



Edge states and edge solitons in counter-rotating trimer helical waveguide arrays

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Abstract: The trimer lattice is an extended Su-Schrieffer-Heeger (SSH) lattice that supports richer topological states than the conventional SSH lattice. Yet, the influence of the Floquet mechanism on these topological states remains largely unexplored. Here, we investigate trimer helical waveguide arrays featuring counter-directional rotations, analyzing both equal and unequal intra-cell coupling regimes. Our findings demonstrate that the effective Floquet mechanism—driven by the helical modulation of waveguide arrays—lifts the parametric Dirac points and opens topological band gaps that host robust pairs of boundary states. This elaborated Floquet scheme significantly expands the existence domains of edge states, as compared to those in straight waveguide arrays. Furthermore, incorporating optical nonlinearity into the model enables these topological states to form stable topological solitons whose propagation constants are continuously tuned by the beam power, while maintaining periodic dynamical stability throughout propagation. Our results establish a versatile photonic platform for robust beam localization and topological control of the nonlinear Floquet edge states.

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1. Introduction

The Su-Schrieffer-Heeger (SSH) model [1] is one of the simplest platforms for achieving topological insulators, and it has been extensively explored in topological photonics [2–5]. The SSH—also called the dimer lattice—contains two sites in one unit cell and is in the topological nontrivial phase when the intra-cell coupling is weaker than the inter-cell coupling. This topological phase transition is monitored using the well-known winding number, which is zero for the trivial phase and nonzero for the nontrivial phase. The underlying physical principle is also responsible for the formation of higher-order topological insulators [6–8] and edge states in strained lattices [9]. Over the past decade, SSH-based photonic topological insulators have been investigated not only in one-dimensional (1D) systems but also in 2D systems [10–13], and not only in Hermitian dynamical models [14,15] but also in general non-Hermitian models [16–19]. Adding an additional site to the unit cell of the SSH lattice yields the trimer lattice [20–27], and even more complex configurations arise in the extended SSH models if more sites are added [28,29]. As the simplest extended SSH model, the linear trimer lattice exhibits intricate band structures and multiple topological band gaps, enabling the emergence of sophisticated topological properties [22,23,26,30,31].

Topological manipulation in conventional and extended SSH lattice models is typically based on two main approaches. The first is the static modulation, relying on the relative variation of intra-cell and inter-cell coupling strengths, while the second involves the Floquet dynamic modulation via longitudinal periodic oscillations [32–35]. Nevertheless, studies combining Floquet modulation with the intrinsic static-coupling tuning of trimer lattices remain scarce in both linear and nonlinear regimes, apart from a handful of reports [25,36,37]. The Floquet mechanism plays a crucial role in topological photonics, because the first photonic topological

insulator realized without genuine magnetic fields relied on Floquet engineering [38] through periodic longitudinal modulation of photonic waveguide arrays [33,34,39–46]. This has emerged as a promising approach to synthesize artificial gauge fields [47–49] and manipulate topological properties in photonics. Extensive research has demonstrated that such gauge fields can break the effective time-reversal symmetry of photonic systems, trigger nontrivial topological phase transitions, and support a variety of intriguing topological phenomena. Benefiting from the Floquet modulation, numerous exotic topological configurations and transport behaviors have been theoretically proposed and experimentally observed, including anomalous topological-insulator transmission [39,40], bimorphic topological phases [50,51], and Rabi-like oscillations of topological edge states [52–55], to name a few. It has also been shown that the Floquet modulation can be connected with the Thouless pumping [56–60], now referred to as the Floquet-Thouless pumping [61–63].

In this work, we investigate linear topological states and their nonlinear counterparts in tunable trimer SSH waveguide arrays subjected to helical Floquet modulation. Unlike previously reported configurations, our helical trimer array features *a straight middle waveguide within each unit cell, flanked by two helical waveguides rotating in opposite directions*. Furthermore, we examine the influence of unequal intra-cell coupling strengths on these topological states. Although the system retains the conventional static coupling modulation mechanism of trimer SSH lattices, helical Floquet driving induces novel topological evolutionary behaviors that are inaccessible in purely static structures. Using a combination of continuing theoretical modeling and a self-consistent iterative numerical scheme [64], we systematically explore the band evolution, edge-state localization characteristics, and nonlinear state dynamics in the modulated system, identifying robust topological solitons with stable propagation ranges against perturbations. Our results demonstrate that integrating static coupling tuning with helical Floquet driving in trimer SSH lattices provides a versatile platform for the flexible manipulation of spatial optical localization.

2. Results and discussion

2.1. Theoretical model

The propagation of paraxial beams in an extended oscillating waveguide array inscribed in a focusing/defocusing nonlinear medium can be described by the paraxial wave equation:

$$i \frac{\partial \mathcal{E}}{\partial \zeta} = -\frac{1}{2k} \left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right) \mathcal{E} - \frac{k}{n_0} [\Delta n(\xi, \eta, \zeta) + n_2 |\mathcal{E}|^2] \mathcal{E}. \quad (1)$$

Here, $\mathcal{E}$ represents the complex amplitude of the light beam, ξ and η represent the transverse coordinates, and ζ is the longitudinal coordinate (propagation distance). $k = 2\pi n_0/\lambda$ is the wavenumber, with λ being the wavelength and n_0 the ambient refractive index. Δn is the refractive index change induced by the optical lattice and n_2 is the Kerr nonlinear refractive index coefficient that is determined by the material. To facilitate theoretical analysis and numerical simulations, we perform the dimensionless normalization of Eq. (1) by setting $x = \xi/r_0$, $y = \eta/r_0$, $z = \zeta/z_d$, $\mathcal{R}(x, y, z) = k^2 r_0^2 \Delta n/n_0$, and

$$\psi(x, y, z) = \mathcal{E} \sqrt{k^2 r_0^2 \frac{|n_2|}{n_0}},$$

where r_0 is the beam scaling width and $z_d = kr_0^2$ is the corresponding diffraction length. As a result, we end up with the dimensionless paraxial wave equation:

$$i \frac{\partial \psi}{\partial z} = -\frac{1}{2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi - \mathcal{R}(x, y, z) \psi - \sigma |\psi|^2 \psi, \quad (2)$$

which is mathematically equivalent to the cubic nonlinear Schrödinger equation in quantum mechanics, with the propagation distance z acting as an effective evolution time t .

Experimentally, helical waveguide arrays can be fabricated in fused silica via femtosecond laser direct writing [34,38–40,65–72]. We adopt the following typical parameters: $\lambda = 800$ nm, $r_0 = 10$ μ m, $w = 0.5$, $a = 2$ (corresponding to 20 μ m), $n_0 = 1.45$ and $p = 8$ corresponds to $\Delta n \approx 9.0 \times 10^{-4}$. The function $\mathcal{R}(x, y, z)$ models the waveguide array, whose transverse landscape is defined by a lattice of Gaussian profiles:

$$\mathcal{R}(x, y, z) = p \sum_m \left(\exp \left(-\frac{(x - x_{m-1} - r \sin \omega z)^2 + (y - r \cos \omega z + r)^2}{w^2} \right) + \exp \left(-\frac{(x - x_m)^2 + y^2}{w^2} \right) + \exp \left(-\frac{(x - x_{m+1} + r \sin \omega z)^2 + (y - r \cos \omega z + r)^2}{w^2} \right) \right), \quad (3)$$

where $p = k^2 r_0^2 \Delta n / n_0$ represents the depth of the potential, which is proportional to the refractive index change Δn and reflects the ability of the waveguide to confine the optical field. Furthermore, w is the width of the Gaussian function, $r = 0.3$ is the radius of the helical waveguide array with period $Z = 2\pi/\omega = 8$, and x_m denote the transverse lattice site positions at $z = 0$. The parameter $\sigma = \pm 1$ specifies the nonlinearity type: focusing for $\sigma = +1$ and defocusing for $\sigma = -1$.

In the above analysis, the slowly-varying envelope approximation—along with the adiabatic approximation—has been applied, but in the following analysis, the solutions to Eq. (2) are expressed as:

$$\psi(x, y, z) = u(x, y, z) \exp(ibz), \quad (4)$$

where $u(x, y, z)$ is the spatial envelope profile of the light beam and b is the corresponding propagation constant. Substituting Eq. (4) into Eq. (2) yields the general equation for the steady beam profiles:

$$bu = i \frac{\partial u}{\partial z} + \frac{1}{2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) u + \mathcal{R}(x, y, z)u + \sigma |u|^2 u \quad (5)$$

that is of the primary interest here.

2.2. Lattices under periodic boundary conditions and quasi-band structures

We present two types of deformed trimer lattices, in which the central site within each unit cell remains unchanged. The first type is shown in Fig. 1(a), where the separation between the other two sites is fixed, while they are displaced along the same direction simultaneously. The second type is illustrated in Fig. 1(b), where the two sites undergo displacements of equal magnitude in opposite directions. For convenience, we refer to them as Lattice 1 and Lattice 2 in the following text. Note that Lattice 1 loses its inversion symmetry after deformation, while Lattice 2 is unaffected. Since Lattice 1 lacks the inversion symmetry after deformation, the topological invariant associated with its Zak phase is non-quantized [73]. Numerous studies have confirmed that topological edge states can still exist in such systems [20,21,74]. The Zak phase of Lattice 2 has also been reported previously [22]. Since detailed information on the Zak phase for both types of lattices can be found in the previous literature [20,22], we do not discuss it again here.

For straight waveguide arrays, Eq. (5) can be rewritten as

$$bu = \frac{1}{2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) u + \mathcal{R}(x, y)u + \sigma |u|^2 u, \quad (6)$$

where $\mathcal{R}(x, y)$ is the reduction of $\mathcal{R}(x, y, z)$ to two variables. Neglecting the nonlinear term in Eq. (6), the band structure of the lattice—that is, the relation between the propagation constant b

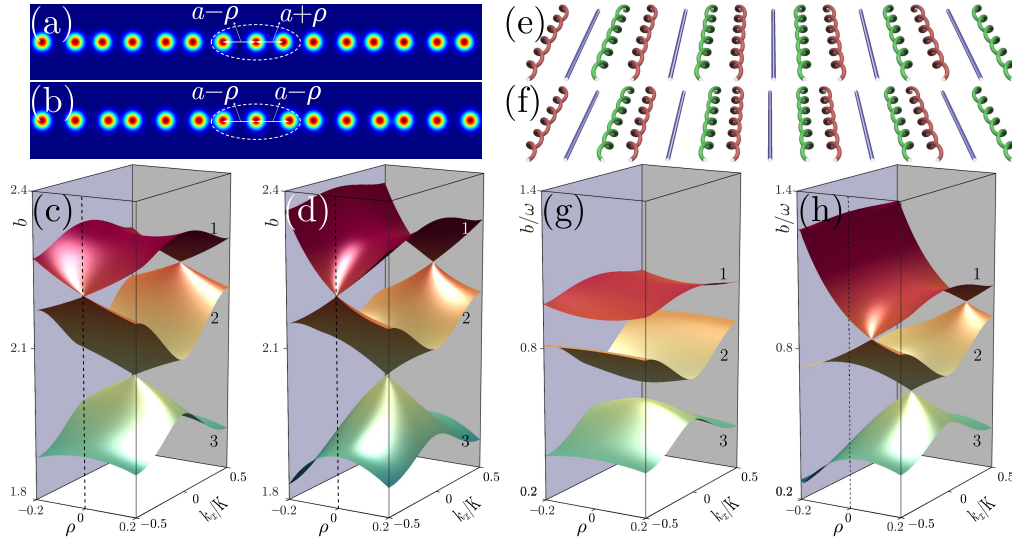


Fig. 1. (a) Trimer lattice with a representative unit cell enclosed by a dashed ellipse. The separation between adjacent unit cells is fixed at $3a$. The intra-cell separations are given by $a - \rho$ and $a + \rho$, where ρ is a real shift parameter (positive or negative). (b) Same setup as in (a), but with both intra-cell separations set to $a - \rho$. (c,d) Band structures for the lattices in (a,b). (e,f) Helical waveguide arrays corresponding to the lattices in (a,b). (g,h) Quasi-band structures for the helical waveguide arrays in (e,f).

and the Bloch momentum k_x —can be obtained using the plane-wave expansion method. The band structures of Lattice 1 in Fig. 1(a) and of Lattice 2 in Fig. 1(b) are displayed in Figs. 1(c) and 1(d), respectively. It is expected that both deformation schemes with $\rho \neq 0$ could open parametric Dirac points in the band structure and give rise to band gaps.

As mentioned above, the helical waveguide array is often employed to break the effective time reversal symmetry of a system, since it generates a gauge field. Neglecting the nonlinear term in Eq. (4), the quasi-band structures of helical waveguide arrays—i.e., Floquet systems—can be obtained numerically, using the propagation and projection method [43,44,75,76]. The quasi-band structures of the helical waveguide arrays shown in Figs. 1(e) and 1(f) are displayed in Figs. 1(g) and 1(h), respectively. As expected, band gaps persist throughout Fig. 1(g), while parametric Dirac points exist in Fig. 1(h), although their positions are shifted away from $\rho = 0$. However, for the helically-modulated waveguide arrays of Lattice 1 and Lattice 2, the broken inversion symmetry renders the Zak phase inapplicable for describing the topological behavior of the system.

2.3. Lattices under open boundary conditions and edge states

We consider 11 unit cells for Lattice 1 and Lattice 2 — as illustrated in Figs. 2(a,b) — with the unit cells indicated by dashed ellipses. In numerical simulations of this finite lattice, we use a computational domain $-40 \leq x \leq 40$, $-5 \leq y \leq 5$ that is sufficiently large to ensure that the results are not influenced by the boundaries. As can be seen from the spectrum of Lattice 1 in Fig. 2(c), which is symmetric about $\rho = 0$, one boundary state is hosted within each of the two band gaps. These edge states never vanish with the variation of ρ , as highlighted by the red and blue curves. These edge states are not in the band gap but close to the bulk bands. The phenomenon observed here is consistent with the Zak phase and spectrum reported in previous literature [20], based on the tight-binding method [77]. Although edge states always exist for

both $\rho > 0$ and $\rho < 0$, their positions are not the same: they localize at the left (right) boundary of the lattice for $\rho > 0$ ($\rho < 0$). Figure 2(e) illustrates the spatial profiles of two edge states (labeled 1 and 2) in the upper band gap and another two edge states (labeled 3 and 4) confined within the lower band gap of the static waveguide array.

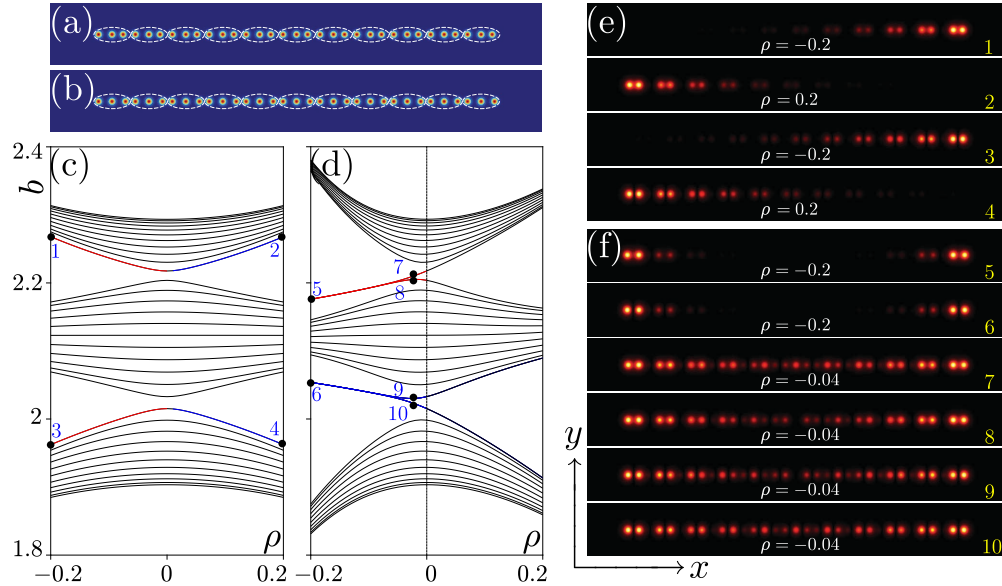


Fig. 2. (a) Lattice 1 under open boundary conditions, with its spectrum b vs ρ shown in (c). (b,d) Same as (a,c) but for Lattice 2. (e,f) Field modulus profiles $|\psi|$ of edge states corresponding to the numbered dots in (c,d).

Results for the straight lattice are shown in Figs. 2(d,f), which are similar to those reported previously [22,23], so that we do not discuss them here. We only note from the field modulus profiles in Figs. 2(e,f), that for both Lattice 1 and Lattice 2, edge states from the upper and lower band gaps differ in their phase configurations: the bright spots at the two outermost sites are in-phase for the edge states in the upper gap, whereas they are out-of-phase for the edge states in the lower gap. Actually, the edge states at different ρ in the same band gap generally share similar phase distributions.

Corresponding to the spectra in Figs. 2(c,d), the quasi-spectra of the helical waveguide arrays under open boundary conditions are shown in Figs. 3(a,b). In Fig. 3(a), there are two pairs of crossing edge states that are well isolated in the band gaps, which are highlighted in red and blue. The edge states marked in red reside on the right boundary of the helical version of Lattice 1, while the ones in blue reside on the left boundary. By comparing the spectra in Figs. 2(c) and 3(a), one clearly sees that Floquet engineering generates the band gap and doubles the existence regions for the edge states marked in red and blue. We select four edge states and present their field modulus profiles in Fig. 3(c). Among them, the edge states labeled 1 and 2 exhibit strong localization, whereas those labeled 3 and 4 are less localized. This difference arises because the states labeled 3 and 4 lie in the proximity of the middle bulk band and thus carry the characteristics of bulk states. We do not show the field modulus profiles of edge states in the bottom band gap, since they are quite similar to those shown in Fig. 3(c), except that the phases of the outermost two spots become out-of-phase.

For the quasi-spectrum of the helical version of Lattice 2 shown in Fig. 3(b), the behavior is quite similar to that in Fig. 2(d), where the edge states remain degenerate. However, the existence region of the edge states is broadened and is no longer confined to $\rho < 0$, particularly for the edge

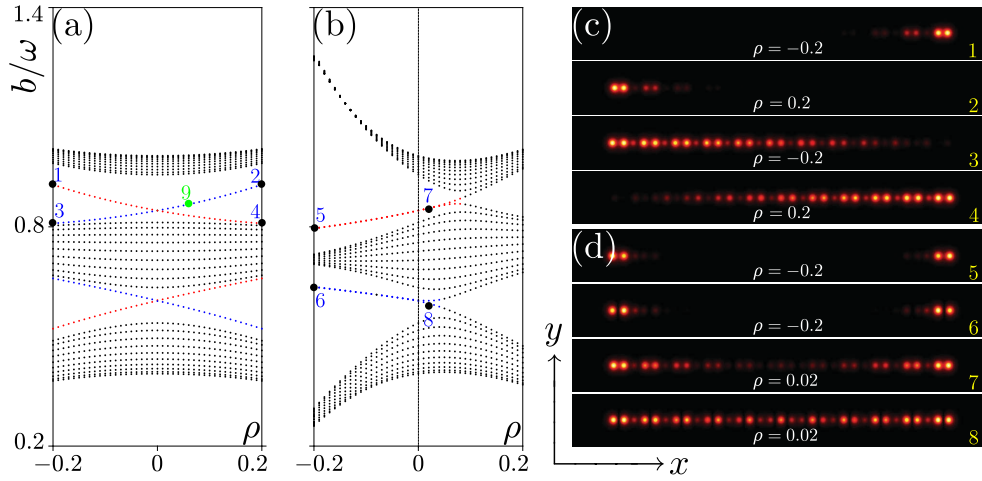


Fig. 3. (a,b) Quasi-spectra of helical waveguide arrays corresponding to spectra in Figs. 2(c,d). (c,d) Field modulus profiles $|\psi|$ of edge states corresponding to numbered dots in (a,b).

states depicted in red in Fig. 3(b). Undoubtedly, this is a consequence of the Floquet engineering enabled by the specific helical modulation adopted here. In Fig. 3(b), the edge states labeled 5 and 6 are selected from the red and blue branches, and their field modulus profiles are shown in Fig. 3(d). They exhibit strong localization but display distinct phase characteristics at the two outermost sites: edge state 5 is in-phase, while edge state 6 is out-of-phase. Since both the red and blue edge state branches extend to the region $\rho > 0$, we choose two edge states at $\rho = 0.02$, labeled 7 and 8 from these branches, and display their field modulus profiles in Fig. 3(d). Even though the localization is weaker, most of the energy still resides on the boundaries of the helical waveguide arrays.

In the straight waveguide array, the bulk-edge correspondence is characterized by a winding number [20,22]. This provides a topological baseline for the bulk-edge correspondence in the Floquet system, where an artificial gauge field [49] opens a topological gap, so that the resulting edge states are spectrally isolated from the bulk bands. However, constructing the topological invariant for the counter-rotating helical waveguide array remains challenging and will be explored in our future work.

2.4. Floquet edge solitons

As shown above, Floquet engineering with counter-rotating modulations gives rise to novel edge states and intriguing phenomena in trimer lattices. In this section, we incorporate nonlinearity and investigate the Floquet edge solitons that bifurcate from the linear edge states shown in Fig. 3. Without loss of generality, we select the edge state labeled 9 in Fig. 3(a) as our representative example.

The soliton family in the helical waveguide arrays can be obtained using the iterative projection method [44,64]. The procedure for calculating the nonlinear Floquet-Bloch modes consists of the following steps, each accompanied by an illustrative MATLAB code:

- (1) We propagate the linear edge state ψ_e^{in} from Eq. (5) with a given power P —which ultimately determines the quasi-propagation constant of the nonlinear Floquet-Bloch state—to obtain the dynamical lattice modified by the nonlinearity, i.e. $\mathcal{R}_e = \mathcal{R} + |\psi_e^{\text{in}}|^2$.

```
%all linear states including the edge state
```

%dx and dy are grid spacings

psi_in=sqrt(P)*psi_in/sqrt(sum(abs(psi_in(:)).^2*dx*dy));

R_e=R+abs(psi_in_e).^2;

- (2) We propagate all linear eigenstates $\psi_{n \in N}^{\text{in}}$ of $\mathcal{R}$ (including ψ_e^{in}) in the modified dynamical lattice $\mathcal{R}_e$ over a full period Z to obtain the corresponding output distributions $\psi_{n \in N}^{\text{out}}$, where N is the number of eigenstates.

%myprop is the subroutine for propagation

%dz is propagation step length

%wx and wy are coordinates in Fourier space

psi_out=myprop(psi_in,wx,wy,dz,R_e);

psi_in_e=myprop(psi_in_e,wx,wy,dz,R_e);

- (3) We calculate the projections $U_{mn} = \langle \psi_m^{\text{in}}, \psi_n^{\text{out}} \rangle$, whose eigenvalues are the Floquet exponents $\exp(iZb_n)$.

for m=1:N

for n=1:N

U_mn(m,n)=sum(conj(psi_in_m(:)).*psi_out_n(:))*dx*dy;

end

end

[V,D]=eig(U_mn); % D=exp(i*Z*b_n)

- (4) For each b_n , we find the index ℓ_n corresponding to the maximum element of the associated eigenvector V_n . The constructed eigenstates $\psi_{n \in N}^r$ can then be obtained according to $\psi_n^r = \sum_{q=1}^N \psi_q^{\text{in}} V_q(\ell_n)$.

for n=1:N

ell_n=find(max(V(n,:))==V(n,:));

psi_r_n=0;

for q=1:N

psi_r_n=psi_r_n+psi_in_q*V(q,ell_n);

end

end

- (5) We isolate the modified edge state ψ_e^r from $\psi_{n \in N}^r$ and normalize it to the given power P .

psi_r_e=sqrt(P)*psi_r_e/sqrt(sum(abs(psi_r_e(:)).^2*dx*dy));

- (6) Steps (1)-(5) are repeated iteratively until the difference between ψ_e^r in two close iterations falls below a predetermined tolerance.

error=max(abs(psi_r_e_loop2-psi_r_e_loop1));

if error < 1e-10

break;

end

The numerically obtained Floquet edge soliton family is displayed in Fig. 4(a), where the power P and the average amplitude $\bar{A}$ are defined as:

$$P = \iint |\psi|^2 dx dy, \quad \bar{A} = \frac{1}{Z} \int_z^{z+Z} A(z) dz, \quad (7)$$

with $A(z) = \max\{|\psi(z)|\}$ representing the peak amplitude of the Floquet edge soliton. Assuming a lossless medium, the power P remains conserved along z . Clearly, when the soliton power P increases, the quasi-propagation constant b within the band gap increases as well, and gradually approaches the bulk band (indicated by the gray region), where we terminate our calculations to prevent coupling with bulk states. When the power approaches zero, the Floquet edge soliton smoothly degenerates into the linear Floquet edge state, thereby confirming that the Floquet edge soliton bifurcates directly from its linear counterpart.

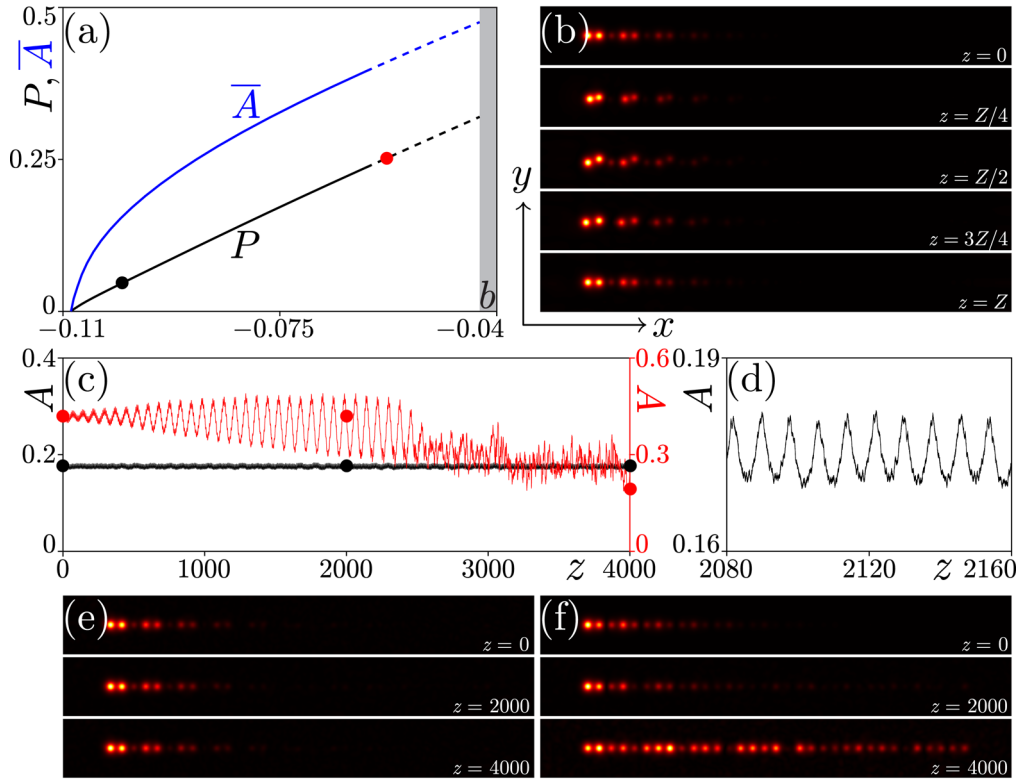


Fig. 4. (a) Edge soliton family bifurcating from the linear edge state at $\rho = 0.06$, corresponding to dot 9 in Fig. 3(a). Blue (black) lines represent the average peak amplitude $\bar{A}$ (power P) of the soliton as a function of the quasi-propagation constant b . The shaded region represents the bulk band, while solid and dashed lines represent stable and unstable solitons. (b) Field modulus distributions $|\psi|$ of the Floquet edge soliton [black dot in (a)] at different distances over one longitudinal period. (c) Peak amplitude A vs propagation distance z for the stable [black curve, left y axis] and unstable [red curve, right y axis] solitons, corresponding to the black and red dots in (a). (d) Magnified view of the black curve in (c), spanning 10 periods. (e) Field modulus distributions of the stable Floquet edge soliton [black dot in (a), $P = 0.05$] at selected integer-period distances [marked by black dots in (c)]. (f) Same as (e), but for the unstable Floquet edge soliton [red dot in (a), $P = 0.25$].

Floquet edge solitons in helically modulated trimer arrays—much like their linear counterparts—are dynamically oscillating states that periodically reproduce their profiles during

propagation. To illustrate this phenomenon, we highlight a specific soliton with $P = 0.05$ [corresponding to the black dot in Fig. 4(a)] and presented its modulus profiles in Fig. 4(b) at five representative distances across a single period: $z = 0$, $z = Z/4$, $z = Z/2$, $z = 3Z/4$, and $z = Z$. The exact profile replication between $z = 0$ and $z = Z$ is evident.

Having identified these Floquet edge solitons, we next analyze their structural stability. Although linear stability analysis provides a more rigorous criterion for determining soliton stability than direct numerical propagation, our previous studies [69,70] have demonstrated that both approaches yield consistent results and agree remarkably well. Therefore, given the complexity of the counter-rotating waveguide array system considered here, we employ direct numerical propagation to examine soliton stability. We impose small random perturbations into the soliton profiles via $\psi(x, y)[1 + \delta(x, y)]$, with $\delta(x, y) \in [-0.10, 0.10]$ being a uniform random noise, and run long-distance propagation simulations of Eq. (2) up to $z \sim 4000$ —a distance far exceeding typical experimental sample lengths. A soliton is classified as stable if it preserves its localized profile over long distances under perturbation; otherwise, it is deemed unstable. In Fig. 4(a), solid curves represent stable Floquet edge solitons, whereas dashed curves designate unstable ones. The evolution of the peak amplitude A along the propagation distance z for both stable and unstable solitons corresponding to the black and red dots in Fig. 4(a), is shown by the black and red curves in Fig. 4(c).

For the stable soliton (black curve), the amplitude neither decays nor brakes down into disordered patterns; instead, it maintains a robust periodic oscillation throughout propagation. Figure 4(d) provides a zoomed-in view of this amplitude over the range $2080 \leq z \leq 2160$, highlighting its periodic behavior. As confirmed by the field modulus profiles in Fig. 4(e), the soliton remains unchanged at selected distances (integer period intervals). This stable dynamics is further illustrated in [Visualization 1](#). In contrast, the amplitude of the unstable state—red curve in Fig. 4(c)]—exhibits periodic oscillations only during the initial stages of propagation. Subsequently, it breaks down into irregular fluctuations and eventually deteriorates. As shown by the field profiles in Fig. 4(f), the unstable soliton experiences significant radiation leakage into the bulk and finally broadens severely in the course of propagation. This unstable evolution is captured in [Visualization 2](#).

3. Conclusions and outlook

In summary, we have theoretically proposed two types of 1D Floquet trimer waveguide arrays driven by counter-rotating modulations, revealing novel edge states hosted by these structures. Thanks to elaborate Floquet engineering, the existence domains of edge states can be greatly expanded by tuning the shift parameter ρ in both configurations. In the presence of nonlinearity, Floquet edge solitons bifurcate from their linear counterparts and exhibit robust stability across a broad range of quasi-propagation constants.

Our findings offer rich opportunities for realizing exotic edge states via periodic helical modulations of waveguide arrays. A natural and promising extension is to generalize the counter-rotating Floquet driving from 1D to 2D settings. In 2D, counter-rotating helical modulations can be applied to different sublattices, rows, or columns in a waveguide array—for example, by displacing two sublattices along circular trajectories with opposite chiralities. Such a driving protocol is expected to generate effective gauge fields [49] and Floquet band structures that support anomalous edge states [47,78], corner states [79], and potentially higher-order topological phases.

The shift parameter ρ —controlling existence domains of edge states in 1D—will play an analogous role in 2D, by tuning effective coupling phases and the localization of edge modes. Moreover, because edge states are protected by driving-induced effective gauge fields rather than strict lattice periodicity, they may persist in aperiodic 2D architectures, such as Penrose tilings and lattices with disclination defects [7,8,80–82]. This could enable reconfigurable

topological edge states and solitons in defect-engineered 2D waveguide arrays. Technically, the main challenges include designing experimentally feasible 2D counter-rotating modulation geometries, suppressing coupling between Floquet replicas, and extending the nonlinear stability analysis to 2D. We leave these interesting problems for future work.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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