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Experimental demonstration of optical Bloch oscillation in electromagnetically induced photonic lattices

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ABSTRACT

The optical Bloch oscillation (OBO) is an optical-quantum analogy effect that is significant for light field manipulations, such as light beam localization, oscillation and tunneling. As an intra-band oscillation, OBO was important for optical investigations in photonic lattices and atomic vapors over an extended period of time. However, OBO in reconfigurable platforms is still an open topic, even though tunability is highly desired in developing modern photonic techniques. Here we theoretically establish and experimentally demonstrate OBO in an electromagnetically induced photonic lattice with a ramping refractive index, established in a coherently-prepared three-level ⁸⁵Rb atomic vapor under the electromagnetically induced transparency condition. This is achieved by interfering two coupling beams with Gaussian profiles and launching a probe beam that exhibits OBO within the resulting lattice. The induced reconfigurable photonic lattice possesses a transverse gradient, due to the innate edges of Gaussian beams, and sets a new stage for guiding the flow of light in periodic photonic environments. Our results should motivate better understanding of peculiar physical properties of an intriguing quantum-optical analogy in an atomic setting.

1. Introduction

Owing to the mathematical equivalence of the Schrödinger equation in quantum mechanics to the paraxial wave equation in photonics, a wide variety of phenomena in quantum mechanics have been put in close connection with the seemingly unrelated effects in optics. Most interestingly, some effects difficult or even impossible to observe in quantum mechanics have been explored in readily available optical systems [1–12]. This happy incident provided a chance for transferring wave dynamics from the temporal domain of wave functions directly to the spatial domain of optical fields. Such a transfer is known as the quantum-optical analogy [1], which offers an improved understanding of coherent quantum effects relevant to atomic or condensed matter physics, like the parity-time symmetry [2, 6], Anderson localization [11], Zeno effect [12], Bloch oscillation [13–32], and Zener tunneling [15, 18, 25, 32]. This is achieved by employing analogous optical effects, thus providing an effective demonstration of the validity of the analogy approach. In this respect, it is now accepted that photonic lattices and optical waveguide arrays play an especially important role in the exploration of various analogy concepts, which in turn help to introduce quantum concepts

as novel investigation tools in controlling the behavior of light. In fact, the imitation of certain coherent quantum transport behavior requires photonic lattices with desired spatial distributions of the refractive index. For instance, a longitudinal modulated optical waveguide array can be used to imitate the quantum anomalous Hall effect [4], which cannot be obtained in straight waveguide arrays. Photonic structures, like the arrays of coupled optical waveguides [13–15] and photonic crystals [14–19], with spatial permittivity gradients have been employed to investigate optical Bloch oscillation (OBO)—the optical analogy of the Bloch oscillation effect in the electronic transport in crystals [7, 20–23]. The introduction of permittivity gradients has been achieved by applying either an additional white light in photorefractive crystals [18, 24], a temperature gradient in thermo-optic materials [13] or sophisticated fabrication technologies in silicon-based materials [25].

Studies of OBO are currently active in newly-developed platforms, such as spin-orbit coupling systems [26], integrated photonic circuits [27, 28], photonic graphene [24], parity-time symmetric systems [29, 30], and photonic topological insulators [31]. OBO was also proposed in photonic lattices established via atomic coherence [32], which opened the door for expanding the study of OBO from commonly adopted solid-

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state materials to coherently-prepared atomic gases. Meanwhile, by taking advantage of the tunable atomic coherence (determining absorption, dispersion and nonlinearity) under the electro-magnetically induced transparency (EIT) [33], many phenomena of high current interest [34–41], for example pseudospin-mediated optical vortices [36, 37] and edge-state solitons [38], have been experimentally investigated in photonic lattices induced in coherent atomic media. These advances demonstrated the applicability of electromagnetically induced photonic lattices (EIPL) as promising candidates for future advancement of the field of quantum-optical analogies.

In this paper we theoretically establish and experimentally demonstrate the OBO in an EIPL with a ramping refractive index change induced in a three-level atomic vapor system. The lattice is constructed by interfering two coupling beams from the same laser source, and modulating the refractive index into a spatially-periodic arrangement under the EIT condition. The refractive index gradient of the lattice arises readily from the spatial intensity gradient of the Gaussian coupling beams themselves, without employing additional light fields or sophisticated fabrication technologies to achieve the same effect. As a result, the output of the incident probe beam exhibits discrete spatial pattern, due to the periodic profile of the refractive index. Further, by controlling the temperature of the atomic ensemble to increase atomic density, which is translated into the extension of propagation path of the probe beam inside the medium [42], the OBO is observed for both cases of exciting single and multiple channels (or waveguides) of the lattice. The current work provides an easily accessible platform and a convenient way for observing OBO in photonic environments.

2. The model

The schematics of the experimental setup is presented in Fig. 1a. Two laser fields are injected into an atomic vapor cell along the z direction, to drive a three-level Λ -type ^{85}Rb atomic configuration [Fig. 1b], which consists of two hyperfine states, $F = 2$ (level $|0\rangle$) and $F = 3$ ($|2\rangle$) of the ground state $5S_{1/2}$, and one excited state $5P_{1/2}$ ($|1\rangle$). The coupling field is introduced by the interference of two Gaussian beams E_2 and E_2' (frequency ω_2 , Rabi frequencies Ω_2 and Ω_2' , from the same external-cavity diode laser), which are injected into the vapor cell symmetrically about the z axis. They intersect at a small angle, forming a one-dimensional photonic lattice along the transverse x direction. Bearing in mind the Gaussian feature of the two coupling beams, the interference pattern exhibits a periodic spatial intensity distribution with a gradient, ramping from the center of the pattern towards both outsides along x direction. With the Gaussian probe field E_1 (ω_1 , 794.98 nm) from another external-cavity diode laser launched into the lattice, a periodic output (discrete diffraction) can be observed at the output surface of the cell in the range $\Delta_1 - \Delta_2 \in [-30, 30]$ MHz, formed by individual lattice waveguides [43, 44].

Theoretically, the propagation dynamics of the probe field E_1 can be described by the Schrödinger-like equation [45, 46]:

$$i \frac{\partial}{\partial z} E_1(x, z) = -\frac{1}{2k_1} \frac{\partial^2}{\partial x^2} E_1(x, z) + \frac{1}{2} k_1 \chi^{(1)} E_1(x, z) \quad (1)$$

where E_1 is the slowly-varying electric-field amplitude of the probe beam with the wavenumber $k_1 = 2\pi n_0/\lambda_1$, with λ_1 being the wavelength of the probe beam, and the susceptibility is defined as:

$$\chi^{(1)} = \frac{iN|\mu_{10}|^2\hbar\epsilon_0}{d_{10} + |\Omega_2 + \Omega_2'|^2/d_{20}} \quad (2)$$

where N is the atomic density, μ_{10} is the electric dipole moment for the transition $|0\rangle \rightarrow |1\rangle$, $d_{10} = \Gamma_{10} + i\Delta_1$ and $d_{20} = \Gamma_{20} + i(\Delta_1 - \Delta_2)$, with Γ_{ij} being the decay rate between states $|i\rangle$ and $|j\rangle$. The interference term $|\Omega_2 + \Omega_2'|^2 = |\Omega_2^2 + \Omega_2'^2 + 2\Omega_2\Omega_2'\cos(2k_c x)|\exp(-x^2/w_1)$, exhibiting a periodic distribution with a ramping intensity along x axis, acts as the coupling field E_c , which can effectively modulate the spatial distribution of the refractive index and the refractive index change

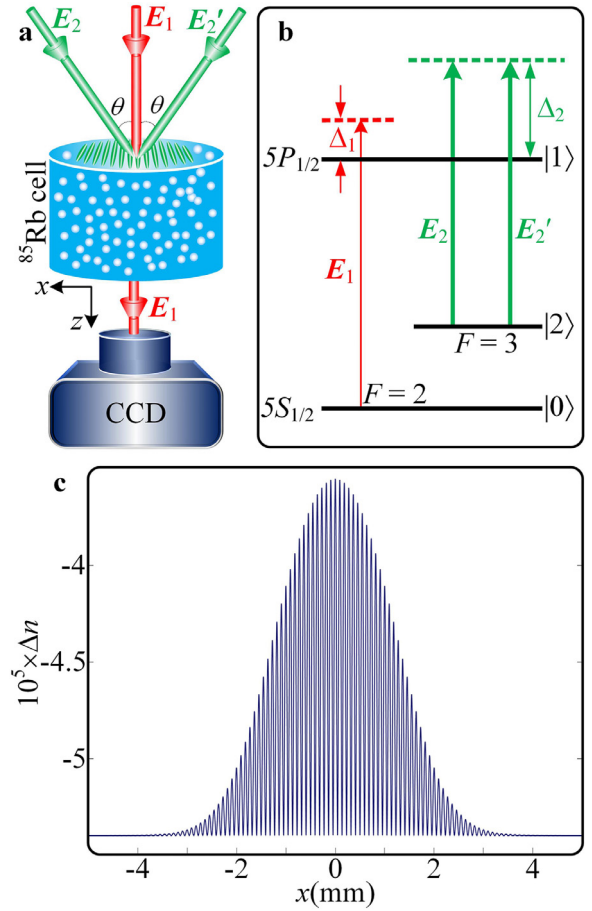


Fig. 1. (a) Experimental setup. The probe field E_1 is horizontally polarized, while the coupling beams E_2 and E_2' are both vertically polarized. The coupling beams intersect inside the rubidium vapor cell at an angle $2\theta \approx 0.5^\circ$, to form a standing-wave coupling field with a periodicity of $\sim 91\mu\text{m}$ along the transverse x direction. The 7.5 cm long cell is wrapped with μ -metal sheets and heated by a heat tape, to control the temperature of the vapor as well as the atomic density. (b) Atomic energy-level configuration. The term Δ_1 (resp. Δ_2) is the detuning between the resonant frequency for transition $|0\rangle \rightarrow |1\rangle$ (resp. $|2\rangle \rightarrow |1\rangle$) and the frequency of E_1 (resp. E_2 or E_2'). (c) The simulated refractive index distribution at $\Delta_1 = 0$ and $\Delta_2 = -4\text{ MHz}$.

$\Delta n = \text{Re}\{\chi^{(1)}\}/2$, is shown in Fig. 1c. Note that the refractive index change will not shift transversely along the longitudinal direction z , since there is no frequency difference between the two coupling beams. Here $k_c = 2\pi \sin(\theta)/\lambda_2$ and w_1 are the wave vector and the width of coupling beams, respectively. As a result, a photonic lattice with a gradient is established in the atomic vapor under the EIT condition, and the index gradient can give rise to the OBO effect during the travelling of the weak probe beam through the induced lattice.

We show the simulated OBO effect in Fig. 2, according to the dimensionless version of Eq. 1:

$$i \frac{\partial E_1}{\partial z} = -\frac{1}{2} \frac{\partial^2 E_1}{\partial x^2} + \frac{1}{2} k_1^2 r_0^2 \chi^{(1)} E_1 \quad (3)$$

where r_0 is the normalization parameter. Here, we still use x and z for the dimensionless coordinates. If we choose $r_0 = 5\mu\text{m}$, the diffraction length $k_1 r_0^2$ in this atomic vapor will be $\sim 0.2\text{ mm}$. Note that one has to launch the probe beam into the negative or positive side of the lattice sufficiently far away from the symmetry axis $x = 0$, to avoid harmonic-like oscillations. We first investigate the propagation of the probe beam exciting a single waveguide, as displayed in Fig. 2a. The probe beam firstly expands in a discrete diffraction way and then reverses its propagation to concentrate energy into one channel (around $z \sim 1700$, so that

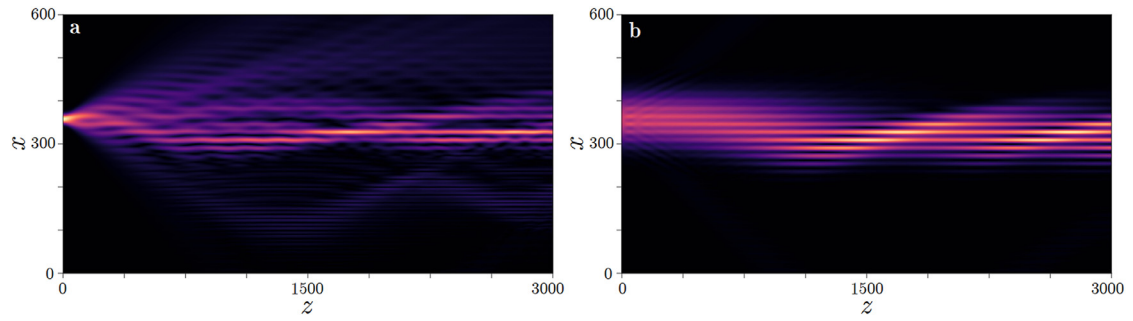


Fig. 2. Simulated OBO in the lattice according to the dimensionless version of the dynamic Eq. 1 corresponding to Fig. 1c, with the maximum Δn located at $x = 0$. (a) Only one lattice channel is excited. (b) Many channels are excited.

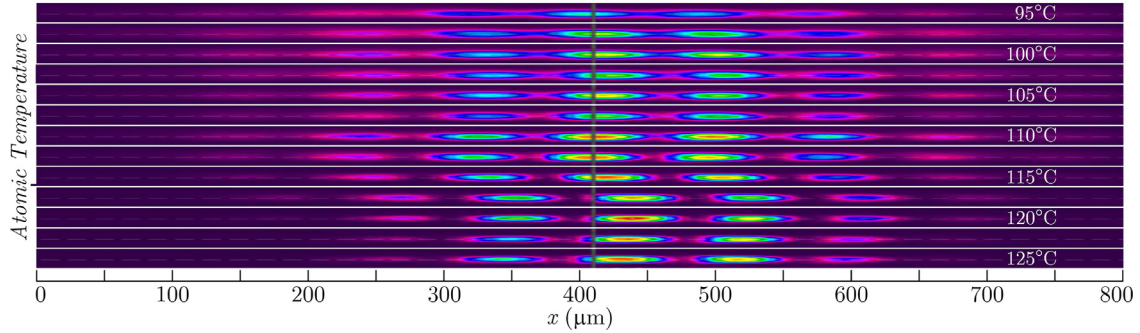


Fig. 3. Observed propagation patterns of a broad probe beam (covering several lattice channels) inside the lattice, demonstrating two cycles of OBO. In the top-to-bottom sequence, the panels show the output probe pattern change upon increasing the atomic temperature.

the period is about ~ 35 cm which is available in our experimental system) during propagation on the side $x > 0$. Further propagation of the beam also exhibits an OBO behavior but in a messy way, the reason being two-fold: the first one is that the gradient comes from the edge of a Gaussian beam which is not constant, and the second one is that the refractive index increases with density during propagation. Different from waveguide arrays fabricated in solid materials with femto-second laser writing technique [47], the longitudinal coordinate in atomic vapors is controlled by the temperature in the experiment. The higher the temperature, the bigger the density of atomic vapor and the longer the equivalent propagation distance. In our simulations, the effect of temperature is indicated in by the change of atomic density N during propagation. The same reasons are also responsible for the radiation during propagation. We boldly speculate that if one can prepare the EIPL using beams with triangular intensity profiles, the phenomenon might be strongly enhanced. When the launched Gaussian beam is wide and covering several channels (about 5 channels) of the lattice at $x > 0$, the OBO effect is displayed in Fig. 2b, with almost the same period as that in Fig. 2a. It is worth noting that during the OBO process, both the narrow and the wide probe beam are “attracted” towards $x = 0$ position, along which the refractive index change is at the maximum. This testifies about the existence of a “restoring” force acting in the system, necessary for the existence of oscillation.

3. Experimental results

As shown in Fig. 3, it shows the experimentally observed OBO of the incident broad probe beam by increasing the temperature of the medium from 95°C to 125°C. According to the observations at different temperatures (which correspond to different effective propagation distances), one can see that the output probe pattern first moves to the right (along the transverse x direction) and then gradually shifts back to the initial x position ($T = 95^\circ\text{C}$) at $T = 112.5^\circ\text{C}$. This shift exactly represents one cycle of the OBO, as predicted in Fig. 2b. It is worth noting

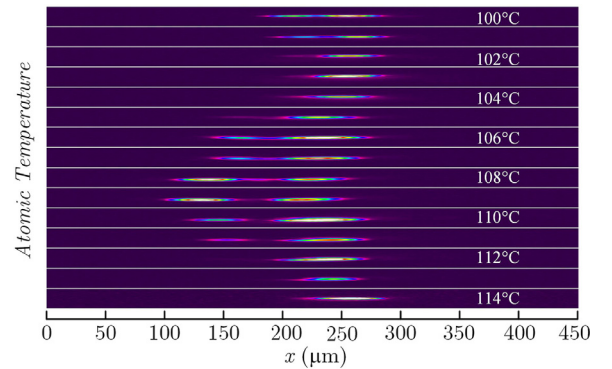


Fig. 4. Observed OBO of a narrow probe beam (covering only one lattice channel). Panels from top to bottom exhibit output diffraction patterns with increasing atomic temperature.

that the observed second oscillation (112.5 – 125°C) shows a perceptibly shorter cycle than the first oscillation (95 – 112.5°C), which agrees with the same tendency of variation of the oscillation cycle as shown in Fig. 2b. It is worth noting that the image at $T = 95^\circ\text{C}$ shows the output patterns of the probe beam that have travelled inside the photonic lattice for a certain distance, instead of $z = 0$ in the simulations of Fig. 2b.

As shown in Fig. 4, it demonstrates the OBO of a narrow probe beam covering only one waveguide at $x > 0$ of the coupling lattice. At $T = 100^\circ\text{C}$ the discrete pattern of the incident probe beam is clearly observed. In addition, the output discrete pattern gradually evolves from multiple fringes to only one fringe, and then jumps back to the multiple again. Also, most of the beam power focuses towards the $x = 0$ position of the highest refractive index change. These observations agree with the predicted OBO shown in Fig. 2a. Note that in the experimental demonstration, starting at position $T = 100^\circ\text{C}$, a diffracted pattern already occurred, instead of the theoretical initial point (the input plane),

because the narrow probe beam had already travelled through the lattice before change in the temperature.

4. Conclusion

In summary, we have theoretically predicted and experimentally demonstrated the OBO in an optically induced photonic lattice with an index gradient formed in a Rb atomic system with EIT. The paper presents the first experimental observation of OBO in a reconfigurable coherent multi-level atomic system. The induced photonic lattice with a permittivity gradient, resulting from the spatial Gaussian intensity gradient of the two coupling beams writing the waveguide lattice in the system, unlocks a new method for manipulating the flow of light in a periodic environment. Such an easily reconfigurable and tunable graded photonic lattice provides a unique platform for investigating analogous concepts in optics and quantum mechanics or condensed matter physics.

Declaration of Competing Interest

The authors declare that they have no conflicts of interest in this work.

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