Azimuthons in weakly nonlinear waveguides

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Abstract

- We show that a weakly guiding nonlinear waveguide supports propagation of stable rotating solitons, so-called azimuthons.
- In a circular waveguide, we calculate the rotation frequencies of the dipole azimuthons analytically and find them in excellent agreement with numerical simulations.
- In a square waveguide, we find dipole solitons either swinging back and forth or rotating during propagation, depending on amplitude and modulation depth.
- Higher-order azimuthons are also investigated in both configurations, we find simple rotation in the circular case, and complex periodic twisting in the square waveguide.

Introduction

Recently, there has been a lot of interest in a generalized type of spatial solitons, so-called azimuthons [1-3]. Those multiple peak ring-shaped solitons, which exhibit angular rotation during propagation, have been studied almost exclusively in nonlocal nonlinear media, because higher order solitonic structures are generally unstable in material with local (Kerr) response. In spite of the fact that there are various physical settings exhibiting nonlocality [4-6], their experimental realization is always quite involved. Moreover, from the theoretical point of view, nonlocal media are quite challenging for numerical modeling and analytical treatment. Here, we propose a much simpler optical system to study the propagation of azimuthons: a weakly nonlinear optical waveguide. We show that in such system azimuthons occur as the natural nonlinear counterparts of linear waveguide modes. Following [7], we can expect that weakly nonlinear azimuthons are stable in multi-mode waveguides.

Mathematical modeling

The following dimensionless equation describes the propagation of the slowly varying optical field envelope Ψ in a weakly-guiding optical waveguide with Kerr nonlinearity

$$i\frac{\partial}{\partial Z}\Psi + \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)\Psi + |\Psi|^2\Psi + V\Psi = 0, \qquad (1)$$

where V represents the waveguide index profile.

- We restrict ourselves to circular and square step-index waveguides.
- The index profile meets the condition $V(X^2+Y^2 \leq 1) = V_0$ (circular) or $V(|X| \leq 1 \cap |Y| \leq 1) = V_0$ (square), and $V = 0$ elsewhere.
- We choose $V_0 = 20$, which guarantees a multi-mode waveguide with stable vortex soliton in the weakly nonlinear regime [7].

Numerical simulations

Circular waveguide

- We seek approximate solutions of the form

$$\Psi(r, \phi, Z) = U(r, \phi - \omega Z)\exp(i\lambda Z), \qquad (2)$$

where r is the radius in the (x, y)-plane and ϕ the azimuthal angle, U is the stationary profile, ω the angular frequency, and λ the propagation constant.

- As stationary profile, we consider the simplest so-called rotating dipole azimuthon with the ansatz

$$U(r, \phi - \omega Z) = AF(r)\left[\cos(\phi - \omega Z) + iB\sin(\phi - \omega Z)\right], \qquad (3)$$

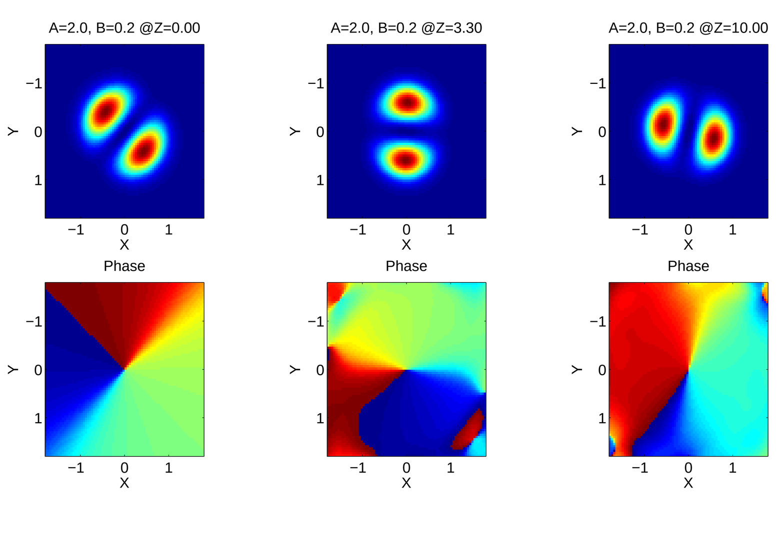
where F is the radial shape of the linear vortex mode and is normalized according to $\pi\int r|F(r)|^2 dr = 1$, A is the amplitude factor, and $1 - B$ is the azimuthal modulation depth.

- According to [8], we insert Eq. (2) into Eq. (1), multiply by Ψ^* and $\partial\Psi^*/\partial\phi$ respectively, and solve for the angular frequency:

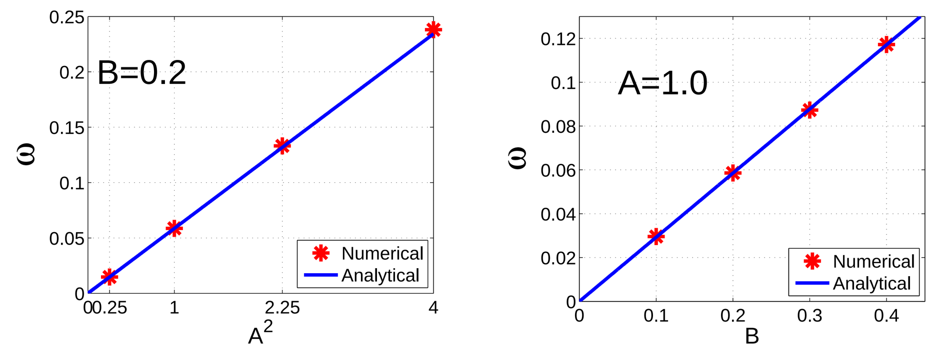
$$\omega = \frac{\pi}{2}\int r|F(r)|^4 dr \cdot A^2 B. \qquad (4)$$

(i) $B = 0$ corresponds to the stationary dipole soliton (two out-of-phase humps), which does not rotate for symmetry reasons.
(ii) $B = 1$ corresponds to the vortex soliton with circularly symmetric amplitude distribution and a phase singularity in the origin.

- Exemplary propagation of the rotating dipole azimuthon in the circular waveguide with $A = 2.0$ and $B = 0.2$.



- Angular frequency ω of the rotating dipole azimuthon versus parameters A and B.



In the range of stability of the azimuthons the angular frequencies predicted by Eq. (4) coincide with those obtained from the numerical simulations (red stars).

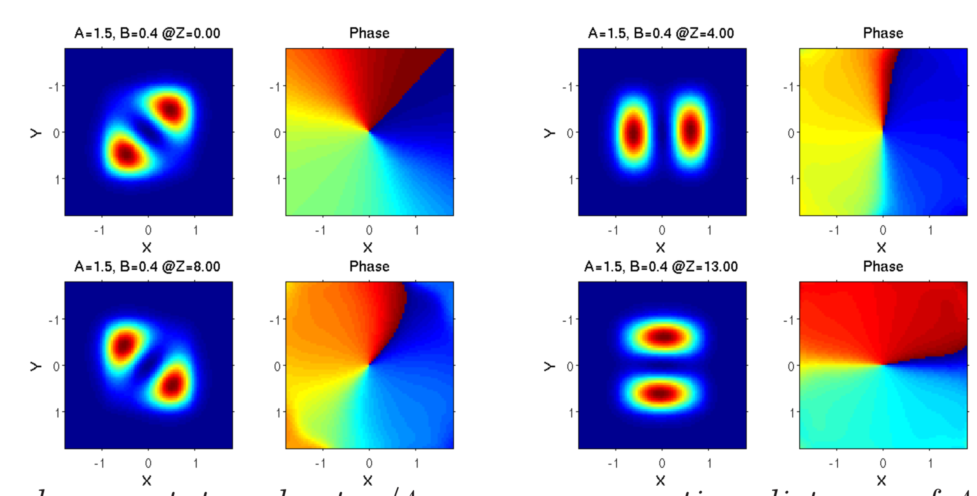
Square waveguide

In a square waveguide, azimuthons in the strict sense of Eq. (3) are not possible, since the beam profile changes considerably during propagation. However, we can still find two degenerated dipole modes D_1, D_2 and superpose them in a similar manner as before to form a "dipole azimuthon",

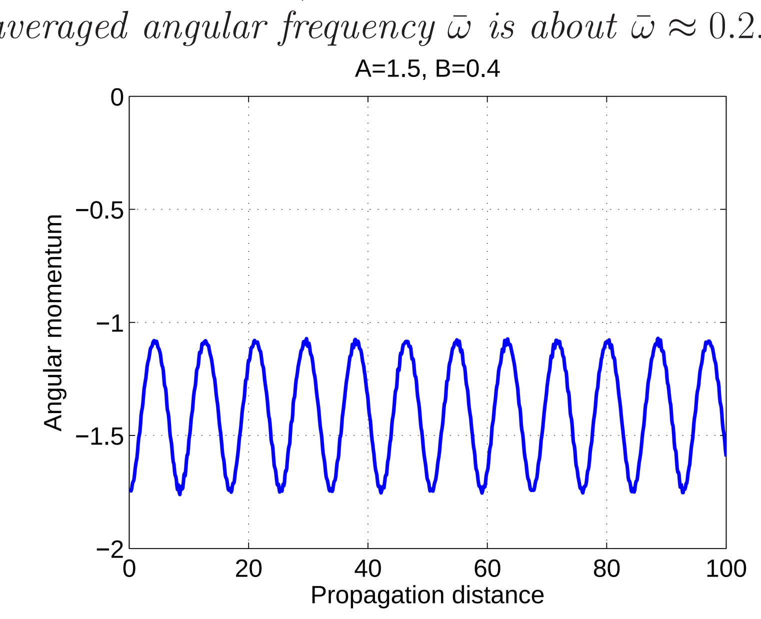
$$\psi(Z = 0) = A\left(D_1 + iBD_2\right), \quad \iint |D_{1,2}|^2 dXdY = 1. \qquad (5)$$

Numerical simulations demonstrate that for given A there exists a threshold value for B above which the solution rotates. Below this threshold, the "azimuthon" swings back and forth.

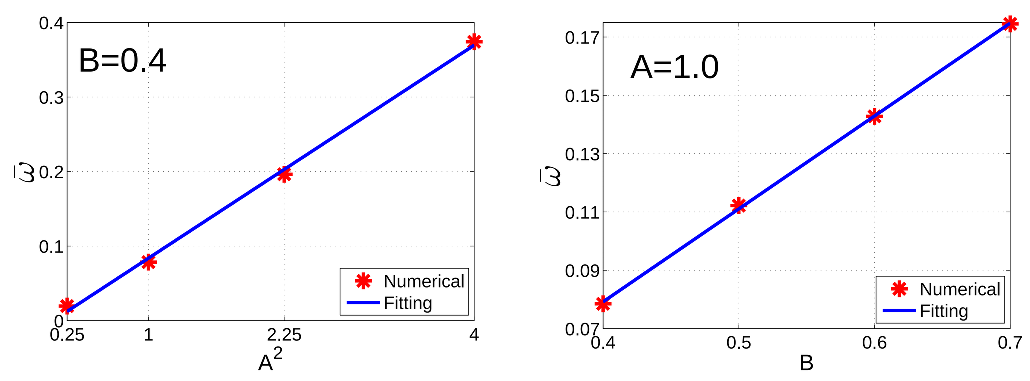
- Rotation of the azimuthon for $A = 1.5$ and $B = 0.4$ (above threshold $B_{th} \approx 0.33$).



The beam rotates about $\pi/4$ over a propagation distance of 4.0, so the averaged angular frequency $\bar{\omega}$ is about $\bar{\omega} \approx 0.2$.

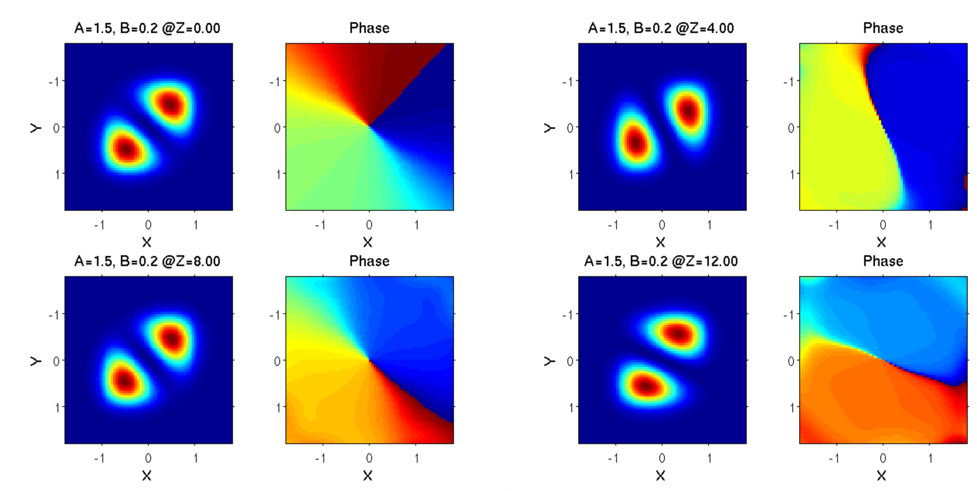


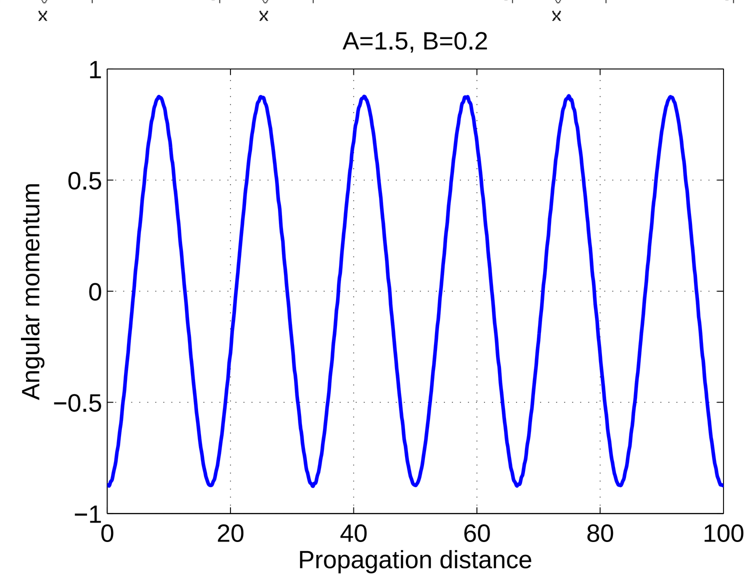
The angular momentum $-i\iint \psi^*\partial_\phi\psi dXdY$ is changing periodically, because the angular frequency of the azimuthon is not constant during propagation.



Red stars represent numerical results, blue lines are fittings. The averaged angular frequency of the azimuthons in the square waveguide is still proportional to A^2 and B.

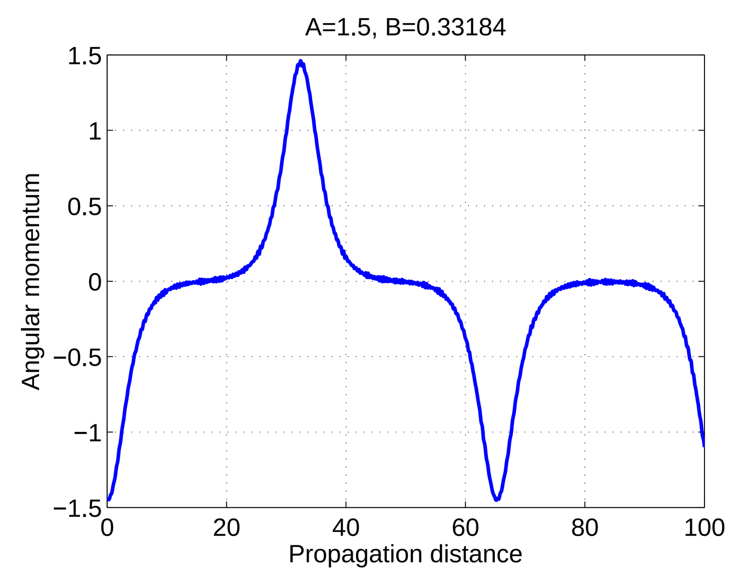
- When B is smaller than the threshold value ($A = 1.5$ and $B = 0.2$), the azimuthon swings back and forth instead of rotating during propagation.





The angular momentum is changing sign periodically, because the azimuthon swings back and forth during propagation.

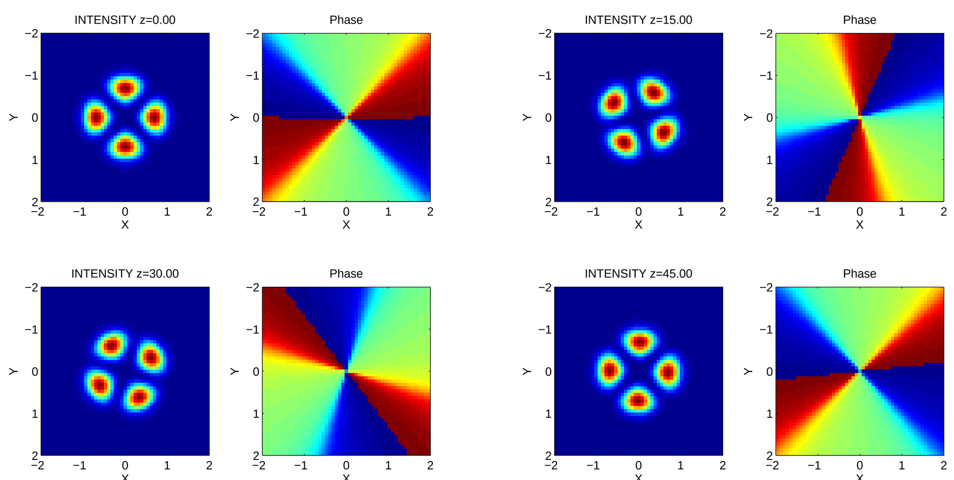
- When B is close to the threshold value $B_{th} \approx 0.33$, the azimuthon may switch between swinging and rotating (see angular momentum below). We attribute this behavior to numerical fluctuations.



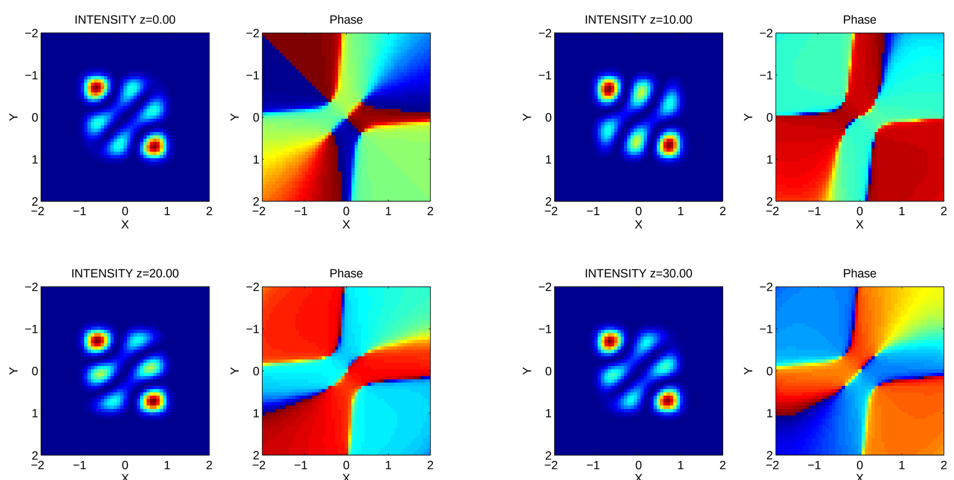
Higher-order azimuthons

Straightforwardly, it is possible to construct higher-order azimuthons from pairs of degenerated waveguide modes, similar to Eq. (5), and study the purely nonlinearity induced propagation dynamics. In order to obtain stable solutions, we increase the normalized index contrast V_0 [7].

- In a circular waveguide with $V_0 = 40$, we can observe a stable rotating quadrupole azimuthon. Here, we show exemplary propagation for $A = 1$ and $B = 0.2$.



- In a square waveguide, higher-order azimuthons always swing back and forth during propagation. Here, the next pair of degenerated waveguide modes are hexapoles. The quadrupole modes are not degenerated, and propagation dynamics of a superposed state in weakly nonlinear regime is dominated by linear mode beating. The following figure shows propagation of the hexapole azimuthon with $A = 1$ and $B = 0.2$.



Unlike in the case of the dipole azimuthon, we do not find a threshold value for B. Numerical simulations demonstrate the azimuthon is always twisting periodically during propagation.

Conclusions

- By means of numerical simulations we have demonstrated stable propagation of azimuthons in weakly nonlinear waveguides.
- Depending on the shape of the waveguide, different nonlinear induced propagation dynamics can be observed.
- Our findings may open a relatively easy route to experimental observations of stable rotating solitons.
- Due to the very simple model equation, we hope to gain a more profound theoretical understanding of rotating nonlinear localized structures in further investigations.

References

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