Application of improved morphological filter to the extraction of impulsive attenuation signals

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Abstract

Rotating machinery response is often characterized by the presence of periodic impulses modulated by high-frequency harmonic components. It can be defined with three parameters, which are natural frequency, fault frequency and decay coefficient. In this paper, we propose an improved morphological filter for feature extraction of the above signals in the time domain. Firstly, an average weighted combination of open-closing and close-opening morphological operator, which eliminates statistical deflection of amplitude, is utilized to extract impulsive component from the original signal. Then, according to the geometric characteristic of impulsive attenuation component, the structure element is constructed with an impulsive attenuation function, and a new criterion is put forward to optimize the structure element. The proposed method is evaluated by simulated impulsive attenuation signals with different natural frequencies and vibration signals measured on defective bearings with outer race fault and inner race fault, respectively. Results show that the background noise can be fully restrained and the entire impulsive attenuation signal is well extracted, which demonstrates that the method is an efficient tool to extract impulsive attenuation component from mechanical signals.

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1. Introduction

There exist a large number of rotating machineries in industry. As a result, the condition monitoring of these equipments is important in terms of system maintenance and process automation. Responses quite often include transient signals generated by impacts, typical cases being the vibration response of machines subjected to detects or wear of certain parts such as bearings and gears. These signals can be characterized by the presence of periodic repetition of sharp peaks, which are further modulated by high-frequency harmonic components.

Many attempts have been made to extract useful information from the response signals, envelope analysis being the most widely accepted one [1–7]. Lou and Loparo [1] employed Daubechies-2 and Daubechies-10 wavelets in fault diagnosis of rolling bearings. The paper claimed that the decomposed details (d3 and d4) are quite different in magnitude between inner race fault and normal condition; and for a ball fault, d3, d4 and a5 are much smaller than those of normal data. Yu et al. [2] applied EMD method and Hilbert transform to the envelope signal of rolling bearings. Results show that the fault characteristics can be extracted by selecting the proper IMFs after EMD.

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Morphological filter, which was first used in image analysis, is also an efficient tool in signal processing. It can decompose the original signal into several physical parts according to certain geometric characteristics. Most applications in signal processing of morphological filters are quite limited to biomedical EEG and ECG signals [8,9]. In 2003, Nikolaou and Antoniadis [10] introduced morphological filter into fault diagnosis of rolling bearings. The author selected closing operation as the basic morphological algorithm and the structure element was defined as a flat line with amplitude of zero. The feature frequency of inner and outer race fault is identified, whereas some helpful information such as the natural frequency of bearing systems is ignored.

Natural frequency is the basis of demodulated resonance and cannot be obtained precisely with traditional methods because of a mass of factors, such as vibration of inner race, outer race, balls and sensors. Following Nikolaou’s idea, this paper presents further consideration on the morphological processing of impulsive vibration signals, which aims to extract the entire impulsive signal including fault frequency, natural frequency and decay coefficient in the time domain. The outline of this study is as follows. We briefly describe the fundamental theory of mathematical morphology in Section 2.1. In the remaining part of Section 2, an improved morphological filter is proposed and two key aspects are concentrated: (a) Morphological operation. Rather than closing operation, an average weighted combination of open-closing and close-opening operation is utilized, which can extract full characteristics of impulsive signals; (b) Design and optimization of structuring element. Structure element constructed by an impulse attenuation signal is designed against the geometric features of the signal. A criterion on optimization of the structure element is put forward in mathematical expression. In Section 3, the proposed method is validated by simulation data, which are composed of a series of periodic impulses with additive noise, modulated by high-frequency harmonic components. The proposed procedure is evaluated in Section 4, using vibration signals from defective bearings with outer race fault and inner race fault respectively. Finally, the conclusions are given in Section 5.

2. Methods

2.1. Fundamental theory of morphological filter

Morphological filter with functional structure element for one-dimensional time series data was first presented by Magaros and Schafer [11] in 1987, which consists of four basic operations:

Erosion:

\[
(f \Theta g^t)(t) = \min_{\tau \in D} \{f(t) - g((-t - \tau))\}
\]  

(1)

Dilation:

\[
(f \oplus g^t)(t) = \max_{\tau \in D} \{f(t) + g((-t - \tau))\}
\]  

(2)

Opening:

\[
(f \circ g)(t) = [(f \Theta g^t) \oplus g](t)
\]  

(3)

Closing:

\[
(f \bullet g)(t) = [(f \oplus g^t) \Theta g](t)
\]  

(4)

where \(f(t)\) is the original signal and \(g(t)\) is the structure element. \(g^t(t)\) is the reflection of \(g(t)\) defined as \(g^t(t) = g(-t)\). \(D\) means the set of real number. The notation \(\oplus, \Theta, \circ\) and \(\bullet\) indicate Minkowski addition, Minkowski subtraction, opening and closing operation, respectively.

The basic properties of the operations can be described as follows: Erosion of \(f(t)\) by structure \(g(t)\) reduces the peaks and enlarges the minima of \(f(t)\), while dilation of \(f(t)\) by \(g(t)\) increases the valleys and enlarges the maxima of \(f(t)\). Opening of \(f(t)\) smoothes the signal \(f(t)\) from below by cutting down its peaks, and closing smoothes the signal from above by filling up its valleys. Thus, closing and opening operation can be applied to detect positive and negative impulses, respectively.

2.2. Improved morphological filter

Morphological operation and structure element constitute the morphological filter. So we try to optimize the traditional morphological filter from the two aspects.

2.2.1. Average weighted combination of open-closing and close-opening operation

Nikolaou and Antoniadis [10] employed closing operation to extract impulsive components from the original signals. According to the property of morphological operation, only positive impulses are detected. However, there are sharp peaks with both positive and negative amplitude in vibration response of machines with defects, such as outer race fault in rolling bearings. Closing operation indeed extracts some useful information from the signal, but loses the geometric characteristics of the signal which may help to fault diagnosis. In order to detect bi-directional impulsive components, the morphological
algorithm can be first applied with opening (or closing) operator followed by closing (or opening) operator, which are defined as follows:

**Open-closing operation:**

\[
OC(f(t)) = f(t) \circ g_1(t) \bullet g_2(t)
\]

**Close-opening operation:**

\[
CO(f(t)) = f(t) \bullet g_1(t) \circ g_2(t)
\]

where \(g_1(t)\) and \(g_2(t)\) are different structure element. However, both open-closing and close-opening operation can lead to a statistical deflection of amplitude, i.e., the result of open-closing operation has lower amplitude than original signal and the result of close-opening operation has larger amplitude than original signal. This maybe will affect the result of fault diagnosis. In this paper, we utilize an average weighted combination of open-closing and close-opening operation as follows:

\[
M_{aw}(f(t)) = \frac{[CO(f(t)) + OC(f(t))]}{2}
\]

With Eq. (7), not only the bi-directional impulses can be extracted but also the statistical deflection of amplitude caused by individual open-closing or close-opening operation is avoided.

### 2.2.2. Design and optimization of structure element

Structure element is another sticking point of morphological filter, which has a decisive effect on the analysis result. In order to extract impulsive-type signals, structure element is constructed with an impulse attenuation signal determined in Eq. (8) and shown in Fig. 1.

\[
g(t) = a \exp(-D_c t) \sin \omega_s t
\]

where \(a\) denotes the maximum amplitude of the structure element and \(\omega_s\) is the natural frequency of the structure element. \(D_c\) accounts for the decay rate of the structure element and indicates the length of the structure element. Since impulsive attenuation signals exist with different amplitude, frequencies and decaying rate, the structure element should be adjusted to proper size. Here, we propose a new criterion to optimize the structure element:

Suppose that \(w(t)\) is the original signal and \(u(t)\) is the extraction result of impulsive attenuation signals, calculated by \(u(t) = w(t) - M_{aw}(w(t))\), the structure element are optimized as follows:

\[
IMA = \frac{Q_{cr}}{R_{pz}}
\]

for which

\[
Q_{cr} = RMS[XCORR(u(t), g(t))]
\]

and

\[
R_{pz} = \frac{N_{pz}}{N}
\]

![Fig. 1. Structure element.](image-url)
where \( \text{RMS}(x) = \left( \int_{-\infty}^{\infty} |x|^2 p(x) \, dx \right)^{1/2} \). \( \text{XCORR}(x,y) \) is the cross-correlation function of \( x \) and \( y \). \( R_{pz} \) points to the zero-pass rate of signal \( u(t) \). \( N \) is total data length of \( u(t) \) and \( N_{pz} \) is the number of zero-pass points in signal \( u(t) \) determined by

\[
N_{pz} = \sum_{n=1}^{N-1} \sigma[u(n)\text{\texttt{=}u(n+1)}]
\]

assuming

\[
\sigma(x) = \begin{cases} 
0, & x > 0 \\
1, & x \leq 0 
\end{cases}
\]

\( Q_{cr} \) indicates the similarity of the structure element and processed data \( u(t) \). \( R_{pz} \) reflects the degree of restraining the background noise. So, a larger \( \text{IMA} \) shows that impulsive-type signals are better extracted and background noise is better restrained. In the present paper, the maximum amplitude, decay coefficient and natural frequency of the structure element are optimized respectively with \( \text{IMA} \) criterion. The process of calculating \( \text{IMA} \) is shown in Fig. 2.

2.3. Impulsive attenuation signal extraction procedure with improved morphological filter

The process for feature extraction of impulsive attenuation signal can be described in detail as follows:

(1) First, the search area on the maximum amplitude, decay coefficient and natural frequency of structure element are decided. These parameters indicate the probable geometric characteristics of the impulsive attenuation signal.

(2) Then, the structure element is optimized with the following steps, which are shown in Fig. 3:

Step 1. At the beginning of the optimization process, parameters including \( \text{IMA}(0), a(0), D_c(0) \) and \( w_s(0) \) are initialized.

Step 2. A structure element set is decided by \( a \) in the search area and the current \( D_c(n), w_s(n) \).

Step 3. For each structure element in the set, \( \text{IMA} \) is calculated. Maximum of \( \text{IMA} \) is marked as \( \text{IMA}_{max1}(n) \) and the corresponding \( a(n) \) is obtained.

Step 4. A new structure element set is decided by \( D_c \) in the search area, the obtained \( a(n) \) and the current \( w_s(n) \).

Step 5. For each structure element in the new set, \( \text{IMA} \) is calculated. Maximum of \( \text{IMA} \) is \( \text{IMA}_{max2}(n) \) and the corresponding \( D_c(n) \) is obtained.

Step 6. A new structure element set is decided by \( w_s \) in the search area, the obtained \( a(n) \) and \( D_c(n) \).

Step 7. For each structure element in the new set, \( \text{IMA} \) is calculated. Maximum of \( \text{IMA} \) is \( \text{IMA}_{max3}(n) \) and the corresponding \( w_s(n) \) is obtained.

Step 8. Let \( \text{IMA}(n) = \max(\text{IMA}_{max1}(n), \text{IMA}_{max2}(n), \text{IMA}_{max3}(n)) \), and return to Step 2.

Step 9. When all the structure elements in the search area are employed, we obtain the optimized \( \text{IMA}_{opt} = \max(\text{IMA}(n)) \). The corresponding \( a(n), D_c(n) \) and \( w_s(n) \) are the optimization result.

(3) After that, the optimized structure element is used to perform \( M_{opt} \) operation on the original signal \( w(t) \). The processed data \( u(t) \) is obtained by subtracting the result of \( M_{opt} \) operation from \( w(t) \).

With the optimized structure element, impulsive component in \( w(t) \) is extracted and background noise can be fully restrained, which represents processed data \( u(t) \).

3. Simulation result

A simulated signal composed of exponentially decaying impulses with additive noise is shown in Fig. 4(a). Improved morphological filter proposed in Section 2 is applied to feature extraction of impulsive attenuation component. The \( \text{IMA} \) curve is displayed in Fig. 4(b). The maximum of \( \text{IMA} \) corresponds to the optimal structure element with the parameters \( a = 6.5 \times 10^{-6}, D_c = 265.1\pi, w_s = 6902.07\pi \). With the optimal structure element, impulse attenuation component in the original simulation signal is extracted in Fig. 4(c). It shows that the pulse repetition frequency is extracted efficiently and noises are fully retained. Detailed information of the fourth and fifth impulse is revealed in Fig. 4(d). The decaying rate and natural frequency are also obtained.
Exponentially decaying impulses with multiple natural frequencies is shown in Fig. 5(a). Three natural frequencies of 30, 50 and 80 Hz are assumed to be excited and the mathematical expressions of the series are defined as follows:

\[ x_1(t) = 0.01 \exp(-5t) \sin(60\pi t) \]  
\[ x_2(t) = 0.01 \exp(-5t) \sin(100\pi t) \]  
\[ x_3(t) = 0.01 \exp(-5t) \sin(160\pi t) \]

The structure element is optimized with IMA criterion and the optimal parameters of the structure element are obtained: \( a = 2.9 \times 10^{-5}, D_c = 240.5\pi, w_s = 6505.02\pi \). Fig. 5(c) refers to the processed data with the optimal structure element and the localization of the result is shown in Fig. 5(d). In Fig. 5(d), helpful information of the original signal, such as pulse repetition frequency, decaying rate, and natural frequencies, is well extracted.

Compared with the original morphological filter employed in Ref. [10], improved morphological filter proposed in this paper utilizes average combination of close-opening and open-closing operation, rather than the closing operation.
More importantly, the structure element is constructed in the form of decaying sinusoid, with the similar characteristic to the object component, and IMA criterion is utilized to obtain the optimal size of the structure element. As a result, it can extract the entire impulsive signal from background noise, including its natural frequency, fault frequency (pulse repetition frequency) and decaying coefficient. Especially, multiple natural frequencies can also be extracted with one structure element.

4. Application on defective bearings

When there exists a defect in a rolling bearing, it produces an impact that excites natural frequencies of the bearing and of the entire machine. Thus, the typical response resulting from these periodic impacts, which are generated by bearing faults, usually consists of a sharp rise and corresponds to the impact between the rolling surfaces at the location of the defect, and an impact response decay. These impulses occur periodically, with a frequency determined by the type of the defective components, its geometry and the rotation speed.

4.1. Defective bearing with an outer race fault

A typical vibration signal of defective bearing with an outer race fault is shown in Fig. 6(a). The measurement is performed on a 308 rolling bearing at rotor workbench with rotating speed of 1572 rpm and sampling frequency of 40 kHz.
The parameters of the bearing are as follows: The pitch diameter is 65 mm and the ball diameter is 15 mm. The number of balls in the bearing is 8 and the contact angle equals to 0°. As a result, the corresponding ball pass frequency in outer race (BPFO) is 80.65 Hz. Fig. 6(b) is IMA curve, which indicates the optimal structure element with \( a = 6.08 \times 10^{-3}, D_c = 280\pi, w_s = 7000\pi \). With the optimal structure element, Maw operation is applied to the original signal. The result signal is shown in Fig. 6(c), which indicates that the impulsive component is extracted in time domain. The time interval of two impulses is approximate 0.0124 s, which corresponds to BPFO. Furthermore, we observe the detailed information of Fig. 6(c), shown in Fig. 6(d). The decaying component and natural frequency are also retained.

4.2. Defective bearing with an inner race fault

The signal presented in Fig. 7(a) is measured on a 308 rolling bearing with an inner race fault. The rotating speed of bearing is 1069 rpm and the sampling frequency is 40 kHz. The parameters of the bearing, including pitch diameter, ball diameter, number of balls and contact angle, are the same as the parameters of the bearing with outer race fault mentioned in 4.1. Thus, the corresponding ball pass frequency in inner race (BPFI) is 87.72 Hz. Fig. 7(b) is IMA curve and the maximum of IMA curve corresponds to the optimal structure element with three parameters \( a = 8.5 \times 10^{-3}, D_c = 307.8\pi, w_s = 5733\pi \). With the optimal structure element, Maw operation is applied to the original signal. The result signal is shown in Fig. 7(c), which indicates that the impulsive component is extracted in time domain. The time interval of two impulses is approximate 0.0114 s, which corresponds to BPFI. Furthermore, we observe the detailed information of Fig. 7(c), shown in Fig. 7(d). Both the natural frequency and the decaying rate are observed.
5. Conclusion and discussion

In this paper, a method for feature extraction of impulsive-type periodical signals by using improved morphological filter is proposed. Compared with the traditional morphological filter, we make improvement in two aspects. Firstly, the morphological operation is decided with an average combination of open-closing and close-opening ($M_{aw}$). With $M_{aw}$, not only impulses can be extracted but also the statistical deflection of amplitude caused by individual open-closing or close-opening operation is avoided. Then, structure element is constructed with an exponentially decaying sinusoid which has the similar feature to the object signal. Since different impulsive-type signals may exist with different frequencies and amplitudes, a new criterion (IMA criterion) is proposed to optimize the structure element to proper size. Three parameters, including the maximum amplitude, decay coefficient and natural frequency of the structure element are optimized, respectively.

As a result, the improved morphological filter can extract the entire impulsive signal from background noise, rather than the envelope of the signal. Three key parameters in the impulsive signal, including fault frequency, natural frequency and decaying rate, are obtained in time domain. The method is validated with both simulated signal and vibration signals of defective bearings.

However, several problems still need consideration. When there exist multiple natural frequencies and these frequencies differ from each other greatly, it is hard to extract them with only one structure element. A possible solution is
to construct a library composed of many structure elements with different parameters [12]. Furthermore, impulsive attenuation is not the unique characteristic in mechanical faults. Structure element should be decided with different shape and size to detect different faults. These are what our research work will focus on next.

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**References**


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