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Direction Estimation of Coherent Signals Using Spatial Signature

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***Abstract*—A computationally efficient spatial signature-based (SS) method is proposed for estimating the directions of arrival of coherent narrowband signals impinging on a uniform linear array. The normalized SS of the coherent signals is blindly estimated from the principal eigenvector of array covariance matrix and then is used to estimate the directions with a modified Kumaresan–Prony method, where a linear prediction model is combined with "subarray" averaging. The proposed method not only has the maximum permissible array aperture and computational simplicity, it also better resolves closely spaced coherent signals with a small length of data and at a lower signal-to-noise ratio.**

***Index Terms*—Array processing, eigenvalue decomposition, linear prediction, multipath environment.**

I. Introduction

In many applications of array processing such as wireless communications, multipath propagation is often encountered due to various reflections, and it causes the direct and reflected signals from a source arriving at an array to be coherent if their delay differences are sufficiently small [2], [5]. Although a linear combination of the array response vectors to the coherent signals (called the "spatial signature (SS)" [8]) is usually not in the array manifold, it is an effective array response to the source and contains sufficient information on the directions of arrival (DOAs) of the signals [5], [6], [8], [13] as the principal eigenvector of array covariance matrix [11]. Recently some new subspace-based methods with "subarray" averaging have been proposed for DOA estimation of coherent signals by using the SS. They include the fourth-order statistics-based forward–backward linear prediction (FBLP) method [5] and extended virtual ESPRIT algorithm [6], the finite-alphabet (FA) property-based method [14], the higher order cyclostationarity-based (HOCS) method [16], and the second-order statistics-based modified Prony (MP) method [7]. Most of these methods exploit the temporal properties of digital communication signals to estimate the SS. Unfortunately, like the ordinary subspace-based methods with spatial smoothing preprocessing (e.g., [3]), their performance of direction estimation is affected by the reduced effective array aperture (i.e., subarray size). Moreover, they suffer serious degradation when the number of snapshots is small, because the fourth-order (cyclic) statistics-based methods [5], [6], [16] require the (cyclic) cumulant to be evaluated; the FA-based method [14] needs an iterative algorithm [15] to solve a nonlinear optimization in SS estimation; and the MP method [7] is sensitive to errors in the estimated SS.

In this letter, we propose a computationally efficient SS-based method for estimating the DOAs of coherent narrowband signals impinging on a uniform linear array (ULA) without any assumption on the temporal structure of incident signals such as the non-Gaussian [5], [6], FA [14], [15], and HOCS [16] properties. First, an FBLP model in terms of SS is combined with "subarray" averaging, and the identifiable condition of DOA estimation of coherent signals is clarified. Furthermore, a blind estimation of the SS from the principal eigenvector of array covariance matrix is investigated. Then, a modified Krumaresan–Prony (KP) method is presented for DOA estimation, where the maximum permissible aperture of a subarray is used, and the effect of estimation error in the SS is alleviated. The effectiveness of the method is demonstrated through numerical examples.

II. Data Model and Basic Assumptions

Consider a ULA of M sensors with spacing d, and suppose that p narrowband signals $\{s_k(n)\}$ with zero-mean and center frequency f_0 are far enough away and impinge on the array from distinct directions $\{\theta_k\}$. The received signal vector can be written as [2]–[6], [8], [10], [11]

$$\boldsymbol{x}(n) = \boldsymbol{A}(\theta)\boldsymbol{s}(n) + \boldsymbol{w}(n) \tag{1}$$

where $\boldsymbol{x}(n)$, $\boldsymbol{s}(n)$, and $\boldsymbol{w}(n)$ are the vectors of the received data, incident signals, and additive noise, respectively, and $\boldsymbol{A}(\theta)$ denotes the array response matrix given by $\boldsymbol{A}(\theta) = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \ldots, \boldsymbol{a}(\theta_p)]$, in which $\boldsymbol{a}(\theta_k) = [1, e^{j\omega_0\tau(\theta_k)}, \ldots, e^{j\omega_0(M-1)\tau(\theta_k)}]^T$, $\omega_0 = 2\pi f_0$, $\tau(\theta_k) = (d/c)\sin\theta_k$, and c is the propagation speed. The additive noises $\{w_i(n)\}$ are temporally and spatially uncorrelated white complex Gaussian noise with zero-mean and variance σ^2 given by $E\{w_i(n)w_k^*(n)\} = \sigma^2\delta_{i,k}$, where $E\{\cdot\}$, $(\cdot)^*$, and $\delta_{i,k}$ denote the expectation, complex conjugate, and Kronecker delta, respectively, and they are uncorrelated with the incident signals $\{s_k(n)\}$.

We assume that the array is calibrated and that the array response matrix $\boldsymbol{A}(\theta)$ is unambiguous. In the frequency-flat multipath propagation, the incident signals are coherent ones expressed by [3]–[6], [8], [13], [14]

$$s_k(n) = \beta_k s_1(n), \qquad \text{for } k = 1, 2, \ldots, p \tag{2}$$

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where β_k is the complex attenuation coefficient of the signal $s_k(n)$ with $\beta_k \neq 0$ and $\beta_1 = 1$. We also assume that the number of signals p is known or has been estimated by the proposed methods (e.g., see [4] and references therein).

III. SS-Based DOA Estimation

A. Identifiablity of Directions From Spatial Signature

From (1) and (2), the signals $\boldsymbol{x}(n)$ can be reexpressed as

$$\boldsymbol{x}(n) = \boldsymbol{A}(\theta)\boldsymbol{\beta}s_1(n) + \boldsymbol{w}(n) = \boldsymbol{a}s_1(n) + \boldsymbol{w}(n) \tag{3}$$

where $\boldsymbol{a}$ is the SS of $\{s_k(n)\}$ given by $\boldsymbol{a} = \sum_{k=1}^{p} \beta_k \boldsymbol{a}(\theta_k)$, and $\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_p]^T$. Obviously, $\boldsymbol{a}$ contains sufficient information on $\{\theta_k\}$ [5]–[8]. By defining the normalized SS $\bar{\boldsymbol{a}}$ as $\boldsymbol{a}/\bar{\beta}$, its components $\{\bar{a}_i\}$ can be expressed as

$$\bar{a}_i = \sum_{k=1}^{p} \bar{\beta}_k e^{j\omega_0(i-1)\tau(\theta_k)} = \boldsymbol{b}_i^T \bar{\boldsymbol{\beta}} \tag{4}$$

where $\bar{\beta} = \sum_{k=1}^{p} \beta_k$, $\boldsymbol{b}_i = [e^{j\omega_0(i-1)\tau(\theta_1)}, e^{j\omega_0(i-1)\tau(\theta_2)}, \ldots, e^{j\omega_0(i-1)\tau(\theta_p)}]^T$, $\bar{\boldsymbol{\beta}} = [\bar{\beta}_1, \bar{\beta}_2, \ldots, \bar{\beta}_p]^T$, and $\bar{\beta}_k = \beta_k/\bar{\beta}$. Evidently, $\bar{\boldsymbol{a}}$ can be interpreted as the vector of the received "signals" for an array of M sensors illuminated by p "signals" $\{\bar{\beta}_k\}$ [5], [6]. Thus, the directions $\{\theta_k\}$ of the signals $\{s_k(n)\}$ can be determined from the "signals" $\{\bar{a}_i\}_{i=1}^{M}$.

Furthermore, we can find that $\{\bar{a}_i\}$ differ only by a phase factor $\omega_0\tau(\theta_k)$ and obey a linear difference equation [1], [2]. By dividing the full array into L overlapping subarrays with m sensors in the forward or backward direction, where $L = M - m + 1$ and $m \geq p + 1$, we obtain the linear prediction (LP) models in terms of the normalized SS for the lth forward and backward "subarrays" [4]

$$\bar{a}_{l+m-1} = \boldsymbol{a}_{fl}^T \boldsymbol{\rho} \text{ and } \bar{a}_{L-l+1}^* = \boldsymbol{a}_{bl}^T \boldsymbol{\rho} \tag{5}$$

where $\boldsymbol{a}_{fl} = [\bar{a}_l, \bar{a}_{l+1}, \ldots, \bar{a}_{l+m-2}]^T$, $\boldsymbol{a}_{bl} = [\bar{a}_{M-l+1}, \bar{a}_{M-l}, \ldots, \bar{a}_{L-l+2}]^H$, and $\boldsymbol{\rho} = [\rho_{m-1}, \rho_{m-2}, \ldots, \rho_1]^T$; $\{\rho_i\}$ are the LP coefficients, and $(\cdot)^H$ denotes the Hermitian transpose. By concatenating (5) for $l = 1, 2, \ldots, L$, we obtain a compact FBLP equation

$$\begin{bmatrix} \boldsymbol{z}_f \\ \boldsymbol{z}_b \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_f \\ \boldsymbol{\Phi}_b \end{bmatrix} \boldsymbol{\rho} \tag{6}$$

i.e., $\boldsymbol{z} = \boldsymbol{\Phi}\boldsymbol{\rho}$, where $\boldsymbol{z}_f = [\bar{a}_m, \bar{a}_{m+1}, \ldots, \bar{a}_M]^T$, $\boldsymbol{z}_b = [\bar{a}_L, \bar{a}_{L-1}, \ldots, \bar{a}_1]^H$, $\boldsymbol{\Phi}_f = [\boldsymbol{a}_{f1}, \boldsymbol{a}_{f2}, \ldots, \boldsymbol{a}_{fL}]^T$, and $\boldsymbol{\Phi}_b = [\boldsymbol{a}_{b1}, \boldsymbol{a}_{b2}, \ldots, \boldsymbol{a}_{bL}]^T$. To investigate the identifiability of the DOAs of coherent signals by using (6), we have to examine the dimension of the signal subspace of the $2L \times (m-1)$ matrix $\boldsymbol{\Phi}$.

Proposition: If the array is partitioned so that $p + 1 \leq m \leq M + 1 - p/2$, the dimension of the signal subspace of matrix $\boldsymbol{\Phi}$ will equal the number of signals p.

Proof: By defining $\boldsymbol{A}_1(\theta)$ and $\boldsymbol{A}_2(\theta)$ as the submatrices of $\boldsymbol{A}(\theta)$ in (1) consisting of the first $m-1$ and L rows, respectively, and by substituting (4) into the vectors $\boldsymbol{a}_{fl}$ and $\boldsymbol{a}_{bl}$ in (5), we can get

$$\boldsymbol{a}_{fl} = [\boldsymbol{b}_l, \boldsymbol{b}_{l+1}, \ldots, \boldsymbol{b}_{l+m-2}]^T \bar{\boldsymbol{\beta}} = \boldsymbol{A}_1(\theta)\boldsymbol{D}^{l-1}\bar{\boldsymbol{\beta}} \tag{7}$$

$$\boldsymbol{a}_{bl} = \boldsymbol{A}_1(\theta)\boldsymbol{D}^{l-1}\boldsymbol{D}^{-(M-1)}\bar{\boldsymbol{\beta}}^* \tag{8}$$

where $\boldsymbol{D} = \text{diag}(e^{j\omega_0\tau(\theta_1)}, e^{j\omega_0\tau(\theta_2)}, \ldots, e^{j\omega_0\tau(\theta_p)})$. By some algebraic manipulations, from (6)–(8), we obtain

$$\begin{aligned} \boldsymbol{\Phi}_f &= \left[\bar{\boldsymbol{\beta}}, \boldsymbol{D}\bar{\boldsymbol{\beta}}, \ldots, \boldsymbol{D}^{L-1}\bar{\boldsymbol{\beta}}\right]^T \boldsymbol{A}_1^T(\theta) \\ &= \boldsymbol{A}_2(\theta)\boldsymbol{B}\boldsymbol{A}_1^T(\theta) \end{aligned} \tag{9}$$

$$\boldsymbol{\Phi}_b = \boldsymbol{A}_2(\theta)\boldsymbol{B}^*\boldsymbol{D}^{-(M-1)}\boldsymbol{A}_1^T(\theta) \tag{10}$$

where $\boldsymbol{B} = \text{diag}(\bar{\beta}_1, \bar{\beta}_2, \ldots, \bar{\beta}_p)$. Then, the matrix $\boldsymbol{\Phi}$ in (6) can be rewritten as

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{A}_2(\theta)\boldsymbol{B} \\ \boldsymbol{A}_2(\theta)\boldsymbol{B}^*\boldsymbol{D}^{-(M-1)} \end{bmatrix} \boldsymbol{A}_1^T(\theta) = \boldsymbol{C}\boldsymbol{B}\boldsymbol{A}_1^T(\theta) \tag{11}$$

where $\boldsymbol{C} = [\boldsymbol{A}_2^T(\theta), \boldsymbol{\Psi}\boldsymbol{A}_2^T(\theta)]^T$, $\boldsymbol{\Psi} = \text{diag}(\psi_1, \psi_2, \ldots, \psi_p)$, and $\psi_i = (\bar{\beta}_i^*/\bar{\beta}_i)e^{-j\omega_0(M-1)\tau(\theta_i)}$.

Under the assumptions that $\beta_k \neq 0$ and $\boldsymbol{A}(\theta)$ is unambiguous, the ranks of the diagonal matrices $\boldsymbol{B}$ and $\boldsymbol{\Psi}$ and the Vandermonde matrices $\boldsymbol{A}_1(\theta)$ and $\boldsymbol{A}_2(\theta)$ are given by $\text{rank}(\boldsymbol{B}) = \text{rank}(\boldsymbol{\Psi}) = p$, $\text{rank}(\boldsymbol{A}_1(\theta)) = \min(m-1, p)$, and $\text{rank}(\boldsymbol{A}_2(\theta)) = \min(L, p)$. Hence, when $2L \geq p$ and $m \geq p+1$, i.e., $p + 1 \leq m \leq M + 1 - p/2$, we can obtain that $\text{rank}(\boldsymbol{A}_1(\theta)) = p$ and $\text{rank}(\boldsymbol{C}) = \min(2L, p) = p$, and we consequently find that the rank of matrix $\boldsymbol{\Phi}$ is given by $\text{rank}(\boldsymbol{\Phi}) = p$, i.e., the dimension of the signal subspace of $\boldsymbol{\Phi}$ equals p. ■

Therefore, if we can obtain the coefficients $\{\rho_i\}$ that satisfy the FBLP model in terms of the normalized SS shown in (6), by forming the prediction polynomial $D(z) = 1 - \rho_1 z^{-1} - \cdots - \rho_{m-1}z^{-(m-1)}$, the directions $\{\theta_k\}$ of coherent signals can be determined from the phases of the p signal zeros of $D(z)$ in the z plane (e.g., see [1], [2], and [4]).

Remark 1: From the proposition, the maximum detectable number of coherent signals is clearly $2M/3$, which coincides with the necessary condition for unique direction estimation with probability one derived in [12]. □

B. Blind Estimation of SS

Under the assumptions on the data model, from (3), the array covariance matrix can be obtained

$$\boldsymbol{R} = E\left\{\boldsymbol{x}(n)\boldsymbol{x}^H(n)\right\} = r_s\boldsymbol{a}\boldsymbol{a}^H + \sigma^2\boldsymbol{I}_M \tag{12}$$

where r_s is the power of $s_1(n)$ given by $r_s = E\{s_1(n)s_1^*(n)\}$, and $\boldsymbol{I}_M$ denotes an $M \times M$ identity matrix. Due to the coherency of signals $\{s_k(n)\}$, the rank of noiseless covariance matrix $\boldsymbol{R} - \sigma^2\boldsymbol{I}_M$ is clearly equal to 1, and its eigenvalue decomposition (EVD) is given by

$$\boldsymbol{R} - \sigma^2\boldsymbol{I}_M = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H = \lambda_1\boldsymbol{u}_1\boldsymbol{u}_1^H \tag{13}$$

where $\boldsymbol{U} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_M]$, $\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_M)$, $\{\lambda_i\}$ and $\{\boldsymbol{u}_i\}$ are the eigenvalues and eigenvectors, and $\lambda_1 > \lambda_2 = \cdots = \lambda_M = 0$.

It is known that the principal eigenvector $\boldsymbol{u}_1$ contains a sufficient statistic for direction estimation [11]. A comparison of (12) and (13) shows that both $\boldsymbol{a}$ and $\boldsymbol{u}_1$ are the eigenvectors corresponding to the solitary signal eigenvector of matrix $\boldsymbol{R} - \sigma^2\boldsymbol{I}_M$ (i.e., $\boldsymbol{R}$) [7]. However, we also find that the SS $\boldsymbol{a}$ or the signal power r_s can be determined only as $\eta\boldsymbol{a}$ or r_s/η^2 with a nonzero-scale η by using a blind estimation technique [9], when information about the signals is not used. Similarly, $\boldsymbol{a}$ can be estimated blindly as $\hat{\boldsymbol{a}} = \mu\boldsymbol{u}_1$ from (12) and (13), where μ is an unknown factor. This scaling ambiguity can be handled by imposing a constraint on the SS. By defining the normalized principal eigenvector $\bar{\boldsymbol{u}}_1$ as $\boldsymbol{u}_1/u_{11}$, where u_{11} is the first element of $\boldsymbol{u}_1$, from (12) and (13), we obtain

$$\gamma \bar{\boldsymbol{a}}\bar{\boldsymbol{a}}^H = \lambda_1 \left|u_{11}\right|^2 \bar{\boldsymbol{u}}_1\bar{\boldsymbol{u}}_1^H \tag{14}$$

where $\gamma = r_s|\bar{\beta}|^2$. Because $\bar{\boldsymbol{a}}$ and $\bar{\boldsymbol{u}}_1$ have their first elements as $\bar{a}_1 = \bar{u}_{11} = 1$, we easily find that $\gamma = \lambda_1|u_{11}|^2$. Thus, we obtain that the normalized SS $\bar{\boldsymbol{a}}$ is equivalent to the normalized principal eigenvector $\bar{\boldsymbol{u}}_1$, i.e., $\bar{\boldsymbol{a}} = \bar{\boldsymbol{u}}_1$.

C. Modified KP Method

From the proposition, we find that the subarray size m must be set to satisfy the inequality $p+1 \le m \le M+1-p/2$. The KP method takes advantage of the maximum possible aperture of array/subarray to improve the estimation performance and simplify the computation in the estimation of the LP coefficients [1], [2]. When the subarray size is set to $m = M+1-\lceil p/2 \rceil$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x, from (6), the minimum-norm estimate of $\boldsymbol{\rho}$ is obtained

$$\hat{\boldsymbol{\rho}}_{\mathrm{MKP}} = \boldsymbol{\Phi}^H \left(\boldsymbol{\Phi}\boldsymbol{\Phi}^H\right)^{-1} \boldsymbol{z} \tag{15}$$

where the $p \times p$ matrix $\boldsymbol{\Phi}\boldsymbol{\Phi}^H$ has only nonzero eigenvalues, and the eigenvectors in noise subspace are eliminated.

The implementation of the proposed method with the finite data $\{\boldsymbol{x}(n)\}_{n=0}^{N-1}$ is summarized as follows.

1) Calculate the sampled array covariance matrix as $\hat{\boldsymbol{R}} = (1/N)\sum_{n=0}^{N-1} \boldsymbol{x}(n)\boldsymbol{x}^H(n)$.
2) Perform EVD on the matrix $\hat{\boldsymbol{R}}$, and estimate the normalized SS as $\hat{\bar{\boldsymbol{a}}} = \hat{\boldsymbol{u}}_1/\hat{u}_{11}$.
3) Calculate the estimate $\hat{\boldsymbol{\rho}}_{\mathrm{MKP}}$ from $\hat{\bar{\boldsymbol{a}}}$ by (15) and (6).
4) Estimate $\{\theta_k\}$ from the phases of p zeros of the polynomial $\hat{D}(z)$ closest to the unit circle in the z plane.

Remark 2: The first three steps roughly take $8NM^2$, $O(M^3)+6M$, and $8p(p+1)(m-1)+O(p^3)$ MATLAB flops. Hence, the number of flops required by the implementation is about $8NM^2$ when $N \gg M$, where the other needed computations are negligible.

IV. Numerical Examples

A ULA has $M = 10$ sensors with a half-wavelength spacing, and two coherent binary phase-shift keying signals that have a raised cosine shape with 50% excess bandwidth come from θ_1 and θ_2 with equal power. The signal-to-noise ratio (SNR) is defined as the ratio of the signal power to that of the noise at each sensor, where the additive noises are temporally and spatially uncorrelated white complex Gaussian noise. The presented results are based on 1000 independent trial runs.

Example A: Performance Versus SNR: Two coherent signals are from $\theta_1 = 5^\circ$ and $\theta_2 = 12^\circ$, and the SNR is varied from -10 to 25 dB. The number of snapshots is $N = 128$. The root mean-squared-error (RMSE) of the estimate $\hat{\bar{\boldsymbol{a}}}$ against SNR is plotted in Fig. 1(a). It is shown that a better estimate $\hat{\bar{\boldsymbol{a}}}$ is provided as the SNR is increased. Fig. 2 shows the RMSEs of the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ versus the SNR, where the stochastic Cramér–Rao lower bound (CRB) [10], [11] is also depicted. The proposed

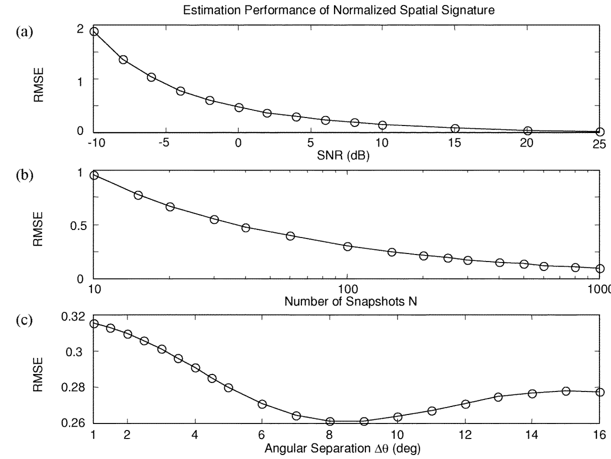

Fig. 1. RMSEs of the estimates of normalized spatial signatures versus (a) SNR, (b) number of snapshots, and (c) angular separation in Examples A, B, and C, respectively.

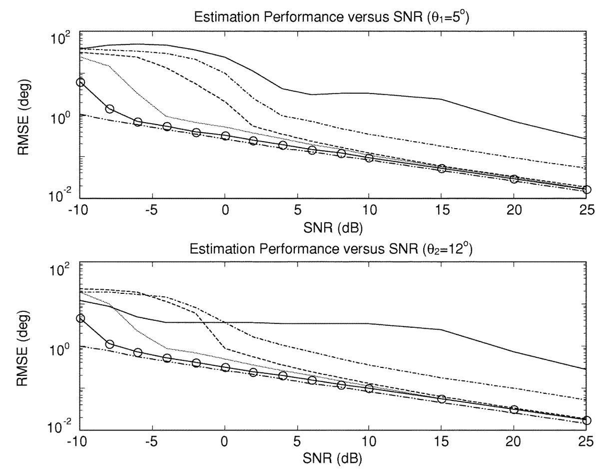

Fig. 2. RMSEs of the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ versus the SNR (dotted line: KP; solid line: SS-based MP; dash-dot line: spatial smoothing-based root-MUSIC; dashed line: TCLS-LP; solid line with "o": proposed method; and dash-dots line: CRB) in Example A ($N = 128$).

method clearly has a much lower threshold of SNR at which the estimation performance drops sharply than the ordinary KP [1], [2], SS-based MP [7], spatial smoothing-based root-MUSIC ($m = 5$) [3], and corrected least squares with truncation-based LP (TCLS-LP) ($m = 5$; see [4] for reference) methods. And its RMSE is close to the CRB like the KP and TCLS-LP methods at higher SNRs.

Example B: Performance Versus Number of Snapshots: The simulation conditions are the same as in Example A, except that the SNR is set at 5 dB, and the number of snapshots N is varied from ten to 1000. The RMSEs of the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ with respect to the number of snapshots are shown in Fig. 3. When N is very small, the proposed method performs as well as the KP method and outperforms the others. As N is increased, the matrix $\hat{\boldsymbol{R}}$ comes to more closely resemble the true one, resulting in a more accurate estimate $\hat{\bar{\boldsymbol{a}}}$ as depicted in Fig. 1(b). Thus, the estimation accuracy of the proposed method becomes better with a much smaller RMSE than that of the other methods.

Example C: Performance Versus Angular Separation: Two coherent signals arrive from $\theta_1 = 0^\circ$ and $\theta_2 = \Delta\theta$, where

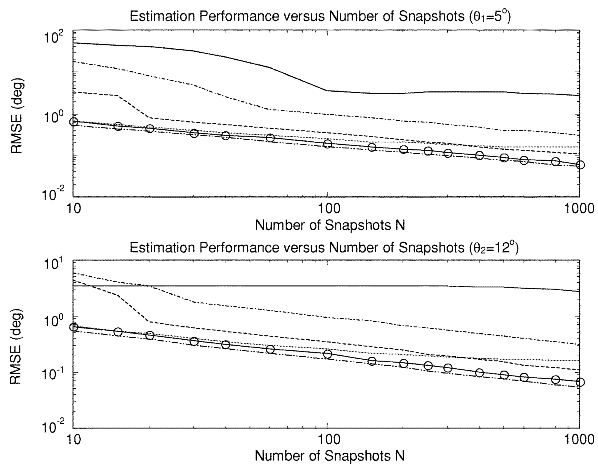

Fig. 3. RMSEs of the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ versus the number of snapshots (dotted line: KP; solid line: SS-based MP; dash-dot line: spatial smoothing-based root-MUSIC; dashed line: TCLS-LP; solid line with "o": proposed method; and dash-dots line: CRB) in Example B (SNR $= 5$ dB).

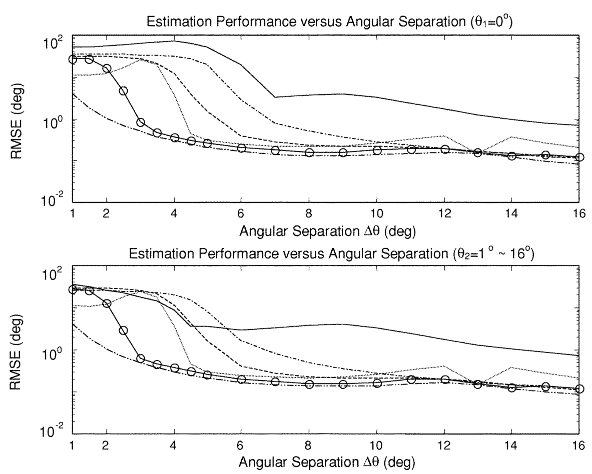

Fig. 4. RMSEs of the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ versus the angular separation (dotted line: KP; solid line: SS-based MP; dash-dot line: spatial smoothing-based root-MUSIC; dashed line: TCLS-LP; solid line with "o": proposed method; and dash-dots line: CRB) in Example C (SNR $= 5$ dB, and $N = 128$).

$\Delta\theta$ is varied from 1° to 16°, and the SNR is fixed at 5 dB. The other simulation parameters are similar to those in Example A. The RMSEs of the estimates $\hat{\boldsymbol{a}}$ and $\{\hat{\theta}_k\}$ versus the angular separation $\Delta\theta$ are shown in Figs. 1(c) and 4, respectively. The proposed method generally estimates the directions of closely spaced signals more accurately than the other methods. It is noted that the RMSE of the proposed method does not decrease monotonically with the increasing angular separation like the CRB [10].

V. Conclusion

We proposed an SS-based modified KP method for estimating the DOAs of coherent narrowband signals impinging on a ULA. The normalized SS is blindly estimated from the principal eigenvector of array covariance matrix, and the maximum permissible aperture of subarray is used to improve the estimation performance. The simulation results demonstrated that the proposed method better resolves closely spaced coherent signals with a small data length and at a lower SNR.

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References

[1] D. W. Tufts and R. Kumaresan, "Estimation of frequencies of multiple sinusoids: Making linear prediction perform like maximum likelihood," *Proc. IEEE*, vol. 70, pp. 975–989, Sept. 1982.

[2] S. Haykin, "Radar array processing for angle of arrival estimation," in *Array Signal Processing* , S. Haykin, Ed. Englewood Cliffs, NJ: Prentice-Hall, 1985, pp. 194–292.

[3] T.-J. Shan, M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-33, no. 4, pp. 806–811, 1985.

[4] J. Xin and A. Sano, "MSE-based regularization approach to direction estimation of coherent narrowband signals using linear prediction," *IEEE Trans. Signal Processing*, vol. 49, pp. 2481–2497, Nov. 2001.

[5] N. Yuen and B. Friedlander, "DOA estimation in multipath: An approach using fourth-order cumulants," *IEEE Trans. Signal Processing*, vol. 45, pp. 1253–1263, May 1997.

[6] E. Gonen, J. M. Mendel, and M. C. Dogen, "Applications of cumulants to array processing—Part IV: Direction finding in coherent signals case," *IEEE Trans. Signal Processing*, vol. 45, pp. 2265–2276, Sept. 1997.

[7] C. Ma, Y. Peng, and L. Tian, "Blind estimation of DOA's of coherent sources," in *Proc. 4th Int. Conf. Signal Processing*, Beijing, China, Oct. 1998, pp. 327–330.

[8] A. L. Swindlehurst, "Time delay and spatial signature estimation using known asynchronous signals," *IEEE Trans. Signal Processing*, vol. 46, pp. 449–462, Feb. 1998.

[9] L. Tong, R. Liu, V. C. Soon, and Y.-F. Huang, "Indeterminacy and identifiability of blind identification," *IEEE Trans. Circuits Syst.*, vol. 38, pp. 499–509, May 1991.

[10] P. Stoica and A. Nehorai, "Performance study of conditional and unconditional direction-of-arrival estimation," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38, pp. 1783–1795, Oct. 1990.

[11] P. Stoica, B. Ottersten, M. Viberg, and R. L. Moses, "Maximum likelihood array processing for stochastic coherent sources," *IEEE Trans. Signal Processing*, vol. 44, pp. 96–105, Jan. 1996.

[12] M. Wax and I. Ziskind, "On unique localization of multiple sources by passive sensor arrays," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38, pp. 996–1000, July 1989.

[13] J. C. Liberti Jr and T. S. Rappaport, *Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications*. Upper Saddle River, NJ: Prentice-Hall, 1999.

[14] G. Xu and H. Liu, "An effective transmission beamforming scheme for frequency-division-duplex digital wireless communication systems," in *Proc. ICASSP*, Detroit, MI, May 1995, pp. 1729–1732.

[15] S. Talwar, M. Viberg, and A. Paulraj, "Blind estimation of multiple co-channel digital signals using an antenna array," *IEEE Signal Processing Lett.*, vol. 1, pp. 29–31, Feb. 1994.

[16] Q. Yin and M. Yao, "Spatial signature estimation in cyclic cumulant domain and multiuser signal separation in uplink SDMA," in *Proc. ICASSP*, Phoenix, AZ, May 1999, pp. 2861–2864.