Nondestructive Testing and Evaluation

Some advances in numerical analysis techniques for quantitative electromagnetic nondestructive evaluation

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Some advances in numerical analysis techniques for quantitative electromagnetic nondestructive evaluation

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In this paper, some progresses in numerical techniques mainly made in our research group for the forward and inverse simulation of electromagnetic nondestructive evaluation (ENDE) signals are introduced.

For the first part, efficient forward analysis schemes for the simulation of eddy current testing (ECT), remote field ECT (RFECT) and magnetic flux leakage testing (MFLT) signals are described respectively, in addition to some numerical examples. Fast and accurate ECT signal simulation is realised by introducing a database type strategy using precalculated unflawed potential field data. To meet the high accuracy requirement of the simulation of RFECT signals, a hybrid scheme using 2D and 3D geometry and a new formula for pickup signal are proposed. To improve the efficiency of MFLT signal simulation, a fast scheme is developed based on a FEM–BEM hybrid code of polarisation method. In addition, a phenomenological method is also described in the first part, which is developed for the qualitative estimation of eddy current distribution and pickup signals.

The second part of this review paper is on the reconstruction of defect from the detected ENDE signals (mainly ECT signals). Reconstruction schemes based on conjugate gradient (CG) method of deterministic category and NN method, metaheuristic methods of stochastic category are developed and sizing of both artificial and natural cracks are performed by using measured signals. It is clarified through applications that a deterministic optimisation method is more efficient for treating simple cracks, while a stochastic way is prefer for defects of complicated geometry such as a stress corrosion crack and multiple cracks. In the crack modelling and parameterisation, an element of discontinuous material property is introduced to treat crack of arbitrary shape based on a given regular mesh. Several numerical models are proposed for natural cracks, which makes the reconstruction of some natural cracks become possible.

Keywords: ECT; numerical simulation; forward analysis; inverse analysis

1. Introduction

Electromagnetic nondestructive evaluation (ENDE) methods, a group of NDT techniques based on electromagnetic phenomena, play important roles in the safety guarantee of metal structural components. Upgrades of probes and inspection systems are being conducted in order to improve the inspection quality and to cover the new needs of industries. Recently, not only the detection, but also the sizing of detected defect become necessary because the evolution behaviours of defect are very important in the optimal structure maintenance.

In the development of new probe and new defect sizing scheme, numerical tools for ENDE signal simulation are indispensable. During the probe development, both the selection of probe

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configuration and the probe size optimisation need a lot of forward analyses to be performed in order to understand the feature of eddy current perturbation and the effect of probe parameters. On the other hand, an optimisation approach is usually adopted in a defect reconstruction strategy to minimise the residual error. For either a deterministic optimisation method or a stochastic way, simulations of ENDE signals due to defects of different profiles are necessary. Therefore, numerical tools of high efficiency and accuracy are very important for the further development of ENDE techniques [1].

The numerical simulation of ENDE signals is a typical electromagnetic (EM) field problem. For instance, the ECT is a low frequency, linear EM field problem, while the simulation of magnetic flux leakage testing (MFLT) signal is a nonlinear EM problem, in which a nonlinear constitutive relation and hysteresis property have to be taken into account. In recent years, many numerical methods have been developed for the ENDE problem and were validated through benchmark problems [2–11]. However, as the ENDE signal simulated is a 3D FEM problem, the computational burden is heavy, especially for the iteration procedure used in the crack reconstruction. In addition, simulation of RFECT signals needs extremely high simulation accuracy and it was difficult with the conventional FEM codes. To solve these problems, efforts have been made in our group to develop new efficient numerical schemes.

On the other hand, to realise the reconstruction of practical cracks from the measured signals, in addition to a proper way for the inversion, the modelling of natural cracks, parameterisation, efficient calculation of gradient functions etc. are also important. Based on the efficient forward solver, improved inverse analysis scheme, proper numerical model, reconstructions of multiple cracks, some SCC and fatigue cracks are successfully realised from measured ECT signals.

In this paper, efficient forward analysis schemes for the simulation of ECT, RFECT, and MFLT signals are given, respectively, with some related numerical examples. At first, a database type scheme for fast and accurate ECT signal simulation is depicted. Then, a hybrid approach using 2D field database is introduced for simulating RFECT signal due to 3D defect. For the simulation of RFECT signals, a new formula that enables the direct calculation of RFECT signals with conventional FEM code is also given. For the MFLT problem, a similar fast scheme is introduced to enhance the performance of the FEM–BEM hybrid code of polarisation method. In addition to the precise numerical analysis methods, a phenomenological method, which is developed for the qualitative estimation of eddy current distribution and ECT pickup signals, is then reported.

In the second part of the paper, the algorithms for the reconstruction of artificial cracks, natural SCCs, closed fatigue cracks and multiple cracks are introduced respectively. Modelling of natural cracks, schemes using CG method of deterministic category and NN method, metaheuristic methods of stochastic category are presented. Reconstruction results of measured ECT signals and the true crack profile observed after destruction are compared for several typical cracks to support the validity of the proposed algorithms.

This paper is arranged as follows: in the following section, the efficient forward analysis methods are introduced for ECT, RFECT, MFLT respectively. The phenomenological way for ECT problem analysis is given in the last part of this section. In section 3, algorithms and examples for crack reconstruction are presented. The concluding remarks and some prospects are given in the last section.

2. Efficient forward analysis schemes for ENDE signal simulation

2.1 A database strategy for fast simulation of ECT signals

A numerical model of ECT problem is shown in Figure 1. The symbol Ω corresponds to the whole unflawed conducting object, Ωc is the region of crack and Ω0 is a selected region in regular
shape that contains the crack. If, we focus on the problem of a slit-like crack, one can choose $\Omega_0$ as a regular shape with a thickness equal to the crack opening, and subdivide it into a grid of small cells, as shown in the figure. In this case, the eddy current perturbation due to crack of arbitrary shape can be evaluated rapidly by using the following knowledge-based scheme [12].

2.1.1 Basic formulation

Upon subtracting the $A - \phi$ formulae about $A^u$, $\phi^u$, the vector and scalar potentials of the unflawed conductor, from those about $A$, $\phi$, the potentials of a conductor with flaw present, one can obtain the following governing equations about the field perturbation $A^f$, $\phi^f$ for a low frequency eddy current problem. In conductor region:

$$\frac{1}{\mu_0} \nabla^2 A^f - \sigma_0 (\nabla A^f + \nabla \phi^f) = -[\sigma_0 - \sigma(\mathbf{r})](\nabla A + \nabla \phi),$$  \hspace{1cm} (1)

$$\nabla \cdot [\sigma_0 (\nabla A^f + \nabla \phi^f)] = \nabla \cdot [(\sigma_0 - \sigma(\mathbf{r})) (\nabla A + \nabla \phi)],$$  \hspace{1cm} (2)

In air region:

$$\frac{1}{\mu_0} \nabla^2 A^f = 0,$$  \hspace{1cm} (3)

where, $A^f = A - A^u$, $\phi^f = \phi - \phi^u$ are the potential perturbations due to the presence of a crack, $\sigma_0$ is the conductivity of the host material and $\sigma(\mathbf{r})$ is the distribution of conductivity that is equal to 0 in crack region and $\sigma_0$ in the conductor. As $[\sigma_0 - \sigma(\mathbf{r})]$ vanishes in the conducting area, after FEM–BEM discretisation, the equations (1)–(3) reduce to

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} q_1^f \\ d_2^f \end{bmatrix} = \begin{bmatrix} K_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_1^f + q_1^u \\ q_2^f + q_2^u \end{bmatrix},$$  \hspace{1cm} (4)

where, $\{q^f\} = (A^f, \phi^f)^T$, $\{q^u\} = (A^u, \phi^u)^T$ are the potentials at every node, $[K]$ is the coefficient matrix of the unflawed conductor, and $[\bar{K}]$ is the coefficient matrix corresponding
to the terms with \([σ_0 − σ(r)]\). The unknown potential vector \(\{q\}\) was divided into two parts: \(\{q_1\}\) are the potential values at the node of crack element and \(\{q_2\}\) are the remained unknowns. As \([σ_0 − σ(r)]\) vanishes in region \(Ω − Ω_c\), \([K]\) is written as in equation (4) where the nonzero submatrix \([K_{11}]\) corresponds to the cells at the crack region. Multiplying equation (4) left by \([H] = [K]^{-1}\), we find,

\[
\begin{pmatrix}
  q_1^f \\
  q_2^f \\
\end{pmatrix} =
\begin{bmatrix}
  H_{11} & H_{12} \\
  H_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
  K_{11} & 0 \\
  0 & 0
\end{bmatrix}
\begin{pmatrix}
  q_1^f + q_1^u \\
  q_2^f + q_2^u
\end{pmatrix},
\]

(5)

The equations related to the unknowns \(\{q_1^f\}\) can be simply separated from the equation (5). If we denote \([H_{11}][K_{11}]\) as \([G]\), and the unit matrix as \([I]\), the relation connecting the unflawed field to the field perturbation in region \(Ω_c\) is finally deduced as follows,

\[
[I - G]\begin{pmatrix}
  q_1^f \\
\end{pmatrix} = [G]\begin{pmatrix}
  q_1^u \\
\end{pmatrix},
\]

(6)

As the unknowns of equation (6) are limited to the crack region \(Ω_c\), calculation of the disturbed field \(\{q_1^f\}\) using this equation is much faster than the conventional FEM–BEM code. This increment of speed comes from the significantly reduced number of unknowns. The information about the conductor and exciting coils are contained in the coefficient matrices \([H_{11}]\) and \(\{q_1^u\}\), that can be calculated \textit{a priori} as they are independent of the crack. Thus, once these fields were calculated and stored in databases, there is no need to compute them again in the actual calculation of crack fields. Consequently, the computational burden can be reduced significantly.

The basic difference between the equation (6) and the conventional schemes using VIM is the different type of unknowns. The application of the potentials makes the interpolation of eddy currents to be of a higher order and makes a new FEM element containing crack edge applicable (this will be explained in 2.1.4).

2.1.2 Establishment of databases of unflawed field information

From ECT signal, some features of a crack such as the position, rough length and inner/outer property usually can be estimated with a classification method. Thus, it is not difficult to choose an area containing the possible crack (as region \(Ω_0\) in Figure 1) and to subdivide it into a grid of small cells. We calculate the unflawed field at \(Ω_0\) with the conventional FEM–BEM code with the excitation current in the probe or a unit potential located at one of the nodes in \(Ω_0\) respectively. Storing this unflawed information in databases, the matrices \([H_{11}]\) and \(\{q_1^u\}\) of equation (6), which are necessary for the fast evaluation of ECT signal, can be extracted directly from the database for any cracks belonging to the region \(Ω_0\). Here, we use the property that the field at the crack region excited by a unit source corresponds to a column of the inverse matrix \([H_{11}]\).

In the case of a conducting straight tube that is sufficiently long compared to the probe dimensions, the size of the databases can be greatly reduced by considering the shifting symmetry property of the eddy current in the material. If the exciting points are in the same layer (same radial coordinate), the fields excited at node of different axial (or circumferential) positions will be the same other than a shifted axial (or circumferential) coordinate. Thus, one only needs to store the field of sources setting at a radial line in the central cross section of the tube. The field due to a unit source at other nodes can be simply found from these data by shifting the coordinates.
2.1.3 Formula for pick-up signal

As the disturbed field $\{q^f\}$ can be considered as the unflawed field induced by current dipoles in crack region, it is convenient to calculate the pick-up signals using the reciprocity theorem [4]. For a self-induction pancake coil, the impedance change is written as,

$$\Delta Z = \frac{1}{I^2} \int_{\text{coil}} E^f \cdot J_0 dv = \frac{1}{I^2} \int_{\text{flaw}} E^u(E^f + E^u)(\sigma_0 - \sigma(\mathbf{r})) dv,$$

where $(\sigma_0 - \sigma(\mathbf{r}))(E^f + E^u)$ is the current dipoles in flaw region and $I$ is the current per turn in the coil. The calculation of the impedance with equation (7) requires very little computational work because it can be integrated with the help of the known element coefficient matrices directly. The electromotive force (EMF) of a mutual induction probe can also be evaluated by using a similar formula. In this case, the fields induced by a virtual current in pick-up coil have to be incorporated.

2.1.4 A special element for FEM discretisation of crack edge

As the database has to be established using the grid of given cells $a$ priori, only cracks make up of the cells can be treated by using these databases if homogeneous conductivity is required in each FEM element. However, as the permeability and permittivity of the nonmagnetic SG tubes are the same as those of free space, the electric charge at a conductor surface is negligibly small like the neglected displacement current for low frequency problem. Hence, the EM fields are continuous and the corresponding potentials $\mathbf{A}$, $\phi$ are approximately one order differentiable at the crack edge. In addition, with the fact that small grid cells are usually selected for subdividing $\Omega_0$, it is reasonable to apply the shape function of a normal FEM element in the interpolation of an unknown field even for a cell containing crack edge, i.e. with different media (Figure 2). Introducing such a new element will enable us to simply treat cracks of arbitrary shape in signal simulation. In such a case, the difference of the material is taken into account in the element coefficient matrices, as the integration over the whole element is reduced to the part related to the crack region.

![Figure 2. Concept of new element with different material.](image-url)
2.1.5 Numerical example of fast forward solver

In numerical calculation, the ECT signal of the axial scanning along the crack was evaluated for a straight tube model as an example. The size of the tube and the pancake coil were chosen as the same in step 3 of benchmark models [2]. The crack was chosen as an axial EDM notch with 0.2 mm width, though this method is suitable for other complex-shaped cracks as well. A region of dimensions $12 \times 1.27 \times 0.2$ mm along the axial direction was chosen as the possible crack region $\Omega_0$ and was subdivided into $24 \times 10 \times 1$ cells. Potentials at a region two times as long as the region $\Omega_0$ were calculated and stored in the database for the inverse matrix while a region with 32 mm length was used for the database of unflawed field for the axial scan from $-10$ to $10$ mm.

The impedance results for an elliptic crack are shown in Figure 3, with the exciting frequency applied as 300 kHz. The crack was located at the inner side, with a maximum depth of 60% of the tube wall thickness and a length of 10 mm. In order to compare with the measured data, the step 6 benchmark models [2] were considered. As depicted in the figure, good agreement was obtained. The small difference of two numerical results seems also to be caused by the mesh difference. The mesh used for FEM–BEM code was not as fine as that used for the present method.

2.2 A database type algorithm of hybrid 2D-3D geometry for accurate RFECT signal simulation

Though many 3D analysis techniques for ECT problems have been developed in recent years [2–5], they do not have enough precision for a 3D analysis of the RFECT problem of magnetic tube unless a very fine mesh is used, because a much higher numerical accuracy is necessary for the simulation of RFECT signals [8–10,15,17].

The database type fast forward solver can use a fine mesh in principle as its analysis region is localised in the defect region, and is usually very small, however, the database type fast solver is also not applicable for efficient RFECT signal simulation, because the unflawed potential data due to coils and unit sources has to be precalculated by using a 3D FEM code.

Figure 3. Comparison of the impedance results of the present method, the FEM–BEM code and the experiment for step 6 of benchmark problems.
However, as the excitation and pickup coils of an RFECT probe are mainly of bobbin type, the RFECT problem is usually in an axisymmetric geometry for an unflawed situation. Therefore, the unflawed potential database due to coils can be calculated by using a 2D axisymmetric code. In addition, though the database of unit sources has to be calculated by using a 3D code, the simulation precision can be assured because the field due to unit source is localised around the source point, which means a small analysis region is applicable. By using such a hybrid geometry strategy, i.e. calculating the database of coils using 2D code and calculating the database of unit potential with 3D code, the fast forward scheme of database type will become suitable for the simulation of the RFECT signal due to a 3D defect.

2.2.1 Numerical formulation of the hybrid approach

The governing equations of the RFECT problem are the same with the normal ECT problem. Therefore, the formulation presented in the last subsection is also valid for the RFECT problem. However, the FEM formulation of edge element will be adopted in the discretisation of the governing equations of Ar method [8] because the inspection target of RFECT problem is usually magnetic.

After subtracting the unflawed governing equations of Ar method from the flawed ones and applying the Galerkin’s discretisation, the following system of linear equations can be obtained for the RFECT problem:

\[
\begin{bmatrix}
\frac{1}{\mu_s} \sum_{e \in E_f} \nabla \times [N_e] \nabla \times [N_e]^T dV + j \omega \sigma \sum_{e \in E_f} [N_e]\cdot[N_e]^T dV \\
\frac{1}{\mu_a} - \frac{1}{\mu_0} \sum_{e \in E_f} \nabla \times [N_e] \nabla \times [N_e]^T dV + j \omega \sigma \sum_{e \in E_f} [N_e]\cdot[N_e]^T dV
\end{bmatrix}
\begin{bmatrix}
A_f \\
A_e
\end{bmatrix} = \begin{bmatrix}
0 \quad 0 \\
0 \quad 0
\end{bmatrix}
\begin{bmatrix}
A_f \\
A_e
\end{bmatrix},
\tag{8}
\]

where the suffixes \(f\) and \(u\) denote flawed and unflawed variables, \(\Omega_e\) is the volume of the corresponding hexahedral element, \(E_f\) and \(E_e\) are the edges at the defect and at the whole analysis region, \(j\), \(\omega\) and \(\sigma\) are the imaginary unit, frequency and conductivity, \([N_e]\), \([A_e]\) are the shape function matrix and the vector of magnetic potential respectively. In this work, the edge elements are chosen as the edges of the hexahedral volume elements.

As the right hand is vanished outside the crack region, equation (8) can be rewritten as the following matrix form by separating the potential vector \([A]\) into subvector \([A_1]\) the potentials in the crack region, and subvector \([A_2]\), the potentials at the other edges:

\[
\begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix}
\begin{bmatrix}
A_1' \\
A_2'
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix},
\tag{9}
\]

where \([P]\) is the coefficient matrix for the unflawed material, and \([Q]\) is the coefficient matrix related to the crack.

From equation (9), one can obtain the system of equations for \([A_1]\)—the potential in the crack region when the crack presents as:

\[
[I - R_{11}Q_{11}][A_1] = \{ A_1^u \},
\tag{10}
\]

where \([R]\) is the inverse matrix of the matrix \([P]\) and \([I]\) the unit matrix.
Equation (10) shows that if the unflawed potential \( A_u^1 \) due to the exciting coil and the inverse matrix \([R]\) (the potential due to unit sources) are known, \( A_1 \) can be solved with a very small computational burden as the problem is limited at the crack region.

In case of a RFECT problem, the calculation of database for \( A_u^n \) is a typical axisymmetric problem if the exciting coil is in a bobbin geometry. With use of an axisymmetric code, the database \( A^1 \) can be calculated with a high precision. On the other hand, the database \([R]\) can be obtained by calculating the potential at each crack element due to a unit source at one of the crack elements. A 3D code can give an accuracy good enough for these calculations as a small analysis region is applicable.

For the signal evaluation, the approach based on the reciprocity theorem has been applied in the database approach. However, only the formulation for a nonmagnetic material has been presented in Ref. [12]. For a magnetic material, the term of field due to the magnetisation is also necessary. Based on the formulation given in Ref. [6], the pickup signal for a magnetic material is found as:

\[
\Delta V = \frac{-\omega^2 \sigma^a}{I_p} \int_{\Omega_{\text{line}}} A_u^p \cdot A^1 dV + \frac{j\omega}{I_p} \left( \frac{1}{\mu^a} - \frac{1}{\mu_0} \right) \int_{\Omega_{\text{line}}} \left( \nabla \times A_u^p \right) \left( \nabla \times A^1 \right) dV,
\]

where \( A_u^p \) is the vector potential induced by a virtual current \( I_p \) in the pickup coil. To apply equation (11), in addition to the database for the exciter coil, a database for potentials due to a virtual current in the pickup coils is also necessary. For a pickup coil of bobbin type, the method for the exciting coil can be applied directly.

In practice, the 3D simulation is performed by using an edge element FEM code, while the 2D simulation is by a nodal element code. To calculate the potentials for edge element from the 2D nodal data, the following formula is applicable:

\[
A_{\text{edge}} = \frac{1}{2} \left( A_{\text{node}1} + A_{\text{node}2} \right) \cdot t,
\]

where \( A_{\text{node}} \) is the vector potential on an edge element, \( A_{\text{node}} \) is the vector potential at a node, \( t \) is the unit direction vector, and \( l \) is the length of the edge element.

### 2.2.2 Validation of the hybrid strategy

Based on the proposed scheme, a numerical code is developed, and the RFECT signals of problem shown in Figure 4 is calculated. The RFECT probe consists of one bobbin type exciting coil and two bobbin pickup coils and its output is the difference of the pickups. The conductivity

![Figure 4. Calculation model for validation.](image-url)
and the magnetic permeability are taken as $3 \times 10^6$ S/m and 100. The inner and outer diameters of the tube are 23.4 and 31.8 mm respectively. The excitation frequency is chosen as 500 Hz.

Figure 5 depicts a comparison of the signal due to a full length circumferential outer wall thinning defect (length: 10 mm, depth: 50% of the tube wall thickness) calculated by using the 2D axisymmetric code and the hybrid code newly developed. A good agreement was found. As the precision of the 2D code is known as high accuracy tool for RFECT simulation, it is not difficult to conclude that the proposed approach is efficient.

2.3 New EMF formula for RFECT problem

In this subsection, a new formula for the pickup signal calculation, which enables the 3D edge element FEM code of Ar method to be applied to the 3D RFECT problem, is introduced. The validity of the formulation and the implementation are verified through comparing numerical results of the 3D Ar code with the measured signals and with those of the axisymmetric code [16].

The numerical results show that, although both the conventional formulation and the proposed way are valid for a nonmagnetic tube, the upgraded code gives a more satisfactory simulation for a tube of ferromagnetic material. It is also proved that the upgraded Ar method code is high efficiency in the 3D simulation of the RFECT problem.

2.3.1 New EMF formula for RFECT problem

In the Ar method, the reciprocity theorem is also valid for calculating the total signal. In fact, by substituting

\begin{align}
\mathbf{J}_e &= -j \omega \sigma \mathbf{A}_r, \\
\mathbf{E}_s &= \frac{1}{\sigma} \nabla \times \frac{1}{\mu} \nabla \times \mathbf{A}_s,
\end{align}

Figure 5. Comparison of the results of the 2D code and the hybrid database approach (full length circumferential outer defect).
\[ V = -\frac{1}{I_s} \int_{V_s} J \cdot E \, dv \] (15)

and taking into account that \( \mathbf{A}_s = \mathbf{A} - \mathbf{A}_s \), one can obtain

\[ V = -\frac{j \omega}{I_s} \left[ \int_{V_s} \mathbf{A} \cdot \nabla \times \frac{1}{\mu} \nabla \times \mathbf{A}_s \, dv - \int_{V_s} \mathbf{A}_s \cdot \nabla \times \frac{1}{\mu} \nabla \times \mathbf{A}_s \, dv \right]. \] (16)

With use of the Green's first formula and the FEM discretisation, the following equation can be derived further,

\[ V = -\frac{j \omega}{I_s} \left[ \int_{V_s} (\nabla \times [\mathbf{N}]) \cdot (\nabla \times [\mathbf{N}]) \, dv \, \mathbf{A}_s \right] + \frac{1}{\mu} \int_{S_s} ([\mathbf{N}] \times \mathbf{H}_s) \cdot \mathbf{n} \, ds \} \{ A \} \]

\[ -\frac{j \omega}{I_c} \left[ \int_{V_c} (\nabla \times [\mathbf{N}]) \cdot (\nabla \times [\mathbf{N}]) \, dv \, \mathbf{A}_s \right] + \frac{1}{\mu} \int_{S_c} ([\mathbf{N}] \times \mathbf{H}_p) \cdot \mathbf{n} \, ds \} \{ A \}. \] (17)

Equation (17) has been proved efficient for calculating the pickup signal of a self-induction ECT sensor [23]. From the derivation procedure, however, it is not difficult to find that the equation is also applicable for a mutual induction probe. In equation (17), the information of the pickup coil is represented by the field \( \mathbf{A}_s \) and \( \mathbf{H}_s \) in case of a self-induction probe. Therefore, if one changes \( \mathbf{A}_s \) and \( \mathbf{H}_s \) of equation (17) into the magnetic vector potential \( \mathbf{A}_p \) and the magnetic field intensity \( \mathbf{H}_p \) induced by a virtual unit current source flowing in the pickup coil, equation (17) will be valid for a mutual induction probe. On the other hand, the third and the fourth terms of the right hand of equation (17) can be neglected for defect signal calculation as it is independent of the inspection target. Based on these considerations, the crack signal of a mutual induction sensor can be written as,

\[ V = -\frac{j \omega}{I_c} \left[ \int_{V_c} (\nabla \times [\mathbf{N}]) \cdot (\nabla \times [\mathbf{N}]) \, dv \, \mathbf{A}_s \right] + \frac{1}{\mu} \int_{S_c} ([\mathbf{N}] \times \mathbf{H}_p) \cdot \mathbf{n} \, ds \} \{ A \}. \] (18)

2.3.2 Validation by comparing with measured RFECT signals

The conventional 3D edge element code of Ar method [8] is updated based on equation (18). The validity of the new formula and the upgraded code are verified by calculating a problem with experimental results. Figures 6 and 7 show simulation results of frequency 150 Hz with comparison to the measured ones for a full length circumferential wall thinning defect and a local wall thinning defect respectively. From Figures 6 and 7, one can see that the simulated and the measured signals are in a satisfactory agreement.

2.4 A fast scheme for MFLT signal calculation

For the MFLT problem [14,18], there is also a need for an efficient signal simulator to evaluate probe performance and to reconstruct the defect based on an optimisation approach. Simulation of MFLT problem is more difficult as the large applied magnetic field may cause nonlinearity in the problem. In addition, the hysteresis behaviours also have to be considered in order to calculate the residual magnetisation. Recently, a FEM−BEM polarisation method has been proposed for the MFLT problem [11]. Because an iteration procedure is employed in the
In the following, a fast scheme to improve the FEM–BEM polarisation method is presented.

2.4.1 The fast scheme to predict the perturbation of magnetic flux leakage

The new scheme is based on the same basic idea applied in the previous sections, that can predict the perturbation of the MFLT signals with greatly reduced computational resources but can keep a similar precision with the conventional method.

Figure 6. The RF sensor signal due to a full circumference outer wall thinning OD50, with the length of 5 mm, frequency of 150 Hz.

Figure 7. The RF sensor signal due to a local circumference outer wall thinning OD50, with the length of 5 mm and the circumference width of 59.5°, frequency of 150 Hz.
The governing equation for an unflawed inspection object can be written as follows by denoting $A^u, M^u$ as the magnetic vector potential and magnetisation vector in case no defect is presented.

In material:

$$\frac{1}{\mu_0} \nabla^2 A^u = \nabla \times M^u.$$  \hfill (19)

In air:

$$\frac{1}{\mu_0} \nabla^2 A^u = -J_0.$$  \hfill (20)

Subtracting equations (19) and (20) from the corresponding equations of object with flaw, one obtains:

In material:

$$\frac{1}{\mu_0} \nabla^2 A^f = \nabla \times M^f.$$  \hfill (21)

In air:

$$\frac{1}{\mu_0} \nabla^2 A^f = 0.$$  \hfill (22)

where, $A^f = A - A^u$ is the difference of the vector potentials in cases with a crack and without a crack, and $M^f = M - M^u$ is the difference of the magnetisation vectors in cases with a crack and with the crack absent. Moreover, the B–H curve of the material was written in a scalar form of $H = F(B)$ that can express the nonlinear property of the material. In other words, the constitutive relation in terms of $M^f$ can be written as

$$M^f = \frac{B^f}{\mu_0} + F(B^u) - F(B^u + B^f).$$  \hfill (23)

Discretising equation (21) with finite element method, equation (22) with boundary element method and coupling the discretised equations with the boundary conditions, one can obtain the discretised system equations of equations (21) and (22) as

$$[K][A^f] = [S][M^f],$$  \hfill (24)

where $[K], [S]$ are the coefficient matrices again.

For a MFLT problem of yoke type magnetiser and pickup sensor of Hall element without magnetic core, the perturbation of the magnetic field (the difference of the fields with and without flaw) is significant only at the region in the inspection target near the crack. Figure 8 shows an example of numerical results of the distribution of the magnetic field perturbation due to a crack in a plate of magnetic material when the yoke poles are far from the crack. One can observe that the perturbation due to the presence of a crack is localised around the vicinity of the crack. Therefore, it is not necessary to solve the equations (21) and (22) with the same analysis region for equations (19) and (20). A reduced analysis region can be used for the discretisation of equations (21) and (22), which enables significant reduction of computational time.
On the other hand, as $\mathbf{B}^0$, $\mathbf{M}^0$ are needed in the constitutive relation, the magnetisation distribution in the unflawed material is required to solve the equation (24) with the polarisation method. In other words, a forward problem for calculating the unflawed field data needs to be solved in order to reduce the analysis region with the scheme described above. Fortunately, once the magnetic field of the unflawed material has been calculated, it is not necessary to be recalculated again for treating a different crack. This feature is a very important factor to apply the present fast scheme into the model based inverse analysis scheme or into the database construction of the Neural Network approach that needs a fast prediction of MFL signals for cracks with different shapes.

2.4.2 Numerical examples

In Figure 9, results using the conventional code with the full analysis region are compared with the results using the proposed scheme with reduced analysis regions. The distribution of the $x$ component of the leakage magnetic field at a line ($y = 0$ mm, $z = 0.5$ mm) are illustrated in the figure. The CPU times for the three reduced regions are respectively about 3, 5 and 8 min while it is about 3 h for the full system. From these results, one can find that the analysis results of the new method approach to the results of the full system when the reduced region is getting larger. Even for the smallest region in the three cases, the difference with the results of the conventional code is less than 10%. These results show that the proposed algorithm is feasible both from the point of view of the calculation speed and the numerical accuracy.

2.5 A phenomenological strategy for ECT analysis

During the design of a new probe, it is very important to know the pattern and distribution of induced eddy current for a selected coil arrangement. Conventionally, it is difficult to get a clear image about the eddy current distribution in terms of density and pattern from the conventional methods without complicated numerical computations. In this part, a phenomenological

Figure 8. An example of the distribution of the perturbation of magnetic field due to a crack.

On the other hand, as $\mathbf{B}^0$, $\mathbf{M}^0$ are needed in the constitutive relation, the magnetisation distribution in the unflawed material is required to solve the equation (24) with the polarisation method. In other words, a forward problem for calculating the unflawed field data needs to be solved in order to reduce the analysis region with the scheme described above. Fortunately, once the magnetic field of the unflawed material has been calculated, it is not necessary to be recalculated again for treating a different crack. This feature is a very important factor to apply the present fast scheme into the model based inverse analysis scheme or into the database construction of the Neural Network approach that needs a fast prediction of MFL signals for cracks with different shapes.
approach proposed to fulfill this requirement is described. The two key ideas of the method, i.e. a phenomenological relationship between the source magnetic field and the induced eddy current distribution, and a ring current model for eddy current perturbation description, are introduced in detail [13].

2.5.1 Relation between the source magnetic field and the induced eddy current

Through numerical calculations of the eddy current distribution for many frequencies and arrangements of excitation coils, it is found that the following simple relation exists for the induced eddy current and the applied external magnetic field when a crack and abrupt geometrical change are absent in the inspection target and the probe is placed at a position free of the edge effect:

\[
J_e(r, \theta, z, t) = a(r, \theta) \hat{n} \times B_0 \left[ \frac{r_0(\theta)}{r_{in}} \right] \cos \left[ \omega t + \phi(r, \theta, z) \right],
\]

where \( J_e(r, \theta, z, t) \) is the eddy current at a point \((r, \theta, z)\) within the conductor at a time \(t\), \( \hat{n} \) is the normal unit vector of the tube surface at point \((r_{in}, \theta, z)\), \( B_0 \) is the source magnetic flux density of the driver coil(s), \( r_{in} \) is the inner radius of the tube, and \( r_0(\theta) \) is a constant for a given frequency that needs to be determined empirically with use of numerical results. As described later, the outer radius of the tube is a good selection for frequencies usually used in the SG inspection (100–500 kHz).

In equation (25), the coefficients of the magnitude, \( \alpha \), and the phase difference of the eddy current density, \( \phi \), are approximated by the following formulae for the simple analytical solution of the eddy currents in a conducting half space,

\[
\alpha(r, \omega) = \alpha_0 e^{-\frac{(r-r_{in})}{\delta}},
\]

\[
\phi = \frac{0.75(r-r_{in})}{\delta} + \frac{\pi}{4} \left[ 1 - \cos \frac{d \pi}{6R} \right],
\]

where \( \delta = \left( \frac{2}{\omega \mu_0 \sigma} \right)^{1/2} \) is the skin depth, \( R \) is the outer radius of the driver coil, and \( d \) is the in-plane distance between the field point and the centre of the driver coil. Though \( \alpha_0 \) in equation (26) is an unknown constant, it will be cancelled during the S/N ratio calculation. The constants 0.75 and 6 in equation (27) were calibrated with use of the numerical results for a pancake probe.

Figure 9. Comparison of scanning signal (Box) along x-direction for the full system and those of different analysis areas (line \( y = 0, z = 0.5 \text{ mm} \)).
and were verified as suitable for other probes by comparing their numerical results in the conductor region near the centre of the excitation coil, and for the frequencies usually used in the inspection of SG tubing. As the pick-up coils are usually arranged around the centre of the excitation coil, equation (27) is especially suitable for the calculation of pick-up signals.

Figure 10 shows comparisons of the eddy current distributions calculated by the FEM–BEM hybrid code and by equation (25) at the layer near the outer surface of the tube. The real part of the eddy current distribution in a quarter of the tube are compared in (a) and (b). The eddy currents of 400 kHz were excited by a square pancake coil which was placed over the centre of the segment with the coil plane arranged perpendicularly to the surface of segment. Figure 10(c) is a comparison of the eddy current distribution along the line $y = 0$, where the eddy current is normalised. From these examples, one can see that the simplified calculation for the $J_e \sim B_0$ relation is valid for the tube geometry.

![Figure 10](image-url)

**Figure 10.** Eddy current distribution at the outside layer excited by a perpendicular square coil, (a) result by the FEM–BEM method (b) result using the simple relation (c) eddy current distribution on the line $y = 0$ (The x and y coordinates are normalised, 400 kHz, Tube geometry, coil side length = 3.0 mm).
2.5.2 Modelling of eddy current perturbation due to a crack

To consider the effect of the crack opening which cannot be neglected for shallow artificial cracks, a new model with two sets of ring currents is proposed in Figure 11 (hereafter, we call it the ring current model). The ring current set (b) in the figure is considered as the approximation of the eddy current perturbation perpendicular to the crack plane while the other set (c) is equivalent to the eddy current component parallel to the crack. The effect of the component $i_{e2}$ (see Figure 11), which flows in the two sides of the crack plane, was neglected because the corresponding current loops always exist in pairs and in opposite directions.

To verify the validity of this ring current model for a tube geometry, the magnetic field produced by the ring currents and the scattering magnetic field are calculated by the FEM–BEM code. Several excitation coils (pancake coil, bobbin coil, plus-point etc.) at different locations along an axial crack were calculated. All the results including the results of Figure 12 show good qualitative agreement in distribution with each other. This verified the validity of the ring current model, and enables us to determine the suitable locations of the pick-up coils.

Figure 11. Concept diagram of the ring current model. (a) eddy current flows of the conductors with and without crack. (b) ring current set for the perpendicular component $I_{e2}$ (c) ring current set for the parallel component $I_{e1}$.

Figure 12. Magnetic field perturbation of a square exciting coil parallel to the crack plane, (a) Field computed by the FEM–BEM method, (b) Field computed by using the ring current model.
2.5.3 The impedance change due to a crack

The impedance change due to a crack can be considered as the impedance change caused by the corresponding ring currents. The impedance change due to a crack can be calculated by the following equation,

\[
\Delta z = -\frac{\mu_0}{4\pi I_0} N j \omega \sum_{p=1}^{N} \sum_{k=1}^{N_{dd}} \oint_{r_p} \int_0^{2\pi} \frac{I_k}{r} R_k \, d\Theta \cdot dI_p,
\]

(28)

where \(N_{dd}\) is the number of the ring currents of set (b) which we choose as the number shown in Figure 11, \(N_{dd}\) is the number of the ring currents of set (c), \(I_k\) is the magnitude of the \(k\)-th equivalent ring current, \(R_k\) is the radius of the \(k\)-th ring current, \(r\) is the distance between the source and field points, \(I_0\) is the total current in the pick-up coil which transforms the EMF into impedance, \(j\) is the imaginary unit, and \(N\) is the number of turns of the pick-up coil. The integral route \(\Gamma_p\) corresponds to the \(p\)-th turn of the pick-up coil, and \(\Gamma_{ring,k}\) is the route of \(k\)-th ring current.

The ring currents \(I_k\) can be calculated from the source magnetic flux density \(B_0\) by using equation (25) and the modification coefficients.

2.5.4 Impedance signal from an unflawed tube

Once the eddy current is known, the impedance of the pick-up coils can be easily calculated from the predicted eddy current by using equation

\[
Z = -\mu_0 \frac{\alpha_N}{4\pi I_0} \sum_{p=1}^{N} \oint_{r_p} \int V e^{-(r-r_0)/\delta} n \times B_0 [r_0(\omega), \theta, \zeta] \frac{\delta(r, \theta, \zeta)}{r} \, dv \cdot dI_p,
\]

(30)

where integral area \(V\) is the volume of the whole conductor.

Using equation (30), the noise signals due to an unexpected lift-off change or probe inclination can be predicted by subtracting the standard signal from the disturbed ones (signal with lift-off change or probe inclination). Using these noise signals and the crack signal of equation (30), the \(S/N\) ratio can be finally calculated.

2.5.5 Application examples of the phenomenological strategy

By using the method described above, detectability analysis was carried out for the probe structures consisting of the plus-point or the four-coil type pick-up arrangements and many configurations of excitation coils. The results of \(S/N\) ratios for 12 types of probes are summarised in Figure 13 where the vertical axis is the \(S/N\) ratio against lift-off noise. The numbers on the horizontal scale of the figure represent the probe type that are explained on the right hand of the figure, where ‘coil A–coil B’ means that a probe is excited by the coil A and picked up by the coil B. From Figure 13, it is not difficult to understand that probe No.7 (plus-fourcoil) and probe...
No.11 (spiral-fourcoil) have better S/N ratios. In fact, one can find that the lift-off noise of probe 11 is very small when the spiral exciting coil is very long in the axial direction compared with the tube diameter. The four-coil type pick-up also decreases the inclination noise. Based on these results, these two probe structures were proposed as the candidates of a new high performance ECT probe.

3. Techniques for crack reconstruction from ECT signals

ECT inversion is a typical optimisation problem and is usually ill-posed. To solve this problem, methods in both deterministic and stochastic category are applied. J. Bowler et al. [19] have presented a way based on a thin crack model and the CG method. EDM cracks in a plate are successfully reconstructed by this method. However, the Green function is necessary in this approach. On the other hand, F. Kojima [20] has applied the trust region method to the crack reconstruction. As the conventional FEM–BEM solver is employed for the forward analysis, huge simulation time is consumed. In addition, based on some other forward analysis methods and idealised crack models, reconstruction of EDM crack also has been performed by many other researchers [21–28]. However, great effort is still necessary for the practical application of these methods, especially for natural crack and multiple cracks in different orientations.

In what follows, some inverse algorithms developed by authors and typical reconstruction examples are explained [29].

3.1 Deterministic optimisation methods

Using the fast forward solver, the iteration algorithm becomes suitable for solving crack profile parameters from measured ECT data. In this subsection, the reconstruction scheme for a planar crack of zero conductivity and constant opening is given. In these conditions, the crack reconstruction is equivalent to recognizing the crack edge curve. For more complicated natural cracks, the inversion scheme is given in the next subsection [30–32].

3.1.1 Basic formulation

As the predicted ECT signal due to the crack of true profile shall have a minimum mean-square residual with the measured signals, to reconstruct the crack shape is equivalent to finding
a function of the crack edge curve, in a continuous functional space, which minimises the mean-square error. This functional problem can be simplified to a problem with the objective function in the Riemann space by discretising the edge curve.

To search for the optimal solution, a one order deterministic scheme is suitable for present problem as the gradient of the residual error can be simply obtained from the calculated electric field at crack region. The major procedure of the inversion is as follows.

We define the mean-square residual as,

$$\varepsilon(c) = \sum_{m=1}^{M} |Z_m(c) - Z_m^{obs}|^2,$$

where, \(\{c\}\) is the vector of crack shape parameters. \(z_m(c)\) and \(z_m^{obs}\) are respectively the predicted and observed impedance signals at \(m\)-th sampling point. The total data number \(M\) is equal to the number of scanning points.

In the present problem, as we assume that the initial values of the crack profile can be chosen as values not too far from the true one, it is reasonable to regard the object function as the mean-square error defined by equation (31). Then, the optimal crack shape vector \(\{c\}\) can be solved by minimising the mean-square error with the iteration

$$\{c\}_n = \{c\}_{n-1} + \alpha_n \{\delta c\}_n,$$

where \(\{\delta c\}_n\) is the update direction of \(n\)-th iteration, which is chosen as the direction parallel to the derivative vector \(\{\partial \varepsilon / \partial c_i\}\) in the steepest descent algorithm. \(\alpha_n\) is a step-size parameter determined as the value maximally reducing the residual. In the CG method, the vector \(\{\delta c\}\) in equation (32) is adjusted to a new direction by considering the convergence history in order to accelerate the convergence.

To calculate the gradient \(\partial \varepsilon / \partial c_i\), it is efficient to apply the method using the adjoint field. The formulae of this method are as follows,

$$\frac{\partial \varepsilon}{\partial c_i} = 2Re \left\{ \sum_{m=1}^{M} \left\{ Z_m(c) - Z_m^{obs} \right\} \frac{\partial Z_m(c)}{\partial c_i} \right\}.$$

The perturbation of impedance signal \(\delta Z\) can be expressed as follows by using the adjoint field [19],

$$\delta Z = \sigma_0 \int \mathbf{E} \cdot \mathbf{E} \delta v(r) dV,$$

where \(\delta v(r)\) is vanished outside the varied crack zone due to a perturbation of crack parameter vector \(\delta c\) (along the crack edge). In this case, \(\delta v(r) dV = \delta r \cdot n ds\),

where \(n\) is the normal unit vector at the crack edge point \(r\) and \(ds\) is the corresponding area element of the crack bottom edge.
If denoting the bottom edge surface of the nonconducting planar crack as $r = s(c, t)$, ($t$ is the curve coordinate vector of the surface), one can obtain

$$
\delta Z_m = \sigma_0 \int_S \mathbf{E}_m \cdot \mathbf{E}_m \sum_{\delta c_i} \frac{\delta s}{\delta c_i} \mathbf{n} \delta c_i \, ds,
$$

where the integral region $S$ is a strip in width of the crack opening $h_0$ and along the crack bottom edge.

From equation (36) and taking into account that $\delta Z = \sum \delta Z/\delta c_i \delta c_i$, we can obtain the following equation for the impedance derivatives,

$$
\frac{\partial Z_m(c)}{\partial c_i} = \sigma_0 \int_S \mathbf{E}_m'(r) \cdot \mathbf{E}_m'(r) \frac{\partial s(c, t)}{\partial c_i} \mathbf{n} \, ds.
$$

In equation (37), the electric field was replaced by its tangential component as no current flows across the crack bottom edge surface for a nonconducting crack.

For an ECT problem of uniform material and a crack in perpendicular with the object surface, the electric field is self-adjoint that can simplify the equation (37) further. On the other hand, the step size parameter $a_n$ in equation (32) can be calculated based on the steepest descent method by [19]

$$
a_n = \frac{R_e \left\{ \sum_m \left( Z_m^{n-1} - Z_m^{obs} \right)^* \frac{\partial Z_m^{-1}}{\partial u_n} \right\}}{\sum_m \left| \frac{\partial Z_m}{\partial u_n} \right|^2}.
$$

### 3.1.2 Parameterisation of an EDM crack

We limit our problem further to the surface breaking cracks considering the property of ECT. Assuming inner/outer and axial/circumferential properties were known from the phase property of ECT signal prior to the inversion, it is reasonable to express the crack with an open piecewise line as shown in Figure 14. A constraint condition is required in this parameterisation method to impose the two end points $p_1$ and $p_n$ located at the surface of conductor.

![Figure 14. The discretisation of crack edge.](image-url)
In this case, the edge curve can be expressed as:

\[
x(t) = s_1(p_1, \ldots, p_{n_c}, t) = x_p + \frac{x_{p+1} - x_p}{t_{i+1} - t_i}(t - t_i), \quad t \in [t_i, t_{i+1}] = \Gamma_i, \ i = 1, 2, \ldots, n_c,
\]

\[
z(t) = s_2(p_1, \ldots, p_{n_c}, t) = z_p + \frac{z_{p+1} - z_p}{t_{i+1} - t_i}(t - t_i),
\]

\[
t \in [t_i, t_{i+1}] = \Gamma_i, \ i = 1, 2, \ldots, n_c - 1,
\]

where \(p_1, \ldots, p_{n_c}\) are \(n_c\) points equally spaced at the crack edge. \(x_p, z_p\) and \(t_i\) are the rectangular and curve coordinates of the point \(p_i\) respectively. The first and last points are located at the corresponding surface point with same \(x\) value. The variation of the edge is expressed by the variation of coordinates as follows,

\[
\delta s(c, r) = \delta r = \delta s_1(c, r)i + \delta s_2(c, r)k = \sum_i \frac{\partial s_1(c, r)}{\partial x_p} \delta x_p i + \sum_i \frac{\partial s_2(c, r)}{\partial z_p} \delta z_p k,
\]

where \(i, k\) denote the unit vectors along the \(x\) and \(z\) direction. Substituting equation (41) into equation (37) with \(\partial s_1(c, r)/\partial x_p\) and \(\partial s_2(c, r)/\partial z_p\) derived from equations (40) and (41), the derivatives of the impedance \(Z_m\) are expressed in terms of the electric field by the following integrals,

\[
\frac{\partial Z_m}{\partial x_p} = -\sigma_0 \int_{-h_0/2}^{h_0/2} \left\{ \int_{\Gamma_i} E^2_m \frac{(t_{i+1} - t)(z_{p+1} - z_p)}{(t_{i+1} - t_i)(t_{i+1} - t)} dt \right\} dy,
\]

\[
i = 1, 2, \ldots, n_c,
\]

\[
\frac{\partial Z_m}{\partial z_p} = -\sigma_0 \int_{-h_0/2}^{h_0/2} \left\{ \int_{\Gamma_i} E^2_m \frac{(t_{i+1} - t)(x_{p+1} - x_p)}{(t_{i+1} - t_i)(t_{i+1} - t)} dt \right\} dy,
\]

\[
i = 2, 3, \ldots, n_c - 1,
\]

where the integration along the thickness direction of crack is carried out with the variable \(y\) from \(-h_0/2\) to \(h_0/2\).

If the crack parameter \(c_i\) was chosen as the step length of the modification along the normal direction of the crack edge as shown in Figure 14, the derivative with respect to this parameter can be obtained by imposing the point \(P_i\) move along the normal direction. Thus, the derivative with respect to \(c_i\) is equal to the partial derivative at the normal direction \(n_{pi}\),

\[
\frac{\partial Z_m}{\partial c_i} = \nabla Z_m \cdot n_{pi} = \frac{\partial Z_m}{\partial x_p} n_{xpi} + \frac{\partial Z_m}{\partial z_p} n_{zpi}.
\]

Moreover, we only need to choose the point number \(n_c = 4\) for dealing with a rectangular crack. In this case, the derivatives with respect to the independent crack parameters (such as the depth of the crack and the \(x\) coordinate of the two ends) can be obtained from the derivatives
of equations (43) and (44) using the transform matrix $[T]$ as,

$$
\begin{align*}
\frac{\partial Z_m}{\partial c_i} = [T] \left[ \begin{array}{c} 
\frac{\partial Z_m}{\partial \chi_m} \\
\frac{\partial Z_m}{\partial \chi_p} \\
\frac{\partial Z_p}{\partial \chi_p}
\end{array} \right]^T.
\end{align*}
$$

(45)

3.1.3 Numerical examples

At first, reconstruction results for a crack of complex shape were conducted by using the impedance data computed with the FEM–BEM code as input signal. The reconstructed crack shape is shown in Figure 15(a) with comparison to the true shape and initial one. In this case, the CPU time used was about 10 min for a PC. Both crack depth, length and shape are properly reconstructed. Figure 15(b) is the comparison of impedance due to crack of predicted profile and the true values. The results proved the feasibility of the proposed method for the ECT inversion of SG tube geometry.

As an example of reconstruction with measured signals, the reconstruction was performed using signals of the JSAEM benchmark problem step 6 [2]. Figure 16(a) shows the inversion

![Graph showing comparison of crack shapes](image)

![Graph showing comparison of impedance signals](image)

Figure 15. Results of reconstruction for a crack in complex shape using the impedance calculated with FEM–BEM code(400 kHz, tube).
results for the slope crack with maximum depth being 60% of the wall thickness. Though the shape was not predicted exactly, the length and depth were obtained with good accuracy. Figure 16(b) shows the comparison of reconstructed and true impedance signals. Good agreement was found again.

As a more difficult case, Figure 17 shows a result for a deep crack located in a welding metal zone. A pluspoint coil probe was adopted in this measurement with a low frequency (10kHz). In the present case, the welding bead has been removed by a grinding machining. Though the welding part results in noise in the signal, a crack in the welding zone can be properly reconstructed even if it is as deep as 10 mm.

To consider the reconstruction of a practical crack, the effect of crack inclination and crack width on the reconstruction accuracy is investigated. In practice, an EDM crack with 20 degree inclination from the depth direction and an EDM crack of smaller width (0.05 mm) are reconstructed by using the numerical model of 0.2 mm opening and 0 degree inclination. Figure 18(a),(b) show the comparison of the true and reconstructed crack profile respectively. From these results, one can conclude that the crack profile can be properly predicted even though the crack model used for databases has some difference with the practical case.

Figure 16. Result of reconstruction for a slope crack embedded in conducting plate.
3.2 Reconstruction of stress corrosion cracks

A stress corrosion crack (SCC) has a much more complex geometry than the zero conductivity crack model. A major difference is that the crack faces may close in view of the microstructure of SCC. The closures lead to a non-vanishing conductivity inside the SCC region that makes the numerical simulation more complicated [30,31,37].

To reconstruct such a crack, we propose an idealised SCC model at first assuming that the closures mainly occur along the crack edge. As shown in Figure 19, the SCC model consists of two regions separated by an inner crack edge. The conductivity of the strip along the crack edge is supposed as a value smaller than that of the base conductor, while the region in the inner part of the crack is assumed of vanishing conductivity. The width of the strip zone is not a constant. In this model, the effect of the crack closures is taken into account by employing both the conductivity and shape parameter of the strip region. In practice, the crack parameters are chosen as $\sigma_1$, the conductivity at the closure zone, and $\mathbf{b} = (b_1, b_2)^T$, the coordinate vector of discrete points at the two edge curves. In what follows, the inversion scheme for SCC based on this two-edge model is described.

3.2.1 Forward problem

The calculation of pick-up signals for the crack with closures can be performed based on the database type fast forward scheme and the new element stated in the previous sections.
Figure 18. Affect of modelling noise (crack inclination and width difference, CG method, plate of 20 mm thickness).

Figure 19. Numerical model of the crack with closure.
Concerning different features of crack parameters, however, the formulae for EDM cracks have to be modified.

In the database type fast forward solver, matrix $[H]$ is independent of cracks. Meanwhile, despite being affected by the crack shapes, the matrix $[K]$ can be derived from coefficient matrices of the two cavity cracks defined by the inner and outer crack edge respectively. The gradients with respect to the crack parameters of the idealised crack model can be obtained from the gradients corresponding to the two cavity cracks.

In fact, if we suppose that the shape function $[N]$ for a normal finite element is still valid in case of a distributed conductivity as applied for the new element [12], a submatrix of the element coefficients can be expressed as,

$$
[N_2] = \int_{\Omega_{e_1}+\Omega_{e_2}+\Omega_{e_3}} (\sigma_0 - \sigma(r))[N]^T[N] dv,
$$

$$
= \int_{\Omega_{e_2}+\Omega_{e_3}} \frac{\sigma_0 - \sigma_1}{\sigma_0} [N]^T[N] dv + \int_{\Omega_{e_1}} \frac{\sigma_1}{\sigma_0} [N]^T[N] dv,
$$

where, $\Omega_{e_1}, \Omega_{e_2}$ and $\Omega_{e_3}$ are the regions of an element where conductivity is equal to $\sigma_0$, $\sigma_1$ and 0 respectively. Therefore, one can find the following correlation between the practical element coefficient matrix and those for the two cavity cracks,

$$
[K] = \frac{\sigma_1}{\sigma_0} [K_1]_e + \frac{\sigma_0 - \sigma_1}{\sigma_0} [K_2]_e = \alpha_1 [K_1]_e + (1 - \alpha_1) [K_2]_e,
$$

where, $[K_1]_e$ is an element coefficient matrix for the cavity crack surrounded by the inner edge, $[K_2]_e$ of that defined by the outer edge, and $\alpha_1 = \sigma_1/\sigma_0$.

To predict the gradients, one can apply the equation (37) to the two cavity cracks separately but using the electric field calculated with equations (6) and (48) (true field). Denoting the gradient for the crack surrounded by the inner edge as $\partial Z_{m}/\partial b_{1i}$ and those defined by the outer edge as $\partial Z_{m}/\partial b_{2i}$, (where, $b_{1i}$, $b_{2i}$ are respectively the shape parameter for the inner and outer cavity crack, $Z_{1m}, Z_{2m}$ are impedance signals calculated from the true electric field but with the formula for a cavity crack), the derivatives for the crack with closures can be expressed as

$$
\frac{\partial Z_m}{\partial b_{1i}} - \frac{\partial Z_m}{\partial b_{1j}} = \alpha_1 [T_1] \left\{ \frac{\partial Z_{1m}}{\partial x_{pi}}, \frac{\partial Z_{1m}}{\partial z_{pi}} \right\}^T,
$$

for points located at the edge of inner side, and

$$
\frac{\partial Z_m}{\partial b_{2i}} = (1 - \alpha_1) [T_2] \left\{ \frac{\partial Z_{2m}}{\partial x_{pi}}, \frac{\partial Z_{2m}}{\partial z_{pi}} \right\}^T,
$$

for those at the outer edge, and $[T_1]$ and $[T_2]$ are the linear transform matrices. For $\alpha_1$, one has

$$
\frac{\partial Z_m}{\partial \alpha_1} = \sigma_0 \left\{ \int_{\Omega_{e_2}} |E_{m}|^2 dv - \int_{\Omega_{e_1}} |E_{m}|^2 dv \right\}.
$$
with $\Omega_{ck1}, \Omega_{ck2}$ denoting the volume of the two cavity cracks respectively. Defining $\alpha = \sigma/\sigma_0$, one can find

$$\frac{\partial Z_m}{\partial \alpha_1} = \frac{\partial Z_{2m}}{\partial \alpha} - \frac{\partial Z_{1m}}{\partial \alpha}. \quad (52)$$

### 3.2.2 Inverse procedure

Compared with the case of a nonconductive crack, the most significant difference in the inverse procedure is that the crack parameters are in different types, i.e. those for the crack shape and the conductivity. The gradients with respect to these parameters are usually with a much different magnitude (even in much different order) because of the different units. Therefore, a quite different convergent speed may occur between these crack parameters if a normal algorithm of the CG method is directly applied. To tackle this difficulty, a scheme on the basis of the CG method is proposed.

The basic idea of this procedure is that the direction and step size at each updating step are calculated separately for parameters in different type. In practice, the iteration equation for predicting the crack parameters is taken as,

$$\begin{align*}
\begin{bmatrix} c_1^n \\ c_2^n \end{bmatrix} &= \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \alpha_1 \begin{bmatrix} \partial g^{n-1} \\ \partial c_1^{n-1} \end{bmatrix} + \alpha_2 \begin{bmatrix} \partial g^{n-1} \\ \partial c_2^{n-1} \end{bmatrix}, \\
\alpha_1 &= \frac{\text{Re} \left\{ \sum_{m=1}^M \left[ Z_m^{n-1}(\epsilon) - Z_m^{(\text{obs})} \right]^2 \sum_{i=1}^{N_i} \frac{\partial Z_m^{n-1}}{\partial \epsilon_i} \frac{\partial g^{n-1}}{\partial \epsilon_i} \right\}}{\sum_{m=1}^M \left[ \sum_{i=1}^{N_i} \frac{\partial Z_m^{n-1}}{\partial \epsilon_i} \frac{\partial g^{n-1}}{\partial \epsilon_i} \right]^2}, \quad j = 1, 2. \quad (53)
\end{align*}$$

### 3.2.3 Numerical examples

Figure 20 depicts an example of impedance signals of the reconstructed crack with a comparison to the true ones (simulated data with no noise) where $\Delta R$ and $\Delta \omega L$ denote the resistance and reactance respectively. The true shape of the crack, as shown in Figure 20, is assumed to be an inner one with depth equal to 50% of the tube wall thickness. The reconstructed relative conductivity is 0.102 for a true value of 0.1 and an initial value of 0.5 (the conductivity of the base material was chosen as $10^6$ S/m). The initial values of the crack shape were chosen as the results predicted by the code for cavity crack. Good reconstruction result is obtained for both the shape parameters and the conductivity.

In addition, Figure 21 shows a reconstruction result for a practical SCC initiated in an austenitic stainless steel pipe of 8 mm wall thickness used in a nuclear power plant. By using the two-edge SCC model and the CG scheme, the length and the depth of this natural SCC are satisfactorily recognised.

### 3.3 Artificial intelligent approach

A feed forward neural network that has only one hidden layer [33] is applied to the ECT inversion. The neural network is trained to realise the mapping between the ECT signals and the
As the variables to be recognised are the crack size, profile parameters defining the crack boundary are taken as the output of the neural network, while the ECT signal scanned just over the crack is applied as its input. Considering the feature of an SCC, only surface breaking cracks with known location and orientation are taken as reconstruction targets [33,37].

Upon these assumptions, a vector of integer elements, which indicates the depth of the selected crack region, is proposed for parameterising the crack profile. In practice, a selected region of regular shape, which can be chosen directly based on the effective signal length, is subdivided into small cells and the numbers of crack cells in the depth direction are set as the value of the elements in the crack parameter vector.

To generate the training data sets, the fast forward solver introduced in section 2 is applicable. As input information, the pickup signal due to cracks with their shape and conductivity distribution randomly designated are calculated for a scanning path just over the crack plane. The application of the randomly distributed conductivity to the crack cells is to cope

Figure 20. Comparison of the reconstructed and the true crack shape and signal (10% noise).
with the problem of a natural crack concerning the contacting or bridges between the crack surfaces. A large number of crack signals and the corresponding crack profile vectors are calculated and stored in 3 databases for NN training, validating and testing respectively. Principal component analysis is also applied to the raw eddy current signals as a preprocessing in order to extract the features of the signal and to decrease the amount of input nodes.

Figure 22 shows a result of the neural network approach for an actual SCC detected in an SG tube (1.27 mm thickness) of a PWR nuclear power plant. The true crack shape obtained by destructive test is also illustrated. The percentage number shown in the figure is the reconstructed relative crack depth (ratio between reconstructed crack depth and the plate thickness). Good agreement is obtained again. To predict the crack shape with the trained network, almost no CPU time is consumed.

3.4 Metaheuristic methods—parallel tabu search

Unlike the deterministic approach, the metaheuristics-based optimisation methods minimise the objective function $e$ in a stochastic way, i.e. the best profile vector $c$ is calculated by a random search. The tabu search, genetic algorithm, and simulated annealing method belong to this category. The feature of this approach is that a global minimum can be guaranteed, although a large amount of computational resource is necessary [34].

As the simulation time required by the fast forward solver depends on the size of the crack, starting the searching procedure of a metaheuristic method from a small initial crack and finding
the optimum profile by enlarging the crack step by step is a reasonable way. From this point of view, an algorithm based on the tabu search algorithm is more efficient in the metaheuristic methods for the crack profile reconstruction from ECT signals. In addition, as parallel computation is applicable for the metaheuristic methods, a parallel computation is an efficient way to reduce the simulation time further.

The major procedure of the tabu search is as follows:

1. Randomly select one of the cells that face the surface of the target plate as an initial crack (solution candidate). Generating \( n \) initial cracks in this manner.
2. Calculate residual function \( e \) of each candidate by using \( n \) CPUs in parallel.
3. If the terminate condition is satisfied, output the best solution in the history as the final prediction.
4. If not, add all of the solution candidates evaluated in step 2 to the tabu list.
5. Copy the best solution candidate in the history into all of the \( n \) solution candidates.
   Generate \( n \) new solution candidates for the next iteration by slightly updating the present ones. If a new solution candidate is already in the tabu list, regenerate it.
6. Return to step 2.

Figure 23 shows a reconstruction result of the tabu search algorithm for the same problem as used for the NN approach. The percentage number shown in the figure is also the reconstructed relative crack depth. Better results are obtained. However, the simulation needs much more computational resource than the CG method and the artificial neural network.

### 3.5 Reconstruction of multiple cracks

Figure 24 shows the configuration of the considered multiple cracks problem. The cracks are searched in a selected region \( \Omega_1 \) chosen from the whole domain based on the features of the ECT signals. The domain \( \Omega_1 \) is subdivided into \( n_x \times n_y \times n_z \) parallelepipedic cells that form geometrically the possible cracks. To consider a specimen with EDM artificial cracks, cells that form cracks have zero conductivity, and cells outside cracks have conductivity of the base material [35,36].

In this section, we give a crack model effective for multiple slits reconstruction. All cracks were assumed to be oriented in the same direction. Since the width of an EDM notch does not significantly affect the ECT signals, it is assumed as a given value of 0.2 mm. The cracks did not fill up the cells completely in the direction perpendicular to the cracks (see Figure 25). Though the boundary profiles of defects did not coincide with the cells division, the mesh of finite element to simulate signals was chosen based on the selected crack width. The crack parameters

![Figure 23. Reconstruction results (SCC crack, Tabu search method, 1.27 mm tube).](image-url)
(vector $c$) are defined as the number of starting cells of the crack, the number of final cells of the crack and the crack depth (Figure 25).

In the following, the reconstruction scheme for the parallelepiped slits model, and validation using measured signals are given. The database type forward solver was upgraded for the multiple crack case. For the inverse analysis, the tabu search technique was selected.

3.5.1 Forward analysis

The fast forward FEM–BEM solver using database [12] was upgraded for the computation of the ECT signals due to multiple cracks at first. For the selected 3D possible defect region, the field values for cells with the same width as the cracks are required in the database. As only the fields when the sources are set in the centre of the possible defect region need to be calculated, a coarse mesh was used in the regions far from the centre region. As the field in far regions does not change sharply, a linear interpolation of the field can give sufficient precision.

3.5.2 Reconstruction method using a tabu search

Tabu search is applied for reconstruction of multiple cracks. While the basic tabu search explores the search space of all possible solutions by a sequence of moves, tabu search here
utilises an iterative local search to generate new solution candidates because evaluation of all possible solutions in a neighbourhood is too time consuming. This algorithm, which is based upon the iterative local search superimposed with tabu rules is termed as tabu search in this paper. Therefore, the difference between the tabu search and the iterative local search is in the generation of solution candidates for the next iteration step.

Tabu rules adopted in this study are the two most fundamental ones: explicit and attribute tabu strategies. The explicit tabu strategy stores defect profiles evaluated during inversion procedure, and the generation of a defect evaluated is prohibited thereafter; the attribute tabu strategy prohibits a certain movement for the given iteration epochs, if the reverse of the movement has improved error recently. The stop condition of the iterative process is whether the error function is small enough or the maximum number of iterations is reached.

3.5.3 Numerical examples

The raw C-scan measured data from single and multiple groups of slits were calibrated and used for the crack reconstruction. Figure 26 shows some results for multiple parallel slits (Group B) in comparison with the real shape from measured data. The average time required for one reconstruction was 4 h for the Group B on a PC.

Even if the measured data were used and the two groups of slits (A and B) were modelled with approximation due to the restrictions of the proposed model (the minimum distance between slits was 0.8 mm), the tabu search was able to find with a good accuracy for the features of multiple slits: location, length and depth, without any knowledge about the number of cracks and distance among them in the searched 3D region.

Figure 26. Reconstruction of three parallel slits (Group B)—top view.
4. Concluding remarks

In this paper, some numerical simulation techniques developed in our research group for the forward and inverse analysis of ENDE signals are summarised.

In the first part, efficient forward analysis schemes for the simulation of ECT, RFECT, and MFLT signals are introduced. From the formulations and simulation results shown in this part, the following conclusions are obtained: (1) The database type approach using pre-calculated unflawed field data is very efficient for the fast and accurate ECT signal simulation; (2) With the proposed hybrid approach of 2D and 3D geometry, and the new formula correlating the EMF signal and the eddy current, high precision RFECT simulation becomes possible; (3) Fast simulation of the MFLT signals based on a FEM–BEM hybrid code of polarisation method is realised; (4) The phenomenological strategy, which gives a qualitative estimation of eddy current distribution based on the excitation magnetic field, is efficient for the probe design.

From the techniques and application examples reviewed in the second part, it is found that the reconstruction schemes based on the deterministic and stochastic optimisation strategies are efficient for the sizing of single artificial crack, single natural crack, cracks in welding parts, and multiple cracks from the signals measured in either a laboratory environment or during the practical ISI. Up to now, the reconstruction of artificial single crack, fatigue cracks and some SCC can be realised by the ECT inversion with a good accuracy. For multiple cracks and deep SCC, on the other hand, additional efforts are still being made to improve the sizing precision.

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Note

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