Dynamic and acoustic response of a clamped rectangular plate in thermal environments: Experiment and numerical simulation

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Experiments were performed to investigate the vibration and acoustic response characteristics of a clamped rectangular aluminum plate in thermal environments. Modal tests were carried out to study the influence of thermal environment on natural vibration. With the increment of structural temperature, natural frequencies of the plate decrease obviously. Mode shape interchange was observed for the modes with frequencies very close to each other. The thermally induced softening effect has unequal influences on the plate along the two in-plane directions. Numerical methods were also employed to study the experimental phenomena. Calculated results indicated that the initial deflection has a great influence on the natural vibration of the heated plate. Even a slight curvature can reduce the thermally induced softening effect obviously. Dynamic response tests were carried out under acoustic and mechanical excitations, and the measured results indicate that the variation in damping determines the response amplitudes at resonant peaks in the test.
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I. INTRODUCTION

Thermal environment is a crucial factor in a variety of engineering applications, such as space craft, nuclear energy, and so on. In practice, structures are usually exposed to extreme thermal conditions along with various loads and excitations. Temperature change caused by the thermal environment could induce structural configuration alteration, material property change and variation in stress state of structures. It will affect the performance of structures during working periods.[1]

Analytical and numerical work has been carried out on the dynamic behavior of simple structures[2–7] in thermal environments to reveal the basic characteristics. Besides this research, experimental studies were also performed for thermo-vibration problems. Vosteen and Fuller[8] designed a set of test facilities for heated cantilever plates and tested the low order natural vibrations. The result indicates that natural frequencies of the first bending mode and the first torsion mode reduce with the increment of temperature. Plates with solid double-wedge and circular-arc multiweb-wing sections were also tested.[9] McWithey and Vosteen[10] conducted experimental investigations of the fundamental vibration mode of a prototype wing under non-uniform heating. They found that thermal stresses affect the structural stiffness significantly. Kehoe and Snyder[11,12] carried out modal tests for aluminum plates with free boundaries under thermal loads. Numerical simulations were performed to investigate the problem as well. A decrement of natural frequencies and an increment of damping capacity were detected when the plate was heated. Kehoe and Deaton[13] provided experimental research on modal characters of plates under steady and transient heating. They found that the material property variations and thermal stresses are the primary reasons for the reduction of structural stiffness. Joen *et al.*[14] carried out research on natural vibrations of rectangular aluminum plates with free boundaries under different heating rates. It was found that some mode shapes disappear when structural temperature rises. It is clear from the literature that a lot of experimental work has been done for structures with free boundaries. However, the response characteristics are affected obviously by the boundary condition in thermal environments. And the clamped boundary is a much more common case in engineering applications. As the structure is heated, the constraint will lead to some new variations which could change the response characteristics. On the other hand, most of the studies paid attention to the influence of thermal environments on natural vibrations. Responses under excitations and the environment influence on vibrating structures in thermal conditions are seldom studied.

Accompanying with the vibration procedure, sound is also radiated by vibrating structures simultaneously. Some studies have contributed toward a better understanding of this topic for plates.[15–18] Due to the complexity of the working environment, the dynamic and acoustic characters of structures in thermal environments have recently become a new research point. Jeyaraj *et al.*[19,20] employed the combined approach of finite element method (FEM) and boundary element method (BEM) to study the vibration and acoustic response of isotropic and composite plates in thermal environments. Geng and Li[21] investigated the influence of thermal stresses on a simply supported plate with theoretical and numerical analyses. The results showed a trend of decrement for natural frequencies with the effect of elevated temperature. The fundamental natural frequency is much more sensitive to temperature variations. Compared with the results reported in Ref. 19, the resonant amplitudes showed a contrary variation trend. With the same method, Liu and Li[22] studied the

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response characters of rectangular sandwich plates in thermal environments. Equivalent non-classical theory, which considered the shear and rotational inertia, was used in the work. The influence of variations in structure configuration and material properties was discussed. As described above, it can be found that the existing work on this topic is carried out analytically and numerically. Results obtained from different research shows different or even contrary conclusions.

In this paper, experimental studies were performed on the response characters of a clamped rectangular plate in thermal environments. Modal tests were carried out to study the variation in natural vibrations. The dynamic and acoustic responses were also examined under acoustic and mechanical excitations. To make further investigations on the influence of thermal loads on the behavior of the plate, numerical simulations were also used to discuss the phenomena observed in tests.

II. THEORETICAL FORMULATION

Consider an isotropic rectangular thin plate lying in the plane $z = 0$ with dimensions a in the x-direction, b in the y-direction, and h in the thickness. Subjected to the variation of thermal environment, static thermal membrane forces will be generated in the plate due to the temperature change. Accordingly, the governing equation of plate flexural motion can be given as[23]

$$D_0\nabla^4 w + \rho h\frac{\partial^2 w}{\partial t^2} = N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y} + q, \tag{1}$$

where $D_0 = Eh^3/12(1-\upsilon^2)$ is the flexural rigidity of the plate, E is the Young's modulus, υ is the Poisson's ratio, ρ is the mass density, w is the transverse displacement, q is the dynamic excitation, N_x, N_y, and N_{xy} are the membrane forces.

Dynamic response of the plate can be expressed as the superposition of contributions from each natural vibration

$$w(x,y,t) = \sum_{m,n} w_{mn}\phi_{mn}(x,y)e^{j\omega t}, \tag{2}$$

where w_{mn} is the modal displacement amplitude of the mode (m, n), and $\phi_{mn}(x,y)$ is the corresponding mode shape function, j is the imaginary unit, ω is the response frequency. For simply supported boundaries, the shape function takes

$$\phi_{mn}(x,y) = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}. \tag{3}$$

For the case of uniform temperature variation, the membrane forces can be calculated by thermo-elastic theory as

$$N_x = N_y = -\frac{\alpha E\Delta Th}{1-\upsilon}, N_{xy} = 0, \tag{4}$$

where α is the thermal expansion coefficient, ΔT is the temperature variation, with positive values denoting temperature rise, and vice versa.

Assuming that the plate vibrates as the mode (m, n) with displacement distribution $w = \phi_{mn}(x,y)e^{j\omega_{mn}t}$. Substituting Eq. (4) into Eq. (1) without the excitation term leads to

$$D_0\nabla^4\phi_{mn}(x,y) - \omega_{mn}^2\rho h\phi_{mn}(x,y) + \frac{\alpha E\Delta Th}{1-\upsilon} \times \nabla^2\phi_{mn}(x,y) = 0. \tag{5}$$

Then, one can obtain the natural frequency of mode (m, n) by solving Eq. (5) as[21]

$$\omega_{mn} = \sqrt{\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2\frac{D_0}{\rho h} - \frac{\alpha E\Delta T}{\rho(1-\upsilon)}\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]}. \tag{6}$$

It is clear from Eq. (6) that there are generally two factors affecting the natural frequencies of thermally loaded plates, material property variations, and thermal forces. When the constrained plate is heated, the thermal expansion of structure will be limited by the boundaries. Then, thermal forces act as compressive forces, with negative values. Meanwhile, the Young's modulus will decrease as temperature rises for numerous materials. Both these two variations will reduce the natural frequencies of the plate. On the contrary, when the plate is cooled, material property changes and tensile thermal forces, with positive values, will increase the frequencies.

By applying the weighted residual (Galerkin) method, the integral of a weight residual of mode shape function should be set to zero,

$$\iint_\Omega\left[D_0\nabla^4 w + \rho h\frac{\partial^2 w}{\partial t^2} - \left(N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y}\right) - q\right] \times \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dA = 0, \tag{7}$$

where Ω is the space domain of the plate. Substituting Eqs. (2)–(4) into Eq. (7), a linear equation set for the unknown modal displacement amplitudes w_{mn} of forced vibration can be established as

$$[\mathbf{C}]_{MN\times MN}\{w_{mn}\}_{MN} = \{\mathbf{Q}\}_{MN}, \tag{8}$$

where $[\mathbf{C}]$ is the coefficient matrix with elements generated during the process of integration, $\{\mathbf{Q}\}$ is the excitation vector, M and N are the numbers of modes concerned in the analysis. Modal displacement amplitudes under given excitations can be obtained by solving the equation set. Then, the dynamic response of the thermally loaded plate can be acquired with Eq. (2). Furthermore, the radiated sound pressure will be predicted with Rayleigh integral[24]

$$p(x_p, y_p, z_p, t) = \frac{\mathrm{j}\omega\rho_a}{2\pi}\int_{\Omega}\frac{v(x,y,t)e^{-\mathrm{j}kR}}{R}\mathrm{d}A, \tag{9}$$

where v is the plate velocity response, ρ_a is the air density, R is the distance between the sound observation point (x_p, y_p, z_p) and the integration point on the plate, k is the wave number evaluated by vibration frequency ω and the speed of sound c_0 with $k = \omega/c_0$.

For fully clamped plates, the shape function takes

$$\phi_{mn}(x,y) = \left[(-1)^m\left(\frac{x^3}{a^3}-\frac{x^2}{a^2}\right)+\left(\frac{x^3}{a^3}-2\frac{x^2}{a^2}+\frac{x}{a}\right)-\frac{1}{m\pi}\sin\frac{m\pi x}{a}\right]\left[(-1)^n\left(\frac{y^3}{b^3}-\frac{y^2}{b^2}\right)+\left(\frac{y^3}{b^3}-2\frac{y^2}{b^2}+\frac{y}{b}\right)-\frac{1}{n\pi}\sin\frac{n\pi y}{b}\right]. \tag{10}$$

Applying the same analysis procedure, dynamic and acoustic responses of heated clamped rectangular plates could be solved as well. Due to the complexity of shape function, the derivation is more tedious and complicated. Results can hardly be given in explicit forms analytically. However, the influence of thermal variation on clamped plates is the same as that on simply supported plates.

III. TEST METHODOLOGY

A. Experimental setup

A set of test facilities was established for heated clamped rectangular plates. The specimen was placed horizontally in the test system supported by steel foundations. The heating equipment composed of four quartz lamps was set on the lower side of the plate, and measuring devices were set on the upper side. Heat flux generated from the lamps was controlled by a voltage regulator so as to change the plate temperature and keep it constant during each test. Thermal insulation blanket material was placed on the inside wall of the experimental frames to minimize the thermal radiation to the support frames.

The test plate under consideration was made of aluminum with dimensions of $0.4 \times 0.3 \times 0.003\,\mathrm{m}^3$. The central region with an area of $0.3 \times 0.2\,\mathrm{m}^2$ was the test region, which was equally divided into 50 areas by nine split lines along the shorter edge and four lines perpendicularly. Thirty six test points were marked on the intersections of the lines. Margins with the width of 0.05 m on all edges were fastened to the foundations with bolts to achieve the clamped boundary condition. Material properties of the specimen were Young's modulus 65 GPa, Poisson's ratio 0.27, mass density $2810\,\mathrm{kg\,m^{-3}}$, and thermal expansion coefficient $2.3 \times 10^{-5}\,\mathrm{m\,m^{-1}\,K^{-1}}$. The elastic parameters were determined via impact modal tests at room temperature during preliminary tests.

B. Test procedures

First, modal tests were carried out to investigate the variation in modal parameters of the plate. The test procedure was implemented via single-input single-output hammer based impact method (PCB hammer 086C04). The accelerometer (PCB 333B32) was located on a proper location, so that the first five modes can be discriminated. The plate was impacted on all the 36 points to obtain the mode shapes. Each point was excited repeatedly, and five impact frequency responses with good coherence were finally selected. The coherence check was implemented in the test system during impact tests. The impact responses were acquired within one second after the impact in the bandwidth of 4096 Hz with a resolution of 0.5 Hz. The Impact Testing module of the software package LMS TEST.LAB was used for signal acquisitions and data analyses (see Fig. 1).

Dynamic response measurements were also made under acoustic and mechanical excitations, respectively. As

FIG. 1. (Color online) Photograph of the modal test setup.

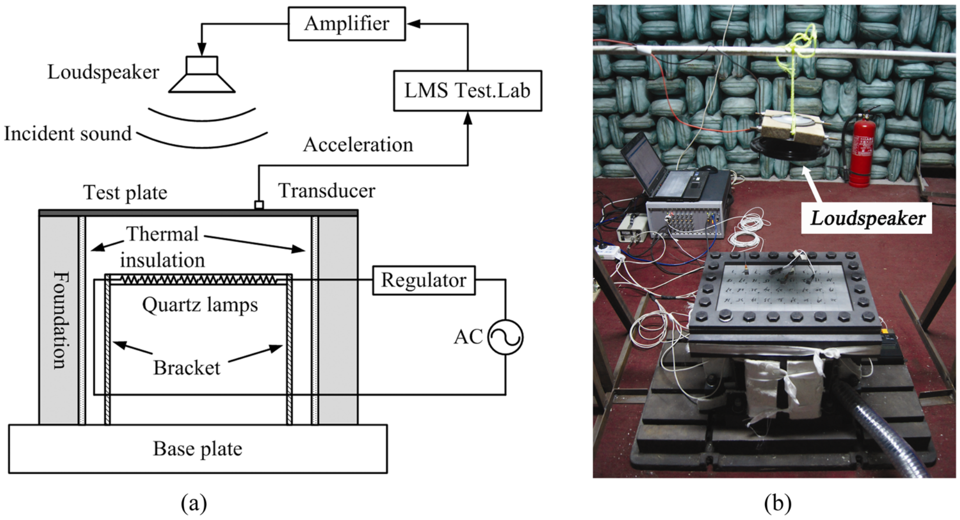

FIG. 2. (Color online) Experimental setup for acoustic excitation tests: (a) schematic of the setup; (b) photograph of the setup.

depicted in Fig. 2, the acoustic source was generated by a loudspeaker (Alpine SPS-170A) installed 0.3 m above the plate center, and was applied on the specimen through a power amplifier (B&K type 2718). The vibration responses of some observation points on the plate were acquired by accelerometers (PCB 333B32). Mechanical excitations were generated by a vibration exciter (MB Exciter Modal 2), which was placed under the plate and covered with thermal insulation blankets. A blower was used to cool the exciter during heating procedures. And it was shut down as the tests were conducted. Acceleration responses on the plate were detected, and the radiated sound pressure was acquired with a microphone (B&K type 4958) set 0.5 m vertically above the plate center (see Fig. 3). To minimize the interference from the circumstance against the radiated sound, all the instruments, except the microphone, were arranged on the lower side of the plate. As being quite close to the lamps if a force transducer was installed, it was not used due to the high environment temperature.

The spectral responses under periodic chirp signals can be obtained more rapidly than single-frequency signals, and the responses are much clearer than other frequency-swept signals provided in the test system, such as random or burst signals. Periodic chirp signals were selected as the excitation form in the tests. This was confirmed in preliminary tests. Responses were all detected in the bandwidth of 4096 Hz with a resolution of 0.5 Hz. Dynamic tests were implemented by the Spectral Testing module of the software package LMS TEST.LAB to achieve the excitation control and data processing.

The experimental setup was originally assembled at room temperature for each test item, and all the tests were first carried out in this condition as reference. The specimen could be assumed to be stress free without thermal influence at the beginning. After heating periods, tests were implemented in two different thermal conditions with structural temperature rises of 10 °C and 20 °C higher. The plate temperature was measured by thermocouples with an instrumental error of about ± 1 °C. All the tests were operated in a semi-anechoic room.

IV. EXPERIMENTAL RESULTS

A. Natural vibration

Table I gives the first five natural frequencies obtained from impact tests. It is clear that the natural frequencies of the plate decrease when temperature is elevated. As seen

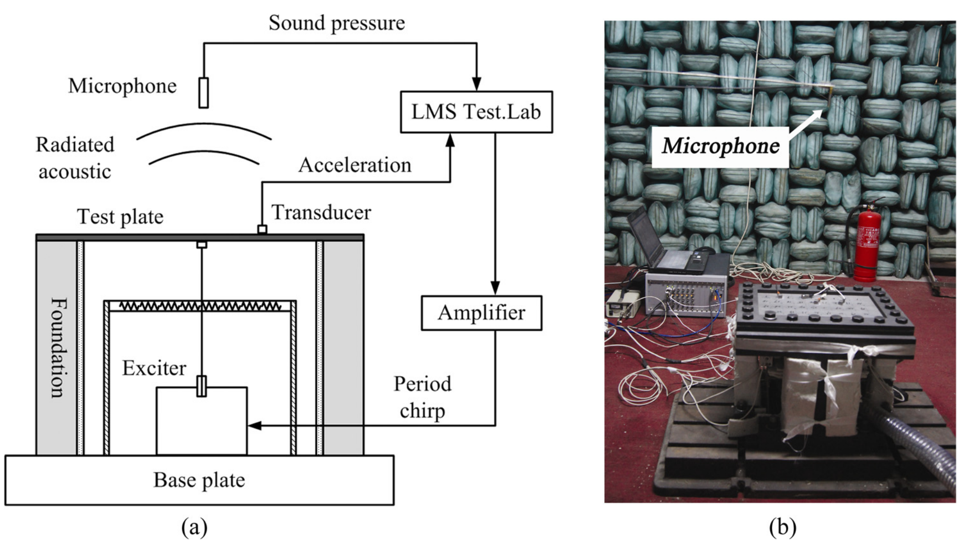

FIG. 3. (Color online) Experimental setup for mechanical excitation tests: (a) schematic of the test setup; (b) photograph of the test setup.

TABLE I. Measured natural frequencies and mode shapes of the plate (Hz).

Test state	Natural frequencies				
	1st	2nd	3rd	4th	5th
Room temperature	447.6 (1, 1)	681.4 (2, 1)	1100.5 (1, 2)	1119.2 (3, 1)	1326.9 (2, 2)
$\Delta T = 10\,^{\circ}\mathrm{C}$	369.6 (1, 1)	581.8 (2, 1)	996.6 (3, 1)	1021.2 (1, 2)	1230.6 (2, 2)
$\Delta T = 20\,^{\circ}\mathrm{C}$	322.7 (1, 1)	522.6 (2, 1)	936.0 (3, 1)	974.2 (1, 2)	1179.3 (2, 2)

from the table, the first natural frequency reduces by 27.9% from the value at room temperature when the plate temperature is elevated by 20 °C. The reduction ratio is much higher than the values of other natural frequencies. This trend is the same as the result of heated simply supported plates reported in Ref. 21. For fully clamped boundaries, the in-plane thermal expansion of the plate is almost completely constrained. When the plate is heated, compressive stresses (with negative values) accumulate in the structure, since thermal expansion is limited. Known from Sec. II, both the decrement of elastic modulus and compressive thermal stresses will make the natural frequencies lower. Test results reported in Ref. 13 indicated that the changes in material properties reduce the natural frequencies of aluminum plates quite slightly in the temperature range of this work. Therefore, thermal stresses can be regarded as the major reason for the reduction of natural frequencies of the heated clamp plate.

The first five mode shapes of the plate obtained from the impact test carried out at room temperature are illustrated in Fig. 4. Configurations of the mode shapes stay the same when the plate temperature is elevated, since they are mainly determined by the boundary condition. Meanwhile, the results from impact tests indicate that the order of the modes (1, 2) and (3, 1) can be interchanged, as shown in Table I. By comparing the natural frequencies related to the two modes, it can be found that the values, 1100.5 and 1119.2 Hz, are quite close to each other with a relative difference no more than 2%. After the plate is heated, the two frequencies are still quite close. And the value of mode (3, 1) becomes lower than mode (1, 2). This indicates that the influence of thermally softening effect on the structural stiffness along the longer edge is higher for the rectangular plate.

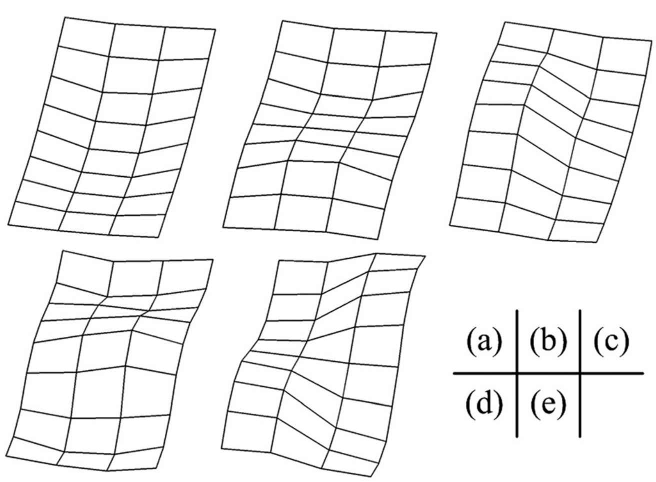

FIG. 4. Representation of measured mode shapes at room temperature: (a) fundamental mode (1, 1); (b) second mode (2, 1); (c) third mode (1, 2); (d) fourth mode (3, 1); (e) fifth mode (2, 2).

The measured damping ratios are listed in Table II for the first five modes of the test plate in different thermal environments. In the test temperature range, the modal damping ratios of odd−odd modes mainly increase with the increment of plate temperature. For the first mode, the modal damping ratio increases to the triple value with 20 °C plate temperature change. The damping ratio of mode (3, 1) rises from 0.38 to 0.6 as well. For other modes, the variations show fluctuation.

B. Acoustic excitation

Responses of the plate were detected in the test under acoustic excitations. Figure 5 shows the acceleration responses on one of the observation points. It is clear that the response curve shifts toward the lower frequency range when the plate is heated, while the global response character almost stays the same. This variation can be explained with the change of natural vibrations in thermal environments. Although the mode shape interchange occurs under thermal loads, the response near 1000 Hz does not vary since the frequencies for the two modes are quite close to each other. Variations in the acceleration responses on other observation points are quite similar.

For the first resonant peak, there is an obvious decrease in amplitude with the increment of temperature. As calculated with the measured acceleration responses, the first resonant amplitudes of velocity responses are about 12.1×10^{-6}, 7.9×10^{-6}, and $4.7 \times 10^{-6}\,\mathrm{m\,s^{-1}}$ for the three test cases, respectively. And the values of displacement responses are about 4.2×10^{-9}, 3.3×10^{-9}, and $2.4 \times 10^{-9}\,\mathrm{m}$, respectively. It is clear that all the response amplitudes at the first resonant peaks decrease as temperature rises. There are generally two alterations when the plate is heated. First, the effective stiffness of the plate decreases due to the existence of thermal loads. This will lead to an increment in the resonant amplitudes of displacement. Second, the damping character of the plate will be changed with thermal variation. Results from modal tests show that the modal damping ratio for the first mode rises greatly when the temperature is elevated, which means that the reduction of vibration goes

TABLE II. Measured modal damping ratios of the plate (%).

Test state	(1, 1)	(2, 1)	(1, 2)	(3, 1)	(2, 2)
Room temperature	0.47	0.53	0.89	0.38	1.09
$\Delta T = 10\,^{\circ}\mathrm{C}$	0.69	0.20	0.41	0.58	1.17
$\Delta T = 20\,^{\circ}\mathrm{C}$	1.59	0.32	0.41	0.60	0.86

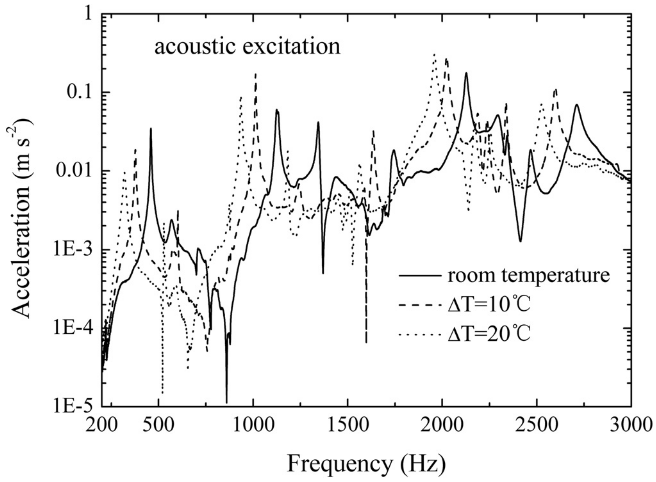

FIG. 5. Measured responses of the plate subjected to acoustic excitation under different thermal loads.

higher. With the combined effect of the two variations, the reduction in amplitude indicates that damping plays the dominant role in the tests.

For the second resonant peak (near 1000 Hz) of the responses, the amplitudes mainly increase. It is determined by the third and the fourth mode. The data listed in Table II show that the damping ratio of mode (1, 2) reduces obviously while the value of mode (3, 1) almost stays the same. So the combined influence of the two modes leads to an incremental variation in amplitude. For other resonant peaks, the variations generally follow this rule as well.

C. Mechanical excitation

The measured vibration responses of the plate in different thermal conditions are plotted in Fig. 6. Similarly with the acoustic excitation tests, the response curves shift toward the lower frequency range with an increment of temperature as well. Meanwhile, there are also some differences between them. First, the locations of the resonant peaks obtained in the two tests are not quite identical, and the frequencies corresponding to the response peaks here are also different from the measured values showed in Sec IV A. In the three thermal conditions, the first natural frequencies of the plate are 447.6, 369.6, and 322.7 Hz, respectively, while the resonant frequencies are about 410, 340, and 310 Hz in current tests. This is caused by the exciter. It makes the resonant frequencies lower than non-contact tests, and this is also mentioned by Amabili *et al.*[25]

Second, the variations in resonant amplitudes are not similar with that observed in acoustic excitation tests. Figure 6 shows that the first resonant amplitudes on all the observation points almost stay the same with a slight decrease in thermal environments, while the variation is quite obvious as observed in the acoustic excitation tests (see Fig. 5).

In mechanical excitation tests, the exciter applies excitations via a rod connected to the plate, and it becomes an additional part of the specimen. There will be an additional mass and stiffness at the exciting point. Damping character of the plate will be changed to a certain degree as well. After the plate is heated, this additional effect would be more or less changed because of the variation in the thermal condition of the plate. As mentioned in Sec III B, the force transducer was not used in the test due to the high temperature of the space adjacent to the plate. Therefore, the influence of the test apparatus on the plate responses cannot be eliminated from the test results. This would be avoided by using force transducers in future work with better instruments.

Figure 7 displays the sound pressure level (SPL) responses at the observation point. The shift of SPL resonant peaks shows the same variation trend with plate acceleration responses, since the radiation character is influenced by the plate vibration directly. Compared with the results shown in Fig. 6, the first resonant amplitude of SPL shows a contrary variation tendency, increasing with the increment of temperature. As calculated from the acceleration response in Fig. 6, it can be obtained that the velocity responses at the first resonant peak are about 4.83×10^{-5}, 5.59×10^{-5}, and $5.65 \times 10^{-5}\,\mathrm{m\,s^{-1}}$ for the three test cases, respectively. The variation tendency of the first resonant amplitude of SPL is the same with the trend of plate velocity responses.

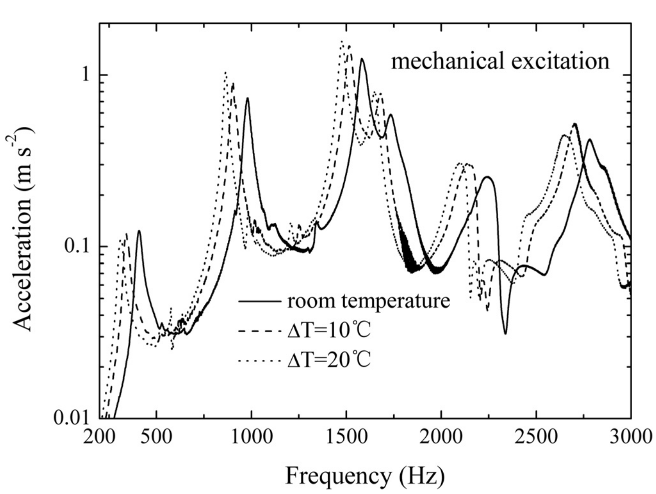

FIG. 6. Measured responses of the plate subjected to mechanical excitation under different thermal loads.

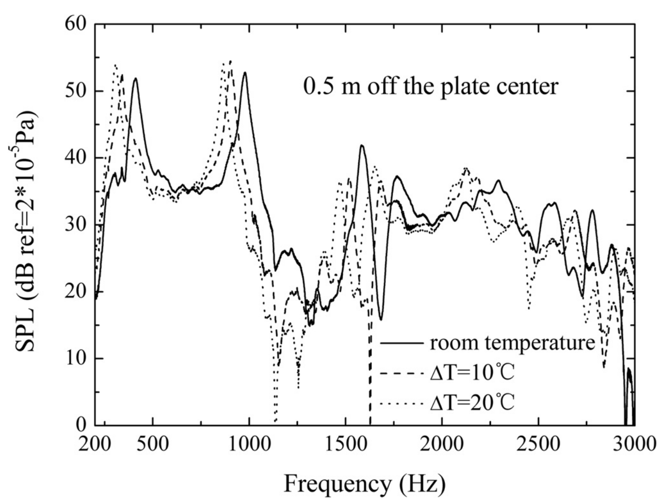

FIG. 7. Measured SPL responses at the observation point under different thermal loads.

V. SIMULATIONS AND DISCUSSIONS

In this section, numerical simulations are employed to study the phenomena observed in experiments. In finite element formulation, pre-stressed modal analysis and dynamic analysis of structures with membrane forces leads to the equations[26]

$$\left([\mathbf{K}+\mathbf{K}_\sigma]-\omega^2[\mathbf{M}]\right)\{\mathbf{U}\}=\{0\}, \tag{11}$$

$$[\mathbf{K}+\mathbf{K}_\sigma]\{\mathbf{U}\}+[\mathbf{M}]\{\ddot{\mathbf{U}}\}=\{\mathbf{F}\}, \tag{12}$$

where $[\mathbf{K}]$ is the conventional stiffness matrix, $[\mathbf{M}]$ is the mass matrix, ω is the angular frequency, $\{\mathbf{U}\}$ is the amplitude vector of nodal degree of freedom (d.o.f.), $\{\ddot{\mathbf{U}}\}$ is the acceleration vector of nodal d.o.f., $\{\mathbf{F}\}$ is the excitation vector, $[\mathbf{K}_\sigma]$ is the stress stiffness matrix which can be defined for plate model as

$$[\mathbf{K}_\sigma]=\sum_i\int_{A_i}[\mathbf{G}]^{\mathrm{T}}\begin{bmatrix}N_x & N_{xy}\\ N_{xy} & N_y\end{bmatrix}[\mathbf{G}]\mathrm{d}A, \tag{13}$$

where N_x, N_y, and N_{xy} are the static membrane forces, $[\mathbf{G}]$ is the strain-displacement matrix, A_i is the area of the ith element.

In this work, the membrane forces are induced by plate temperature variation, and can be obtained by static thermo-elastic analysis at first. Then, the dynamic response of plates under thermal loads could be calculated via pre-stressed dynamic analysis.

A. Modal results

The software package MD NASTRAN was used to carry out normal modes analyses. The plate was modeled by 2400 quadrilateral plate elements (CQUAD4) with 60 elements along the length and 40 elements along the width, respectively. Linear static solver (sol 101) was selected to conduct thermo-elastic analyses first. And then the thermal stresses were used as pre-stresses in normal modes calculations with solver sol 103. As analyzed in Sec. IV A, material properties could be assumed constant in the calculations.

Figure 8 gives the errors of numerical solutions related to the test ones. It is obvious that good agreements are obtained for the first two thermal conditions, with errors about 5% or less. However, the discrepancy of frequencies becomes greater when the plate temperature goes higher. The absolute value of error of the fundamental natural frequency reaches about 30%, and the value of the second frequency exceeds 10%. Mode shape interchange is also reflected in numerical calculations.

Due to the thickness of the specimen, it is impossible to make 3 mm thick aluminum plates stay absolutely flat. There will be an inevitable initial transverse deflection, which is also observed on the test plate. On the other hand, the test plate is heated on one side (the lower side) in this experiment. Before the temperature distribution is stable, there is a transition stage during which the temperature is not quite uniform along the plate thickness. Affected by the non-uniform thermal distribution, the plate will deflect slightly. After the temperature is stable, the thermally induced deformation will not disappear. The plate still remains curved.

Consequently, a set of hypothetical transverse deflections of the plate is used in mode analyses. The procedure of the simulation is expressed below.

(1) Apply a hypothetical uniform surface pressure to the plate and carry out static analyses to obtain the initial deflection.
(2) Import the deformed model obtained in step 1 and apply thermal loads to calculate thermal stresses and deflections. Only the configuration of the curved plate is extracted.
(3) Carry out thermally pre-stressed normal modes analyses.

In this work, three hypothetical initial deflections are used. Deformations on the center of the plate are about 0.4, 0.8, and 1.2 mm corresponding to the initial configurations No. 1–No. 3, respectively. The errors between the curved models and the specimen are plotted in Fig. 9. Results show that natural frequencies are getting higher in all thermal conditions with the increment of initial deformation. The first two frequencies are much more sensitive to the initial deflection than others, especially the fundamental one. Considering this, the relative errors of the heating state with 20 °C temperature increment reduce from about 30% to 1% for the first natural frequency and from 11.5% to 1.5% for

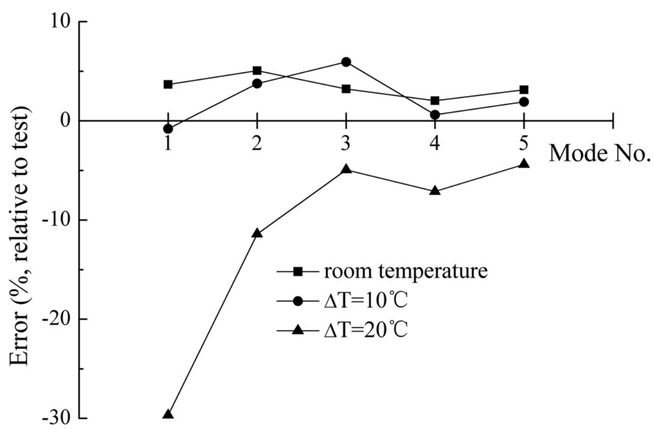

FIG. 8. Frequency errors between the measured and numerical results.

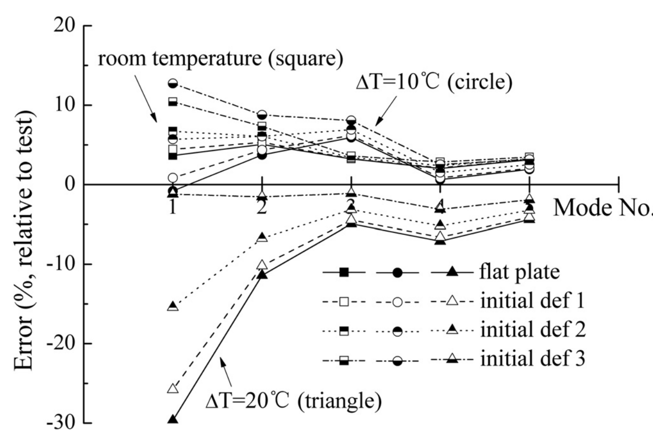

FIG. 9. Frequency errors between the measured and numerical results with different hypothetical initial deflections.

the second one. All the frequencies get closer to the test results. In calculations at room temperature, the first two frequencies get away from test results. For the other three frequencies, errors almost stay the same. In the case of 10 °C temperature variation, the error of the fundamental frequency converts to positive value, and errors of other modes all get little higher.

In the tests, dynamic behavior of the plate is quite similar to a flat one at room temperature. The influence of initial deflection can be neglected. After being heated, the initially curved plate keeps on deflecting. Part of the thermal stresses will be released compared with the ideal flat plate. And part of the thermally softening effect is released. Therefore, the calculated frequencies are lower than the measured ones. During the tests, the plate temperature is elevated gradually. The plate deflection gets larger and the loss of softening effect becomes higher. The discrepancies between the test and numerical results for the flat model become more obvious as temperature rises. In engineering applications for hot structures, the influence of initial deflection should be considered for some particular conditions.

B. Dynamic responses

The combined FEM/BEM solver of the software package VA ONE was employed to calculate the dynamic responses. Plate models were selected according to the modal results. The flat model was used for the room temperature case. Initially curved models with center deflections of 0.4 and 1.2 mm were chosen for the cases with temperature increases of 10 °C and 20 °C, respectively. Modal parameters were imported into VA ONE from NASTRAN result files. The acoustic excitation was applied via monopole source 0.3 m above the FEM plate model in the BEM fluid, and the

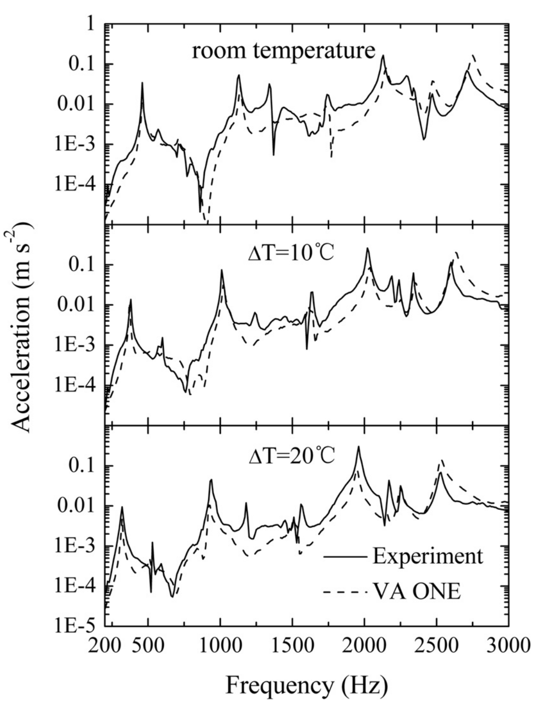

FIG. 10. Comparisons between the measured and calculated acceleration responses under acoustic excitation.

pressure spectrum was measured during the tests. To simplify the simulations, the material damping is modeled with a constant damping loss factor valued 0.5%, which is assumed to be constant in the bandwidth and in the temperature range.

Test results in Fig. 5 show that all the modes of the plate are excited under the acoustic excitations. The loudspeaker acted a little different from a monopole. In the tests, it is impossible to set the loudspeaker perfectly on the center line of the plate. So an offset value of 3 mm was set to the monopole in both the two in-plane directions. Figure 10 displays the comparisons of acceleration responses between the measured and numerical results on point 8. It can be seen that the simulation results coincide with the test ones well in all the thermal conditions. Good agreements between the measured and calculated results are also obtained for other observation points. Comparisons are not displayed here for briefness. Numerical predictions generally reflect the practical response characters of the plate.

VI. CONCLUSIONS

The influence of thermal effect on the dynamic and acoustic response characters of a plate was studied with experiments and numerical simulations in this work. Modal test results indicate that the clamped plate is softened evidently in thermal environments. Simulations show that the initial deflection affects the natural frequencies of the heated plate greatly. The irregularity of plates will reduce the thermally softening effect. The mode shape interchange indicates that the thermal effect has unequal influences on the rectangular plate in the two in-plane dimensions. For the first five modes, modal damping ratios of the odd−odd modes increase with the temperature increases, and the values of other modes fluctuate in the test temperature range.

When the plate is heated, the structural acceleration and the acoustic radiation response curves shift toward the lower frequency range, and the global characters of the responses almost stay unchanged. The variation in the resonant amplitudes of acceleration response shows that the modal damping ratio plays a crucial role. And the first resonant amplitude of SPL varies in the same trend with plate velocity response in the test.

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