall heat flux on the inner liquid layer thickness. (b) Effects of outer wall heat flux on the outer liquid layer thickness. (c) Effects of inner wall heat flux on the inner liquid layer thickness. (d) Effects of inner wall heat flux on the outer liquid layer thickness.
Fig. 7. (Continued).
Fig. 8. The two-phase heat transfer coefficient in annular gap. (a) Effects of outer wall heat flux on the two-phase heat transfer coefficients of inner tube. (b) Effects of outer wall heat flux on the two-phase heat transfer coefficients of outer tube. (c) Effects of inner wall heat flux on the two-phase heat transfer coefficients of inner tube. (d) Effects of inner wall heat flux on the two-phase heat transfer coefficients of outer tube.
Fig. 8. (Continued).
inner liquid film will be entrained and become thin. Thus, its temperature gradient will decrease slightly. Fig. 5b shows the effects of outer wall heat flux on the temperature in the outer liquid layer with different annular gap. The effects are very clear and obvious. The temperature gradient in the outer liquid film will increase with an increase of outer wall heat flux. Although the liquid layer will become thin with increasing outer wall heat flux, the effects on temperature gradient in outer liquid layer is larger than those on liquid film thickness. Fig. 5c and d show the effects of inner wall heat flux on the temperature in the liquid layer. The effects are similar to those of Fig. 5a and b. There are very slight influences of inner wall heat flux on the temperature in the outer liquid layer. The effects on the temperature in the inner layer are very large. It can also be found from Fig. 5 that the temperature gradient will increase with increasing the gap with same mass flow rate and heat flux. The thickness of liquid layer will increase with an increase of gap, thus the temperature gradient will increase.

Fig. 6 shows the velocity profiles in the liquid films. As described above, \( \gamma^+ \) is the non-dimensional distance from the wall. The velocities of liquid films at the wall are zero due to the effect of viscosity. The effects of heat flux on velocity profiles are very small. With increasing outer wall heat flux, the velocity in the inner liquid layer will increase slightly (Fig. 6a), and the velocity in the outer liquid film will also increase slightly (Fig. 6b). The evaporation of the outer liquid film will increase with an increase of outer heat flux, thus more liquid evaporates, the vapor velocity in the core will increase, and the shear stress on the liquid/vapor interface will increase, the drag force to liquid layer will increase. Therefore, the inner liquid film velocity will increase. The thickness of outer liquid layer will decrease obviously with increasing outer wall heat flux. This effect is smaller than that of liquid film acceleration, because the wall heat flux (the maximum value is 50 kW m\(^{-2}\)) is low relatively. Thus the velocity in the outer liquid layer will increase slightly. The effects of inner wall heat flux on the velocity profiles of the inner and outer liquid films are very similar to those of outer wall heat flux on the velocity profiles of outer and inner liquid films (Fig. 6c and d). That is, the change of inner wall heat flux will almost have no effects on the liquid layer velocity. As observed in Fig. 6, the velocity gradient of the outer liquid layer is larger than that of the inner liquid layer, and the velocity gradient in the liquid layer will increase with an increase of gap. The influence of wall heat flux on velocities becomes small with increasing the gap size.

Fig. 7 shows the liquid layer thickness will decrease along axial length. The inner liquid layer is thinner than the outer liquid layer under same conditions. Because: (1) in single-phase flow, both in the laminar and turbulent flow regimes, \( \tau_i > \tau_o \) (Knudsen and Katz, 1958). This is also assumed to be true in two-phase annular flow at the liquid/vapor interface (Doerffer et al., 1997). (2) \( D_{\text{lv}} > D_{\text{g}} \) (Saito et al., 1978; Doerffer et al., 1997). The thickness will increase with increasing annular gap with same mass flow rate, pressure, and heat flux. It will decrease with increasing heat flux with same mass flow rate. The influences of outer wall heat flux on inner wall heat flux on the inner liquid layer thickness or outer liquid layer thickness are very slight. These are similar to the influences of heat flux on temperature and velocity in the liquid layers. With same mass flow rate and an increase of wall heat flux, the liquid layer which covers on the correspondent wall will clearly become thin due to the increase of evaporation. At the same time, the velocity of vapor core will increase, the drag force to liquid film will become large, therefore the liquid film covers on the another wall will become thin.

Fig. 8 shows the effects of heat flux on two-phase heat transfer coefficient and the curves of heat transfer coefficient along axial length. The two-phase heat transfer coefficient increases along the axial length. The liquid layer will become thin and the temperature gradient in the liquid layer will decrease along the axial length, thus the heat transfer coefficient will increase continuously with same heat flux. As described above, the thickness of liquid layer and temperature gradients in the liquid layer will decrease with increase of heat flux, thus, the heat transfer coefficient will increase with increasing heat flux. The two-phase heat transfer coefficient of outer wall will increase obviously with increasing outer wall heat flux, while the amplitude of increase will increase with increasing heated length. The effect of outer wall heat flux on inner wall heat transfer coefficient is very small. Similarly, the inner wall heat transfer coefficient will increase with increasing inner wall heat flux. And its effect on outer wall heat transfer coefficient is also slight. The heat transfer coefficient will increase with
Fig. 9: Effects of critical quality on critical heat flux. (a) Effects of critical quality on CHF occurs at inner tube with $P = 3$ MPa. (b) Effects of critical quality on CHF occurs at outer tube with $P = 3$ MPa. (c) Effects of critical quality on CHF occurs at inner tube with $P = 5$ MPa. (d) Effects of critical quality on CHF occurs at outer tube with $P = 5$ MPa. (e) Effects of critical quality on CHF occurs at inner tube with $P = 10$ MPa. (f) Effects of critical quality on CHF occurs at outer tube with $P = 10$ MPa.
Fig. 9. (Continued).
decreasing annular gap with same mass flow rate and heat flux. Therefore, the smaller the annular gap, the stronger heat transfer.

6. Predicting for critical heat flux (CHF)

The present model may be used to predict CHF in the narrow annuli with bilateral heating. The heat flux at \( \delta = 0 \) will be the CHF. Although the CHF occurs when the liquid film is enough thin and breaks down easily around the dry patches, it is reasonable to assume that the CHF occurs when the liquid film vanishes, that is, \( \delta = 0 \). The local liquid film mass flow rate will be zero at \( \delta = 0 \). Based on this conception, the CHF may be predicted.

Fig. 9 plots the CHF as a function of critical quality in five different annuli gap and for three values of pressure. At a fixed value of \( q_o \), the CHF of the inner wall increases with the decrease of the critical quality (Fig. 9a, c, and e). At the same time, Fig. 9a, c, and e show the effects of the outer wall heat flux on the CHF of the inner wall. At a fixed CHF of the inner wall, the critical quality increases with the increase of the outer wall heat flux. With the increase of the outer wall heat flux, the more liquid in the liquid films will evaporate. Thus, the critical quality will increase.

Fig. 9b, d, and f show the profiles of the outer wall CHF with the change of the critical quality. The results are similar to those of the inner wall. As shown in Fig. 9, the critical quality decreases with the increase of the gap width. In the same annular channel, at a certain CHF, the critical quality when CHF occurs at the outer wall is larger than that when CHF occurs at the inner wall.

Fig. 10 plots the critical quality as a function of mass flow rate in five different annuli gap and for two values of pressure. For the same outer wall heat flux and inner wall heat flux, the critical quality decreases with the increase of the mass flow rate. The velocity of the vapor core will increase with the increase of the mass flow rate, and the shear stress on the liquid/vapor interface will increase, the more liquid droplets will be entrained into the vapor core. Therefore, the critical quality will decrease. Fig. 10 also describes the effect of the annular gap width on the critical quality. For the same mass flow rate, the critical quality decreases with increase of the annular gap width. The larger the gap width of the annular channel, the thicker the liquid films on the channel walls. Under the same conditions, the liquid film in the large gap is thicker than that in the small gap, the vapor core can more easily entrain the liquid droplets from the liquid film into the vapor core. Thus, the critical quality will decrease.
Fig. 11 shows the effects of the pressure on the critical quality. Under lower mass flow rate conditions, the critical quality increases slowly with the increase of pressure. While for higher mass flow rate, the critical quality increases slightly with the increase of pressure and then has a maximum value at about 5 MPa. Above this pressure, critical quality decreases slowly with the further increase of pressure.

Fig. 12 shows the effects of tube radius (curvature of heating surface) on CHF with the same annular gap size (2 mm) under the same pressure and same mass flow rate conditions, in which \(D_{IN}\) and \(D_{OUT}\) are the outer diameter of inner tube and the inner diameter of outer tube, respectively. Under the same CHF conditions, the larger the tube radius, the smaller the critical quality. Curvature of the heating surface usually has a significant effect on distribution of liquid film. Curvature radius is its radius for circular tube. Concave surface which has smaller radius keep liquid film on the surface much
better than that which has larger radius or convex surface.

7. Conclusions

A separated flow model of annular upward flow in a vertical annuli gap with bilateral heating is presented in this study. By numerically solving the proposed model, the liquid film thickness, velocity, and temperature fields in the liquid layer, two-phase heat transfer coefficient and pressure gradient can be obtained. The following conclusions have been found through the analysis of the results of this model:

(1) With same mass flow rate and heat flux, the thickness of liquid film in the annular narrow channel will decrease with decreasing the annular gap, the two-phase heat transfer coefficient will increase with the increase of heat flux and the decrease of the annular gap. That is, the heat transfer will be enhanced with small annular gap.

(2) The effects of outer wall heat flux on velocity and temperature in the outer liquid layer, thickness of outer liquid film and outer wall heat transfer coefficient are clear and obvious. The effects of outer wall heat flux on velocity and temperature in the inner liquid layer, thickness of inner liquid film and the inner wall heat transfer coefficient are very small, the similar effects of the inner wall heat flux are found.

(3) Under same conditions, the outer liquid film is thicker than the inner liquid film.

(4) The present model can be used to calculate the CHF and critical quality in the narrow annular gap.

Appendix A. $\rho_c$, $\alpha_c$, and $u_c$

The momentum conservation equation for vapor core can be written as:

$$\frac{dP}{dz} = -\left(\frac{dP}{dz}\right)_{fric} - \left(\frac{dP}{dz}\right)_{acc} - \left(\frac{dP}{dz}\right)_{elev}$$

$$= \frac{1}{\rho_c \alpha_c} \left[ \frac{d}{dz} \left( \rho_c \alpha_c u_c^2 \right) \right] + \rho \dot{g}$$

(A.1)
where, \(-\frac{dP}{dZ}\) is the total pressure drop; 
\[-(\frac{dP}{dZ})_{hac}\] is the frictional pressure drop due to the frictional loss; 
\[-(\frac{dP}{dZ})_{ac}\] is the acceleration pressure drop due to the momentum change; and 
\[-(\frac{dP}{dZ})_{elev}\] is the elevation pressure drop due to the effect of gravitational force field. \(\rho_c, u_e,\) and \(u_c\)

are the density, void fraction, and average velocity of vapor core, respectively.

The frictional pressure drop can be obtained by:

\[- \left( \frac{dP}{dZ} \right)_{frict} = \frac{2\pi(\rho_c + \rho_v)\nu_c + 2\pi(\rho_c - \rho_v)\nu_v}{\nu_c - \nu_v} \]  

(A.2)

The elevation pressure drop can be calculated by:

\[- \left( \frac{dP}{dZ} \right)_{elev} = \rho_c g = \rho_c \left[ \frac{\rho_c (1 - x)}{x} + \frac{\rho_c (1 - x)}{\rho_v / \rho_c} \right] \times g \]  

(A.3)

where, \(x\) is equilibrium quality.

\[- \left( \frac{dP}{dZ} \right)_{acc} = \frac{G^2}{\alpha_c} \frac{d}{dZ} \left[ \frac{\rho_c (1 - x)}{x} + \frac{\rho_c (1 - x)}{\rho_v / \rho_c} \right] \]  

(A.4)

\[- \left( \frac{dP}{dZ} \right)_{ac} = \frac{1}{\alpha_c} \frac{d}{dZ} \left[ \frac{\rho_c (1 - x)}{x} + \frac{\rho_c (1 - x)}{\rho_v / \rho_c} \right] \]  

(A.5)

where, \(G\) is mass flow velocity.

\[ \frac{dA}{dZ} = \frac{d}{dZ} \left[ \frac{x^2 + \rho_c (1 - x)\frac{\rho_c}{\rho_v} m}{\rho_c \alpha_c} \right] \]  

(A.6)

Eqs. (A.4) and (A.6) are substituted into Eq. (A.5), we get:

\[ \frac{dA}{dZ} = \frac{1}{\rho_c \alpha_c} \left[ \frac{\rho_c (1 - x)}{x} + \frac{\rho_c (1 - x)}{\rho_v / \rho_c} \right] \]  

(A.7)

\[ \frac{dA}{dZ} = \frac{1}{\rho_c \alpha_c} \left[ \frac{\rho_c (1 - x)}{x} + \frac{\rho_c (1 - x)}{\rho_v / \rho_c} \right] \]  

(A.8)

Eqs. (A.8) and (A.9) are substituted into Eq. (A.7), we get:

\[ \frac{dA}{dZ} = \frac{1}{\rho_c \alpha_c} \left[ \frac{\rho_c (1 - x)}{x} + \frac{\rho_c (1 - x)}{\rho_v / \rho_c} \right] \]  

(A.9)

The following assumptions are made to calculate the void fraction in the vapor core:

(1) There is no slip between the vapor and the droplets, as described in Section 2;
(2) Keep thermodynamic balance between the vapor and the droplets. The mass quality can be obtained by energy balance;
(3) The dynamic pressure of liquid phase is equal to that of vapor core. This assumption is also called as equivalent velocity head model. Then,

\[ \left( \frac{dA}{dZ} \right)_{l} = \frac{1}{\rho_l} \]  

(A.10)

\[ \left( \frac{dA}{dZ} \right)_{v} = \frac{1}{\rho_v} \]  

(A.11)
Based on its definition, \( \psi \) can be expressed as:

\[
\psi = (W)_l/(W)_l + (W)_g
\]  
(A.12)

where, \((W)_l\) is the liquid film mass flow rate.

Also, based on its definition, the mass quality can be written as:

\[
x = \frac{W_g}{(W)_l + (W)_g}
\]  
(A.13)

Eq. (A.4) is substituted into Eq. (A.11), then

\[
\alpha = \frac{\rho_l}{\rho_g} \left( d/\alpha \right)
\]  
(A.14)

If the total flow area is denoted as \( A \), it may be the sum of the flow areas of every part: liquid film, droplets, and vapor. That is:

\[
\psi = (W)_{fl} + (W)_l + (W)_g
\]  
(A.15)

Based on its definition of void fraction, it can be written as:

\[
\alpha = \frac{A_g}{A} = \frac{W_g/(\rho_g \alpha_g)}{A} = \left[ 1 + \frac{\rho_g}{\rho_f} \right] \left( \frac{1 - \psi}{\alpha_g} + (1 - \psi) \frac{\rho_f}{\rho_g} \left( \frac{1 - \alpha}{\alpha} \right)^{-1} \right)
\]  
(A.16)

The void fraction in vapor core can be obtained by:

\[
\alpha_v = \frac{A_v}{A} = \frac{1 + (\rho_f/\rho_l)(1 - \alpha)/x}{1 + (\rho_f/\rho_l)(1 - \alpha)/x}
\]  
(A.17)

\[
\alpha_v = \frac{d(\alpha)_{DD}}{dx} = \frac{d(\alpha)_{DD}}{dx} = (\alpha)_{DD} \frac{d(\alpha)_{DD}}{dx}
\]  
(A.18)

where,

\[
\frac{d(\alpha)_{DD}}{dx} = -\frac{\rho_f}{\rho_l} \frac{1}{x^2}
\]  
(A.19)
\[
\frac{d(CC)}{dx} = (EE) \frac{d(BB)}{dx} + (BB) \frac{d(EE)}{dx} \tag{A.20}
\]
\[
\frac{d(EE)}{dx} = -(1 - \phi) \rho_f \frac{1}{\rho_l} \frac{1}{x^2} \tag{A.21}
\]
\[
\frac{d(BB)}{dx} = \frac{1}{2} \left[ \frac{(\rho_f/\rho_l) - \phi}{x} \left| \left[ \frac{(\rho_f/\rho_l) - \phi}{x} + \frac{1}{x} \right] - \left[ \frac{(\rho_f/\rho_l) - \phi}{x} + \frac{1}{x} \right] \left| \left(1 - \phi\right)/\left[1 + (1 - \phi)\right] \right| \right| \right]^{1/2} \tag{A.22}
\]

Eqs. (A.21) and (A.22) are substituted into Eq. (A.20), then we get
\[
\frac{d(CC)}{dx} = \frac{1}{2} \left[ \frac{(\rho_f/\rho_l) - \phi}{x} \left| \left[ \frac{(\rho_f/\rho_l) - \phi}{x} + \frac{1}{x} \right] - \left[ \frac{(\rho_f/\rho_l) - \phi}{x} + \frac{1}{x} \right] \left| \left(1 - \phi\right)/\left[1 + (1 - \phi)\right] \right| \right| \right]^{1/2} \tag{A.23}
\]
\[
\frac{d(DD)}{dx} = \frac{d(AA)}{dx} + \frac{d(CC)}{dx} \tag{A.24}
\]

References


Krause, J.F., Chao, B.T., Soo, S.L., 1991. An improved correlation for two-phase frictional pressure drop in boiling and