# The *Zhang lattice*: a simple lattice naturally has type-II Dirac points

Yiqi Zhang (张贻齐)[*]
*Key Laboratory for Physical Electronics and Devices of the Ministry*
*of Education & Shaanxi Key Lab of Information Photonic Technique,*
*School of Electronic and Information Engineering,*
*Xi'an Jiaotong University, Xi'an 710049, China*
(Dated: June 13, 2023)

I review the discovery as well as the band structure of the *Zhang lattice*.

Dirac materials [1] are special since there are Dirac points in their corresponding band structures, which means that the excited quasi-particle is fermionic and obeys the massless Dirac equation [2]. Probably, the most frquently used Dirac materials are the graphene (or the honeycomb lattice), the Lieb lattice and the kagome lattice [3]. However, the Dirac point supported by these lattices are type-I [4]. Considering half-filling, the Fermi surface is a point for the type-I Dirac point, a line for the type-III Dirac point and a pair of crossing lines for the type-II Dirac point. In comparison with the materials that support type-I Dirac point, materials that support the type-II and the type-III Dirac points are much more rare, especially those support the type-II Dirac point. To obtain the type-II Dirac point, various methods are developed [5–10]. I would like to state that all the efforts are not direct. One cannot help wondering that is there a kind of material or artificial material that naturally possesses the type-II Dirac point, just like a graphene possessing the type-I Dirac point? To this end, I started pondering on this question since I noticed Ref. [6], and finally succeeded in 2020 [11] after many attempts and efforts [12–14].
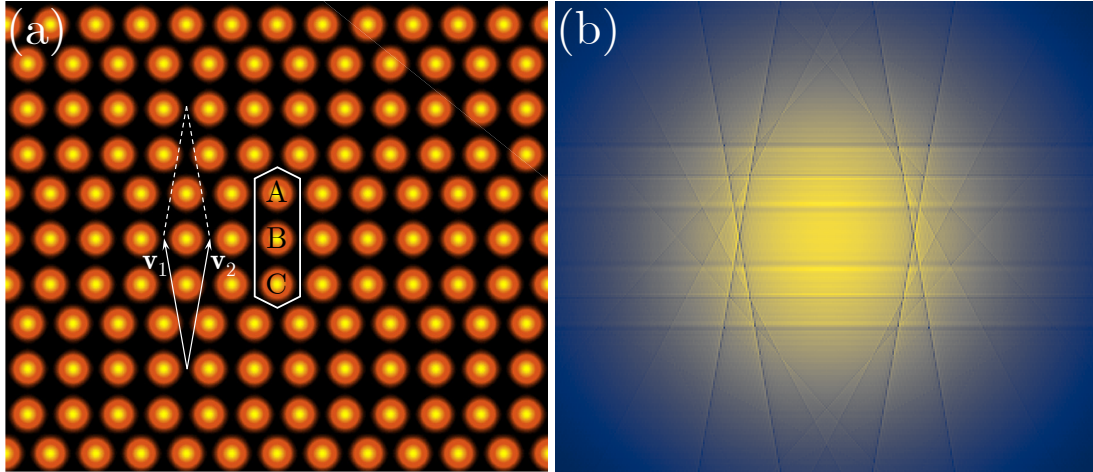


FIG. 1. (a) Landscape of the *Zhang lattice*. In each unit cell (indicated by a hexagon) there are three sites labeled as A, B and C. The basis vectors of the Bravais lattice are $\mathbf{v}_1 = [-a/2, 2a + \sqrt{3}a/2]$ and $\mathbf{v}_2 = [+a/2, 2a + \sqrt{3}a/2]$ with $a$ being the lattice constant. (b) Far-field diffraction pattern with the innermost hexagon being the first Brillouin zone.

The designed lattice, i.e., the *Zhang lattice*, is shown in Fig. 1(a), which has three sites in one unit cell. Its corresponding far-field diffraction pattern in Fig. 1(b) shows the first Brillouin zone (the innermost hexagon) clearly. In Ref. [11], the six corners of the first Brillouin zone are exhibited:

$$\left( \pm \frac{(20 + 8\sqrt{3})\pi}{(19 + 8\sqrt{3})a}, 0 \right) \quad \text{and} \quad \left( \pm \frac{(18 + 8\sqrt{3})\pi}{(19 + 8\sqrt{3})a}, \pm \frac{2\pi}{(4 + \sqrt{3})a} \right). \tag{1}$$

Therefore, I believe the complexity of the *Zhang lattice* is in the same level as that of the Lieb lattice and the kagome lattice, and the *Zhang lattice* is a simple lattice.

The band structure of the *Zhang lattice* was calculated based on the both discrete model (i.e., the tight-binding method) and the continuous model (i.e., the Schrödinger-like paraxial wave equation), and both results demonstrate

* zhangyiqi@xjtu.edu.cn

the existence of the type-II Dirac points. According to the tight-binding method with merely nearest-neighbor hopping being considered, the Hamiltonian can be written as

$$
\mathcal{H} = t \begin{bmatrix} 2\cos[\mathbf{k}\cdot(\mathbf{v}_2-\mathbf{v}_1)] & 1 & \exp(-i\mathbf{k}\cdot\mathbf{v}_1)+\exp(-i\mathbf{k}\cdot\mathbf{v}_2) \\ 1 & 2\cos[\mathbf{k}\cdot(\mathbf{v}_2-\mathbf{v}_1)] & 1 \\ \exp(+i\mathbf{k}\cdot\mathbf{v}_1)+\exp(+i\mathbf{k}\cdot\mathbf{v}_2) & 1 & 2\cos[\mathbf{k}\cdot(\mathbf{v}_2-\mathbf{v}_1)] \end{bmatrix},
\tag{2}
$$

with $\mathbf{k} = [k_x, k_y]$ being the Bloch momentum and $t$ the hopping strength. The band structure, i.e., the eigenvalues of the Hamiltonian versus $k_x$ and $k_y$, is shown in Fig. 2(a). There are three bands in the band structure, and the intersections between each two bands are type-II Dirac points. The appearance of the type-II Dirac point is natural and no additional operation is required onto the *Zhang lattice*.
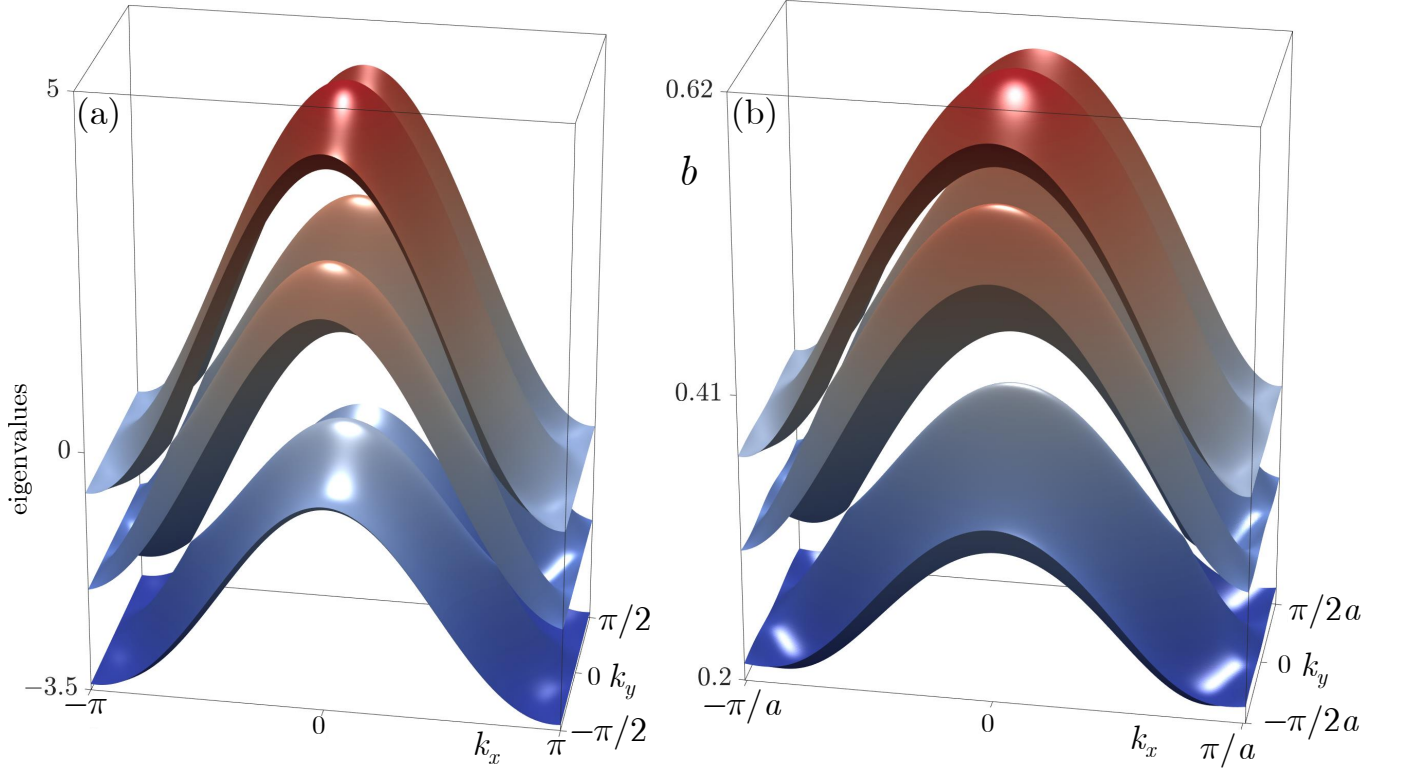


FIG. 2. (a) Band structure of the *Zhang lattice* by assuming $a = 1$ and $t = 1$ based on the tight-binding method. (b) Band structure of the *Zhang lattice* based on the continuous model with $a = 3$ and $p = 5$.

The continuous model includes all hoppings as well as the concrete profile of the sites, and therefore is more accurate than the discrete model. Assuming the *Zhang lattice* is inscribed in a transparent optical medium (e.g., the fused silica) by the femto-second laser direct writing technique, and the dimensionless Schrödinger-like paraxial wave equation should be written as

$$
i\frac{\partial\psi}{\partial z} = -\frac{1}{2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi - \mathcal{R}(x,\gamma)\psi,
\tag{3}
$$

where $\mathcal{R}$ is the *Zhang lattice* that can be depicted by Gaussian functions

$$
\mathcal{R}(x,y) = p\sum_{m,n}\exp\left(-\frac{(x-x_{m,n})^2}{\sigma_x^2}-\frac{(y-y_{m,n})^2}{\sigma_y^2}\right),
\tag{4}
$$

with $p$ being the lattice depth, $(\sigma_x, \sigma_y)$ charging for the beam width, and $(x_{m,n}, y_{m,n})$ being the coordinates of the lattice grids. Considering a set of real experimental parameters [15, 16], e.g., $\sigma_x = 0.25$ ($2.5\,\mu$m), $\sigma_y = 0.75$ ($7.5\,\mu$m), $a = 3$ ($30\,\mu$m), $\lambda = 600$ nm, and $p = 5$ (the refractive index change $\sim 5.5\times10^{-4}$), the corresponding band structure of the *Zhang lattice* is shown in Fig. 2(b), by introducing the ansatz $\psi(x,y,z) = u(x,y)\exp(ibz)$ for Eq. (3) with $b$

being the propagation constant and $u(x, y)$ the Bloch state. The result based on the continuous model is quite similar to that based on the discrete model, and the type-II Dirac points are definitely supported by the *Zhang lattice*.

Note that the *Zhang lattice* has been induced in photorefractive SBN crystals [17], and surely in atomic vapors [18] with the aid of a spatial modulator. In the *Zhang lattice*, the conical diffraction [11], the Klein tunneling [11], the scalar as well as the vector valley Hall edge solitons have been reported [17, 19, 20]. In the future, I believe more and more interesting physical principles and phenomena will be reported and verified based on the *Zhang lattice*.

---

[1] T. Wehling, A. Black-Schaffer, and A. Balatsky, "Dirac materials," Adv. Phys. **63**, 1 (2014).

[2] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, "The electronic properties of graphene," Rev. Mod. Phys. **81**, 109 (2009).

[3] D. Leykam and A. S. Desyatnikov, "Conical intersections for light and matter waves," Adv. Phys. X **1**, 101 (2016).

[4] M. Milićević, G. Montambaux, T. Ozawa, O. Jamadi, B. Real, I. Sagnes, A. Lemaître, L. Le Gratiet, A. Harouri, J. Bloch, and A. Amo, "Type-III and tilted Dirac cones emerging from flat bands in photonic orbital graphene," Phys. Rev. X **9**, 031010 (2019).

[5] H.-X. Wang, Y. Chen, Z. H. Hang, H.-Y. Kee, and J.-H. Jiang, "Type-II Dirac photons," npj Quantum Mater. **2**, 54 (2017).

[6] G. G. Pyrialakos, N. S. Nye, N. V. Kantartzis, and D. N. Christodoulides, "Emergence of type-II Dirac points in graphynelike photonic lattices," Phys. Rev. Lett. **119**, 113901 (2017).

[7] G. G. Pyrialakos, N. Schmitt, N. S. Nye, M. Heinrich, N. V. Kantartzis, A. Szameit, and D. N. Christodoulides, "Symmetry-controlled edge states in the type-II phase of Dirac photonic lattices," Nat. Commun. **11**, 2074 (2020).

[8] C. Hu, Z. Li, R. Tong, X. Wu, Z. Xia, L. Wang, S. Li, Y. Huang, S. Wang, B. Hou, C. T. Chan, and W. Wen, "Type-II Dirac photons at metasurfaces," Phys. Rev. Lett. **121**, 024301 (2018).

[9] X. Wu, X. Li, R.-Y. Zhang, X. Xiang, J. Tian, Y. Huang, S. Wang, B. Hou, C. T. Chan, and W. Wen, "Deterministic scheme for two-dimensional type-II Dirac points and experimental realization in acoustics," Phys. Rev. Lett. **124**, 075501 (2020).

[10] M. Li, N. Han, L. Peng, and S. Zhang, "Type-II Dirac points and topological Fermi arcs in nonlocal metamaterials," Phys. Rev. B **106**, 235303 (2022).

[11] K. C. Jin, H. Zhong, Y. D. Li, F. W. Ye, Y. P. Zhang, F. L. Li, C. L. Liu, and Y. Q. Zhang, "Parametric type-II Dirac photonic lattices," Adv. Quantum Technol. **3**, 2000015 (2020).

[12] H. Zhong, Y. Q. Zhang, Y. Zhu, D. Zhang, C. B. Li, Y. P. Zhang, F. L. Li, M. R. Belić, and M. Xiao, "Transport properties in the photonic super-honeycomb lattice – a hybrid fermionic and bosonic system," Ann. Phys. (Berlin) **529**, 1600258 (2017).

[13] D. Zhang, Y. Q. Zhang, H. Zhong, C. B. Li, Z. Y. Zhang, Y. P. Zhang, and M. R. Belić, "New edge-centered photonic square lattices with flat bands," Ann. Phys. **382**, 160 (2017).

[14] H. Zhong, R. Wang, M. R. Belić, Y. P. Zhang, and Y. Q. Zhang, "Asymmetric conical diffraction in dislocated edge-centered square lattices," Opt. Express **27**, 6300 (2019).

[15] M. C. Rechtsman, J. M. Zeuner, Y. Plotnik, Y. Lumer, D. Podolsky, F. Dreisow, S. Nolte, M. Segev, and A. Szameit, "Photonic Floquet topological insulators," Nature **496**, 196 (2013).

[16] M. S. Kirsch, Y. Zhang, M. Kremer, L. J. Maczewsky, S. K. Ivanov, Y. V. Kartashov, L. Torner, D. Bauer, A. Szameit, and M. Heinrich, "Nonlinear second-order photonic topological insulators," Nat. Phys. **17**, 995 (2021).

[17] H. Zhong, S. Xia, Y. Zhang, Y. Li, D. Song, C. Liu, and Z. Chen, "Nonlinear topological valley Hall edge states arising from type-II Dirac cones," Adv. Photon. **3**, 056001 (2021).

[18] Z. Y. Zhang, R. Wang, Y. Q. Zhang, Y. V. Kartashov, F. Li, H. Zhong, H. Guan, K. Gao, F. L. Li, Y. P. Zhang, and M. Xiao, "Observation of edge solitons in photonic graphene," Nat. Commun. **11**, 1902 (2020).

[19] Y. Q. Tian, Y. Q. Zhang, Y. D. Li, and M. R. Belić, "Vector valley Hall edge solitons in the photonic lattice with type-II Dirac cones," Front. Phys. (Beijing) **17**, 53503 (2022).

[20] Y. Q. Tian, Y. D. Wang, M. R. Belić, Y. Q. Zhang, Y. D. Li, and F. W. Ye, "Vector valley Hall edge solitons in distorted type-II Dirac photonic lattices," Opt. Express **31**, 20812 (2023).