

Optical Bloch oscillation and Zener tunneling in an atomic system

YIQI ZHANG,^{1,2,*} DA ZHANG,¹ ZHAOYANG ZHANG,^{1,2} CHANGBIAO LI,^{1,2} YANPENG ZHANG,^{1,6} FULI LI,² MILIVOJ R. BELIĆ,³ AND MIN XIAO^{4,5}

¹Key Laboratory for Physical Electronics and Devices of the Ministry of Education & Shaanxi Key Lab of Information Photonic Technique, Xi'an Jiaotong University, Xi'an 710049, China

²Department of Applied Physics, School of Science, Xi'an Jiaotong University, Xi'an 710049, China

³Science Program, Texas A&M University at Qatar, P.O. Box 23874 Doha, Qatar

⁴Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA

⁵National Laboratory of Solid State Microstructures and School of Physics, Nanjing University, Nanjing 210093, China

⁶e-mail: ypzhang@mail.xjtu.edu.cn

*Corresponding author: zhangyiqi@mail.xjtu.edu.cn

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Atomic media are important in optics research since they can be conveniently manipulated and controlled due to easy selection of atomic levels, laser fields, and the active adjustment of many system parameters. In this paper, we investigate optical Bloch oscillation, Zener tunneling, and Bloch–Zener oscillation in atomic media both theoretically and numerically. We use two coupling fields to prepare the dynamical optical lattice through interference. To induce a transverse force, we make the frequency difference between the two coupling fields increase linearly along the longitudinal coordinate. These phenomena have potential application for beam splitters and optical interconnects, and are helpful for investigating quantum analogies. © 2017 Optical Society of America

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1. INTRODUCTION

Quantum mechanics posits that electrons in a periodic lattice perform Bloch oscillation if a constant transverse electric field is applied to the system [1]. Such a phenomenon was predicted by Felix Bloch in 1929 and experimentally observed some 60 years later, in a semiconductor superlattice [2]. Study of Bloch oscillation is still an active topic, and to date Bloch oscillations are reported in but not limited to cold atoms [3–6], optical waveguides [7–10], photonic lattices [11–15], integrated photonic circuits [16,17], and non-Hermitian systems [18,19].

Optical Bloch oscillation (OBO), instead of considering the dynamics of electrons or atoms, focuses on the propagation of laser light in arrays of waveguides or in photonic crystals. Such a change in the focus of investigation and systems investigated is facilitated by the formal equivalence between the paraxial wave equation in photonics and the Schrödinger equation in quantum mechanics. If a sufficiently strong transverse force that acts on the beam is introduced [20], optical Zener tunneling (OZT) and the optical Bloch–Zener oscillation (OBZO) between Bloch bands can be observed [10,14,21]. In fact, OBO and OZT are the two typical examples of the so-called quantum-optical analogies [22]. The advantage of such analogies is that they map the temporal evolution of wavefunctions in coherent quantum phenomena onto the spatial propagation of optical fields in photonic

devices. Investigations of OBO and OZT show applicative potential for producing beam combiners, splitters, and interferometers. For a thorough survey of research progress in OBO and OZT, we suggest review papers [22–25]. It should be noted that, as pointed out in [21], BZO is different from Rabi oscillation [26–30], where an ac field is introduced to resonantly induce transitions between Bloch states of different bands.

As mentioned above, previous investigations of OBO and OZT were mainly undertaken in photonic lattices or waveguide arrays that can be prepared by the femtosecond laser writing technique or the multi-wave interference method. These techniques are usually applied to solid materials, such as silicon-based materials or photorefractive crystals. We wonder if one can extend OBO and OZT to non-solid materials, e.g., to atomic systems, in which the lattices are prepared dynamically, by utilizing the multi-wave interference method [31]. Being convenient optical media for various applications, atomic systems have been intensely investigated in the past few decades. To overcome large absorption in atomic systems, electromagnetically induced transparency (EIT) [32] was introduced and played an important role in generating multi-wave mixing processes [33]. It is interesting to note that many phenomena of high current interest, such as optical condensates [34,35], photonic topological insulators [36–38], (anti-)parity-time symmetric systems [39–41], and other phenomena [42,43], have been observed in atomic systems.

Even though the Bloch oscillation of cold atoms was reported before, the questions addressed here will provide a different point of view on light propagation in atomic systems. In this paper, we report OBO, OZT, and OBZO in an atomic system. We believe that most of these effects have not been explored before in atomic media, except for the OBO effect [44]. In comparison with Ref. [44], where nonlinearity is elaborately designed to obtain the OBO effect, we adopt a completely different method. A periodic lattice with a transverse force is constructed using the interference between two coupling fields with a linearly increasing frequency difference. In comparison with the investigations in solid waveguide arrays and photonic crystals, the atomic medium has unique advantages, such as the tunability of beam confinement (the potential depth), the variable spatial extent between optically induced waveguides, and adjustable loss (or even gain), to name a few. Our results can be utilized in fabricating beam splitters and optical interconnects, and also in investigating novel quantum analogies.

2. THEORETICAL MODEL

We consider light propagation in a Λ -type rubidium atomic system [42,43], as shown in Fig. 1(a). For energy levels $|0\rangle$, $|1\rangle$, and $|2\rangle$, we choose the $5S_{1/2}$ ($F=3$), $5P_{3/2}$, and $5S_{1/2}$ ($F=2$) states of ^{85}Rb , respectively. The probe susceptibility in such a system can be written as [45] $\chi^{(1)} = iN\mu_{10}^2[\hbar\epsilon_0(d_{10} + |G_{12} + G'_{12}|^2/d_{20})]^{-1}$, with N being the atomic density, μ_{10} the electric dipole moment, and $d_{10} = \Gamma_{10} + i\Delta_{10}$ and $d_{20} = \Gamma_{20} + i(\Delta_{10} - \Delta_{12})$ the complex decay rates. In Fig. 1(a), the probe field E_{10} connects the transition $|0\rangle \rightarrow |1\rangle$, and the coupling fields E_{12} and E'_{12} connect the transition $|1\rangle \rightarrow |2\rangle$. Γ_{ij} are the decay rates between $|i\rangle$ and $|j\rangle$ states, and $\Delta_{10} = \Omega_{10} - \omega_{10}$, $\Delta_{12} = \Omega_{12} - \omega_{12}$ and $\Delta'_{12} = \Omega_{12} - \omega'_{12}$ are the detunings. They are determined by the transition frequencies Ω_{ij} between $|i\rangle$ and $|j\rangle$, and by the frequencies ω_{10} and ω_{12} (ω'_{12}) of the probe and the coupling fields. G_{12} represents the Rabi frequency of the coupling fields, defined as $G_{12} = \mu_{12}E_{12}/\hbar$. In Fig. 1(b), the geometry of the probe and coupling fields is displayed. A small angle between the two coupling fields that are not exactly counterpropagating is

assumed. It should be noted that the Doppler broadening effect in the EIT window is considered even in hot atomic systems [46].

One can put the coupling fields together, as $E_c = E_{12} \exp[-i(\omega_{12}t - k_{12}x)] + E'_{12} \exp[-i(\omega'_{12}t + k'_{12}x)]$, which will form a standing wave [41,47]—the optically induced lattice (OIL)—as shown in Fig. 1(c1). If one deliberately prepares two coupling fields with a frequency difference $\delta = \omega'_{12} - \omega_{12}$, the OIL will move with a constant speed along the transverse coordinate, as shown in Fig. 1(c2), and the Rabi frequency can be written as $|G_{12} + G'_{12}|^2 = (\mu_{12}/\hbar)^2[E_{12}^2 + E'_{12}^2 + 2E_{12}E'_{12} \cos(\delta t + 2k_c x)]$. The speed v can be written as $v = -\delta/2k_c$ with $k_c = (k_{12} + k'_{12})/2$. We note that the speed of OIL can also be viewed as the slope of OIL along the t direction. If one fixes ω_{12} and changes ω'_{12} to detune the direction of motion and speed, one obtains the effective period of the moving OIL as $D = \pi/(k_{12} + \delta/2c)$, from which one can see that D is bigger for $\delta < 0$ than for $\delta > 0$, due to the Doppler effect. As a result, the reflectivity and the transmissivity of the same probe incidence will be different when launched into the OIL from directions which are nearly perpendicular to the case shown in Fig. 1(b); this property may be useful for producing an optical diode [42,43]. If δ is fixed, the OIL moves with a constant speed, while if δ increases with t linearly, i.e., $\delta = at$, the OIL moves with a linearly increasing speed (δ is much smaller than ω_{12} , and its influence on k_c can be neglected). This means that there is a constant acceleration a [3]; such an OIL is displayed in Fig. 1(c3). In a potential experiment, the frequency difference can be realized by an acousto-optic modulator.

We consider evolution of the probe E_{10} in such a lattice, as shown in Fig. 1(b). The governing equation can be described as a Schrödinger-like paraxial equation:

$$i \frac{\partial E_{10}}{\partial z} + \frac{1}{2k_{10}} \frac{\partial^2 E_{10}}{\partial x^2} + \frac{1}{2} k_{10} \chi^{(1)} E_{10} = 0, \quad (1)$$

where $z = vt$, and for the parameters used in this paper, v is around 1 m/s. For convenience, we set $v = 1$ m/s throughout. Since there is a frequency difference between the two coupling fields that results in the bending of OIL along the evolution direction, we introduce the transformation of coordinates $\tau = t$

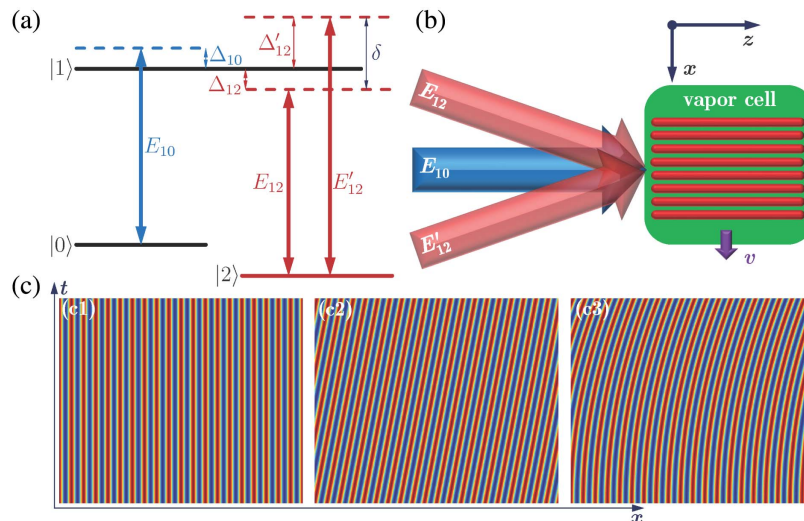


Fig. 1. (a) Λ -type energy system in a rubidium atomic system. (b) Geometry of the beams in (a). (c) Rabi frequency of the two interfering coupling fields for (c1) $\delta = 0$, (c2) δ being a constant, and (c3) δ increasing linearly with t .

and $\xi = x + \zeta(t)$ with $\zeta(t) = \delta t/2k_c$, to transform Eq. (1) in the bending frame of reference:

$$i \frac{\partial E_{10}}{\partial \tau} + i \frac{d\zeta}{d\tau} \frac{\partial E_{10}}{\partial \xi} + \frac{1}{2k_{10}} \frac{\partial^2 E_{10}}{\partial \xi^2} + \frac{1}{2} k_{10} \chi^{(1)} E_{10} = 0. \quad (2)$$

If one introduces the gauge transformation [48,49] $E_{10}(\xi, \tau) = \psi(\xi, \tau) \exp\{ik_{10}[-\xi(d\zeta/d\tau) + \frac{1}{2} \int_0^\tau [d\zeta(\alpha)/d\alpha]^2 d\alpha]\}$, Eq. (2) can be rewritten as

$$i \frac{\partial \psi}{\partial \tau} + \frac{1}{2k_{10}} \frac{\partial^2 \psi}{\partial \xi^2} + \frac{1}{2} k_{10} \chi^{(1)} \psi + k_{10} \frac{d^2 \zeta}{d\tau^2} \xi \psi = 0. \quad (3)$$

The last term in Eq. (3) acts as a transverse force. If δ is a constant [as in Figs. 1(a) and 1(b)], then the transverse force is zero. If δ increases linearly with τ (or t), there will be a fixed transverse force, which leads to the OBO and OZT of light during evolution. The period of OBO is determined by the transverse force and the period D of the OIL [10], as $T = 2k_c/(k_{10}|d^2\zeta/d\tau^2|)$.

3. NUMERICAL SIMULATIONS

We first consider the case $\delta = 0$. As expected, the beam will undergo discrete diffraction during evolution, as shown in Fig. 2(a). If $\delta \neq 0$, the OIL will be oblique in the (x, t) plane, as shown in Fig. 1(c2). Since the transverse force is still zero ($d^2\zeta/d\tau^2 = 0$), one can only observe a bent discrete diffraction, as displayed in Fig. 2(b). As the bending OIL can always be transformed into a straight lattice with a transverse force [Eq. (3)], we consider only the band structure of the straight lattice, which is exhibited in the inset of Fig. 2(a). One can see that there is a small bandgap between the bands.

When the frequency difference changes linearly with evolution time, the OIL will bend along a parabolic curve, as in Fig. 1(c). In this case, we numerically determine the beam evolution, according to both Eqs. (1) and (3). To start with, we consider the case when only one lattice waveguide is excited; the results are displayed in Fig. 3. In Fig. 3(a), the beam bends during evolution, due to the guidance from the OIL. One can note at least two interesting phenomena: the beam oscillates periodically (exhibits breather-like behavior), and the energy of the beam leaks from oscillation—which are the indicators of OBO and OZT. The evolution according to the transformed equation is depicted in

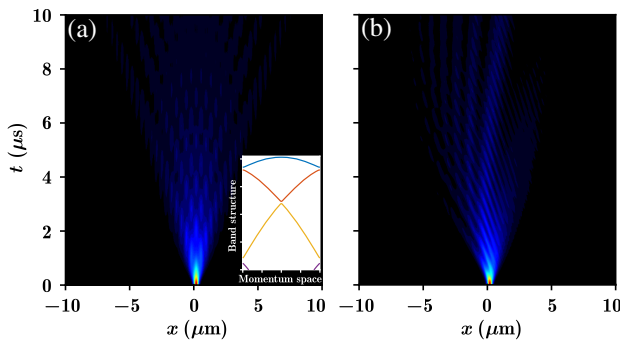


Fig. 2. Discrete diffraction. (a) $\delta = 0$. (b) $\delta = -10$ MHz. Inset shows the band structure. From top to bottom are the first, second, third, and fourth bands, respectively. Other parameters are: $\mu_{10} = 3 \times 10^{-29}$ C · m, $N = 2 \times 10^{13}$ cm $^{-3}$, $G_{12} = 30$ MHz, $G'_{12} = 25$ MHz, $\lambda_{10} = 770.792$ nm, $\lambda_{12} = 770.778$ nm, $\Gamma_{10} = 1$ MHz, $\Gamma_{20} = 2$ kHz, $\Delta_{12} = 0$, and $\Delta_{10} = -1$ MHz.

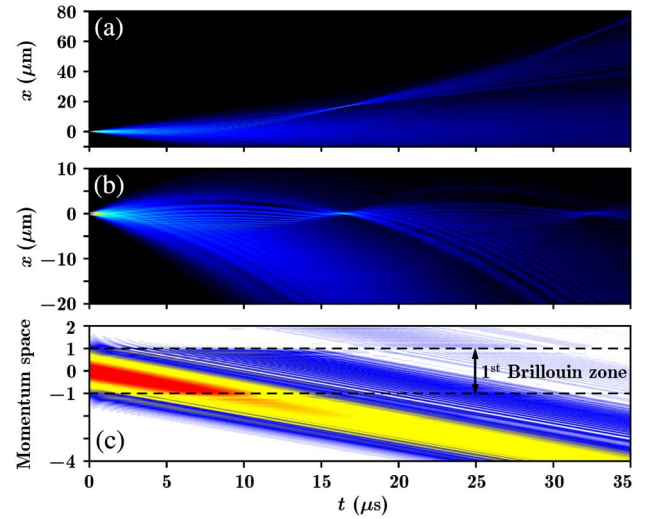


Fig. 3. Optical Bloch oscillation and Zener tunneling when only one lattice waveguide is excited. (a) According to Eq. (1). (b) According to Eq. (3). (c) Same as (b), but in the momentum space, which is normalized with k_{12} . We set $a = 1$ MHz 2 ; other parameters are the same as those in Fig. 2. To see the evolution dynamics explicitly, we show the amplitudes rather than the intensity in (a) and (b).

Fig. 3(b), in which the OBO and OZT are more easily observed. Analytically, the period of OBO is about $T \approx 16.3$ μ s, and the numerical result agrees with it very well. Since the OZT will dampen the OBO, it becomes weaker during evolution. In Fig. 3(c), we also present the evolution of Fig. 3(b) in the momentum space. One finds that the beam excites the whole Brillouin zone and indeed is not confined to it; the energy transfers to other bands over the whole OBO period, which by definition is the OZT.

If one excites more than one lattice waveguide, the evolution dynamics of the input beam is shown in Fig. 4. Again, one observes OBO and OZT quite clearly. However, different from the case in Fig. 3, where OZT happened in the whole OBO period, the OZT here happens only at the edge of the Brillouin zone, as shown by the vertical dashed lines that connect Figs. 4(b) and 4(c).

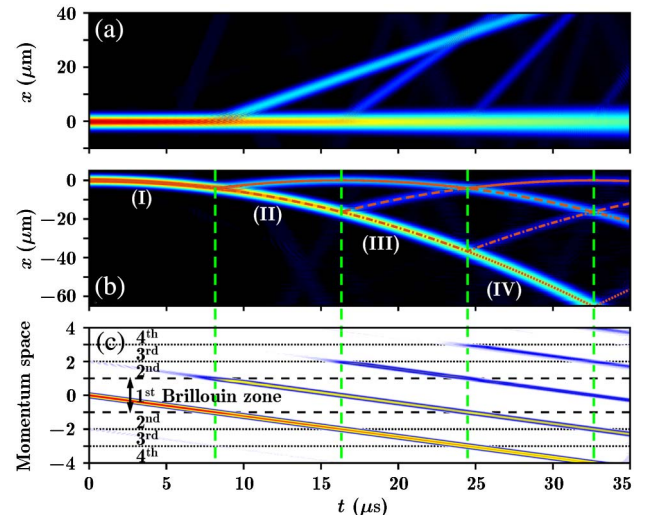


Fig. 4. Same as Fig. 3, but with many lattice waveguides excited.

One can see that the beam moves across the Brillouin zones at the edges successively during evolution. In Fig. 4(b), one can also note that there is OBZO in addition to OBO. We classify the oscillations in Fig. 4(b) into four categories, which are labeled by symbols (I), (II), (III), and (IV) and simultaneously indicated by the solid, dashed, dashed-dotted, and dotted curves. Case (I) is the OBO, while cases (II)–(IV) are the OBZO progressing across different bands. Together with the inset in Fig. 2(a), one can conclude that the beam will escape from one band to the next one at the Brillouin zone edge, due to the large transverse force and mini-gaps between the adjacent bands, to form OZT. On the other hand, the escaped beam will oscillate in each band, to form OBO and OBZO. Finally, a net-like structure is formed in this continuum model during evolution, as shown in Fig. 4(b). From this point of view, the property may have potential applications for on-chip beam splitters and optical interconnect fabrication.

Another evidence for the existence of OZT comes from the projection of the beam during evolution onto an orthogonalized Floquet–Bloch base [50]: $A_n(\kappa) = \frac{D}{2\pi} d_{\kappa n} \exp(-i\beta_{\kappa n}t) \int \Phi_{-\kappa n}^*(-x) E_{10}(x, t) dx$, where κ is the Bloch momentum, n is the band number, $\Phi_{\kappa n}(x) = \phi_{\kappa n}(x) / \sqrt{c_{\kappa n}}$, $c_{\kappa n} = \int_D \phi_{-\kappa n}^*(x) \phi_{\kappa n}(x) dx$, $d_{\kappa n} = \pm 1$, and $\phi_{\kappa n}(x)$ is the Floquet–Bloch mode, with $\beta_{\kappa n}$ being the corresponding eigenvalue. We exhibit the projection coefficients A_n of the beam during evolution in Fig. 5. Clearly, the beam excites the modes of other bands during evolution; in the beginning it excites the modes of mainly the first band and, as it evolves, it starts exciting the other modes. It is interesting to note that the projection coefficients at $t = 25 \mu s$ are quite small almost over the whole Brillouin zone, except at the boundaries. The reason is that this distance is almost 1.5 times the OBO period, which is close to the turning point, where the beam starts to escape from the OBO. From Fig. 4(c), one can infer that the beam is indeed quite close to the boundaries of different Brillouin zones ($t = 25 \mu s$ is close to the vertical dashed line).

We should note that the detunings and the intensity of coupling fields can be adjusted momentarily in atomic systems, which is more advantageous than in the lattices fabricated by femtosecond lasers in silicon-based materials. As a result, the value of $\chi^{(1)}$ can be made to increase, which will improve the confinement of the lattice, and the OZT will decrease to some extent. Therefore, the quality of OBO will become better. Admittedly, this system is also lossy, even in the EIT window. As a result, the beam intensity will decrease during evolution. According to previous literature [41,46], if one introduces an additional pump field into the system (changing it from a Λ -type to an N-type), the pumping

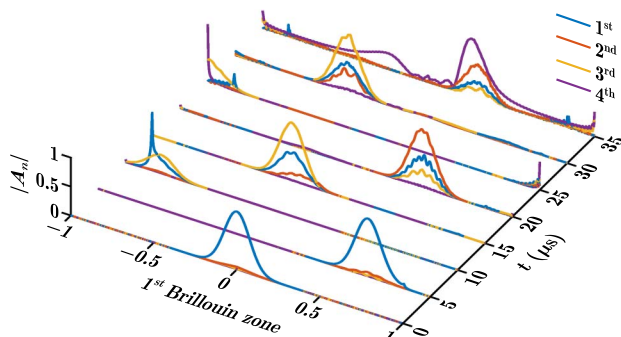


Fig. 5. Projection coefficients of the beam during evolution. Amplitude is normalized by $\max\{|A_{1\sim 4}|\}$.

gain will be obtained, which can compensate for the absorption losses.

4. CONCLUSION

In conclusion, we have shown that OBO, OZT, and OBZT can be observed in an atomic system. Two coupling beams with a linearly increasing frequency difference are used to form a bending OIL via interference. Due to the multi-parameter controllable properties of an atomic system, the phenomena obtained in this paper can be easily adjusted. Also, this investigation can be conveniently transplanted to atomic-like solid media, such as the praseodymium-doped yttrium orthosilicate crystal. We would like to note that, in an experiment, the diffraction patterns can be recorded using a method similar to [51]—that is, one puts a CCD camera at the output plane of the atomic cell to monitor the diffraction patterns in time. Our study not only shows the potential for fabricating beams splitters and optical interconnects, but also provides a new platform for observing optical quantum analogies.

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