Supplemental Document

Optics Letters

$\mathcal{PT}$-symmetric photonic lattices with type-II Dirac cones: supplement

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Supplement DOI: https://doi.org/10.6084/m9.figshare.26153050

Parent Article DOI: https://doi.org/10.1364/OL.530600

$\mathcal{PT}$-symmetric photonic lattices with type-II Dirac cones

For the conservative case in Fig. S1(a), the type-II Dirac cones are always there, even though the inter-cell coupling strength n may increase to a very high value. Correspondingly, there are always type-II exceptional rings in the band structures of the non-Hermitian lattices, as shown in Fig. S1(b). By the way, the band structures in the leftmost column in Fig. 2 in the main text and Fig. S1 are the same (shown in different windows). Increasing n only results in a larger separation between exceptional rings (Dirac cones for the conservative case) that are in $k_x > 0$ and $k_x < 0$ regions, which can be clearly seen from the imaginary part of the band structure in Fig. S1(c).

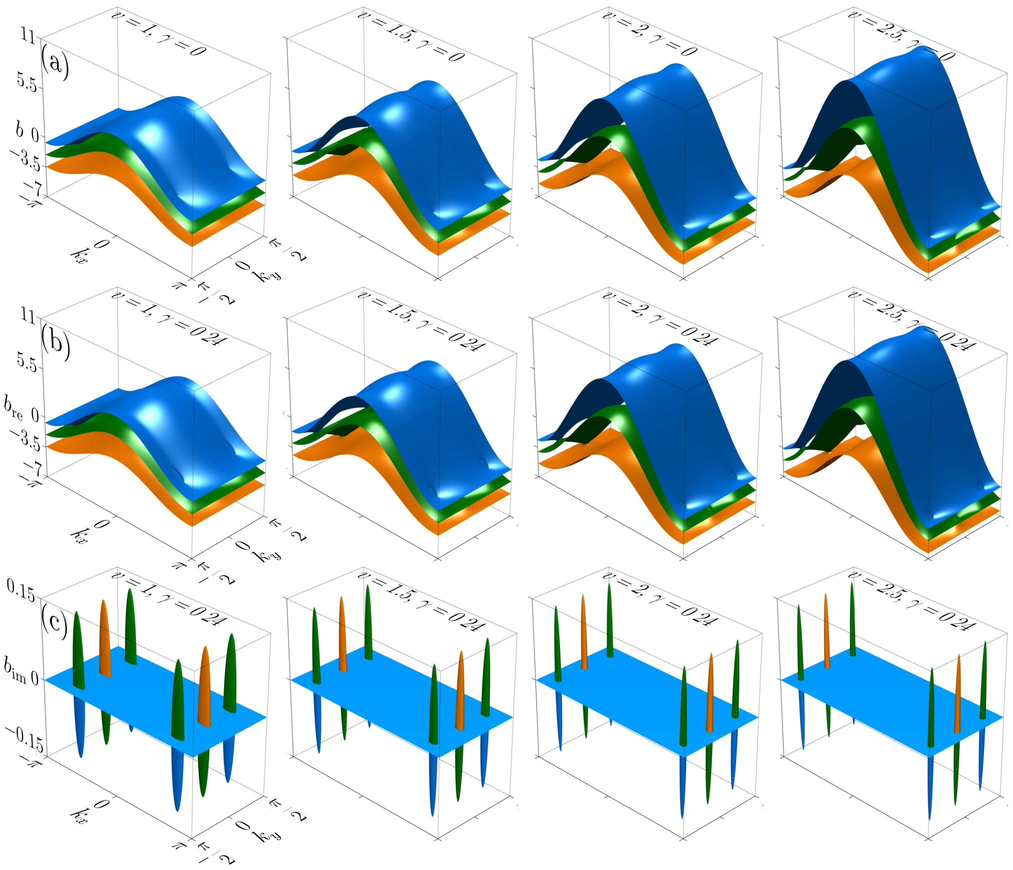

Fig. S1. Same as Fig. 2 in the main text, but under the condition $w \leq n$.

The input Gaussian beam, as shown in Fig. S2(a), is launched into the central waveguide of the array. The propagation dynamics of the light beam launched into the conservative lattice exhibits discrete diffraction, and the modulus profile at $z = 60$ is displayed in Fig. S2(b). Since the intra-cell three sites are closer than the inter-cell sites, the diffraction is faster along the y direction. The corresponding peak amplitude A is indicated by the blue curve in Fig. S2(e), which decreases during propagation. As mentioned above, the non-Hermitian system is $\mathcal{PT}$-symmetric when $p_{\mathrm{im}} = 0.5$. The output beam corresponding to this input beam is shown in Fig. S2(c), and the peak amplitude is shown by the orange curve in Fig. S2(e). One finds that the peak amplitude also decreases with distance, and the reason is that all eigenmodes of this non-Hermitian waveguide system are Hermitian, which cannot be either amplified or damped. However, the $\mathcal{PT}$ symmetry is broken when the value of $p_{\mathrm{im}} = 0.6$, and the input Gaussian beam may excite a gainy eigenmode that will be amplified during propagation. The output amplitude modulus for this case is shown in Fig. S2(d), with the corresponding peak amplitude illustrated by the green curve in Fig. S2(e). The green peak amplitude indeed does not decay and is explicitly higher than both the orange and blue ones. Further numerical simulations demonstrate that the green peak amplitude increases exponentially, which indicates the excitation of the amplified mode.

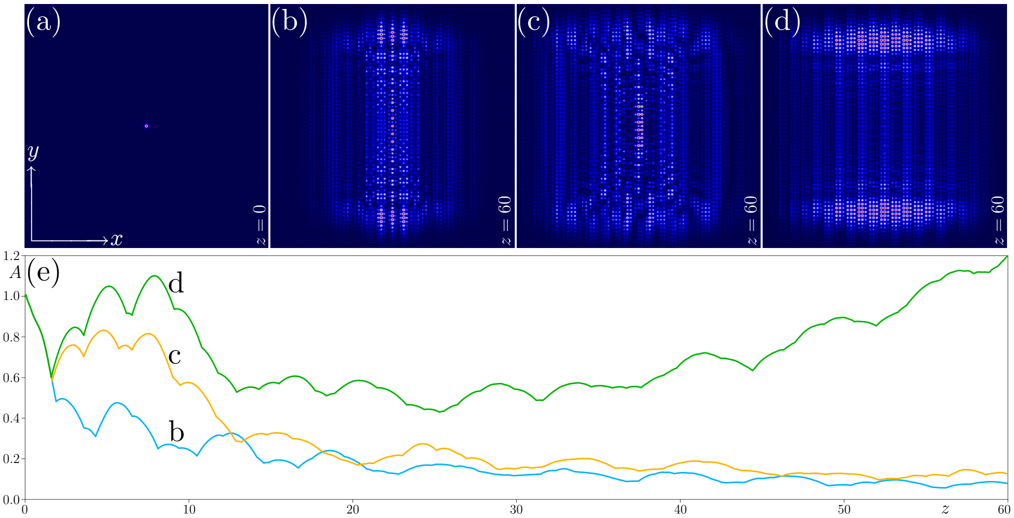

Fig. S2. Propagation dynamics of the light beam launched into the central waveguide. (a) Amplitude modulus of the input Gaussian beam. (b) Amplitude modulus of the beam at $z = 60$ in the conservative lattice. (c,d) Amplitude modulus of the beam at $z = 60$ in the non-Hermitian lattice with $p_{\rm im} = 0.5$ and $p_{\rm im} = 0.6$. (e) Peak amplitude of the beam during propagation. Other parameters: $d = 1.6$, $d_1 = d_2 = 1.4$, $p_{\rm re} = 10$, and $\sigma = 0.5$. Panels in (a)-(d) are shown in the window: $-50 \leq x, y \leq 50$.