动力学系统建模

–阻尼问题

张新华
Email:xhzhang@mail.xjtu.edu.cn

航天学院，西安交通大学

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工程实例：吸粮机

振动系统中的阻尼

工程中最一般的动力学方程

$$M\ddot{X}+C\dot{X}+KX+jDX+G\dot{X}+N(X,\dot{X})=F(X,\dot{X},t)$$

前面已经讨论了建立质量阵 M 与刚度阵 K 的过程中应当注意的问题。
下面我们讨论阻尼的问题。

振动系统中的阻尼

工程中最常用最简单的阻尼模型是比例阻尼。

- 比例阻尼/Rayleigh 阻尼

$$M\ddot{X} + C\dot{X} + KX = F(t)$$

$$C = \alpha M + \beta K$$

这里，α, β 为常数

- 关于阻尼的三个问题
  (1) 如何确定 α, β?
  (2) 阻尼矩阵何时可以 (与 M, K 同时) 解耦?
  (3) 阻尼矩阵不能解耦时该如何处理?

振动系统中的阻尼

- 首先考虑第一个问题。先进行坐标变换 $X = Uq$，然后对原振动方程左乘 U^T，可得

$$U^TMU\ddot{q} + U^T(\alpha M + \beta K)U\dot{q} + U^TKUq = U^TF(t)$$

$$\ddot{q}_i + 2\zeta_i\omega_i\dot{q}_i + \omega_i^2 q_i = f_i(t)$$

- 通过实验测量第 1 阶和第 2 阶模态阻尼比 ζ_1, ζ_2，然后确定常数 α, β

$$\alpha + \beta\omega_1^2 = 2\zeta_1\omega_1, \quad \alpha + \beta\omega_2^2 = 2\zeta_2\omega_2$$

$$\alpha = \frac{2\omega_1\omega_2}{\omega_2^2 - \omega_1^2}(\zeta_1\omega_2 - \zeta_2\omega_1), \quad \beta = \frac{2\omega_2^2(\zeta_2\omega_2 - \zeta_1\omega_1)}{\omega_2^2 - \omega_1^2}$$

- 工程中还可进行更多阶模态阻尼的测量，利用最小二乘法来确定 α, β

振动系统中的阻尼

- 对于第二个问题。有如下结论：
当 M, C, K 为正定实对称矩阵时，当且仅当满足下列三条件之一

$$MK^{-1}C = CK^{-1}M, \; CM^{-1}K = KM^{-1}C, \; KC^{-1}M = MC^{-1}K$$

就一定存在坐标变换，使得 M, C, K 三者可以同时对角化。
- 阻尼矩阵可对角化条件的另一种形式：矩阵 $M^{-1}C$ 与 $M^{-1}K$ 乘法可交换

$$(M^{-1}C)(M^{-1}K) = (M^{-1}K)(M^{-1}C)$$

- 在工程实际中，很少进行这种验证！而是直接采用比例阻尼。

振动系统中的阻尼

- 对于第三个问题。通过引入复模态方法解决。
- 原始的振动方程

$$M\ddot{X} + C\dot{X} + KX = F(t)$$

- 补充下列方程

$$M\dot{X} - M\dot{X} = 0$$

  在状态空间中进行描述:

$$\begin{bmatrix} 0 & M \\ M & C \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \dot{X} \end{bmatrix} + \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} \dot{X} \\ X \end{bmatrix} = \begin{bmatrix} 0 \\ F(t) \end{bmatrix}$$

- 进而可在状态空间中进行解耦。

振动系统中的阻尼

- 状态方程的形式不唯一。如果将补充方程换为

  $$K\dot{X} - K\dot{X} = 0$$

  则状态空间中的振动方程为:

  $$\begin{bmatrix} M & 0 \\ 0 & -K \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \dot{X} \end{bmatrix} + \begin{bmatrix} C & K \\ K & 0 \end{bmatrix} \begin{bmatrix} \dot{X} \\ X \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix}$$

振动系统中的阻尼

- 特征值问题为

$$B\psi = -\lambda A\psi$$

$$\psi = \begin{bmatrix} \lambda\phi \\ \phi \end{bmatrix}, \quad \Psi = \begin{bmatrix} \psi_1 & \psi_2 & \cdots & \psi_{2n} \end{bmatrix}$$

$$\Psi^T A \Psi = A_p, \quad \Psi^T B \Psi = B_p$$

  这里 ϕ 即为复模态，ψ 则为 $2n$ 维复模态。

- 解耦之后的方程为

$$A_p \dot{z} + B_p z = \Phi^T F(t)$$

阻尼的来源与种类

- The damping may be inherent in a structure or material. Unfortunately, the term ”structural damping” has acquired a special meaning: it now appears to mean ”hysteretic damping”, and cannot be used to mean the damping in a structure, whatever its form, as the name would imply.
- Damping in conventional jointed metal structures is partly due to hysteresis within the metal itself, but much more to friction at bolted or riveted joints, and pumping of the fluid, often just air, in the joints.

阻尼的来源与种类

- Viscoelastic materials, such as elastomers (rubber-like materials，弹性体、人造橡胶), can be formulated to have relatively high damping, as well as stiffness, making them suitable for the manufacture of vibration isolators, engine mounts, etc.
- The damping may be deliberately(故意地) added to a mechanism or structure to suppress unwanted oscillations. Examples are discrete units, usually using fluids, such as vehicle suspension dampers and viscoelastic damping layers on panels.

阻尼的来源与种类

- The damping can be created by the fluid around a structure, for example air or water. If there is no relative flow between the structure and the fluid, only radiation damping is possible, and the energy loss is due to the generation of sound. There are applications where this can be important, but for normal structures vibrating in air, radiation damping can usually be ignored. On the other hand, if relative fluid flow is involved, for example an aircraft wing traveling through the air, quite large aerodynamic damping (and stiffness) forces may be developed.

阻尼的来源与种类

- Damping can be generated by magnetic fields
  The damping effect of a conductor moving in a magnetic field is often used in measuring instruments. Moving coils, as used in loudspeakers and, of particular interest, in vibration testing, in electro-magnetic exciters, can develop surprisingly large damping forces.

阻尼的来源与种类

- Coulomb damping: $g(\dot{x}) = \pm v_0$
- Linear velocity damping: $g(\dot{x}) = v_1\dot{x}$
- Velocity squared damping: $g(\dot{x}) = v_2\dot{x}|\dot{x}|$
- nth-power velocity damping: $g(\dot{x}) = v_n\dot{x}|\dot{x}|^{n-1}$
- Structural damping/Hysteretic damping: $g(x) = -i\,\eta x$， η 为迟滞阻尼系数。
  此时， the damping force is proportional to the displacement, x, but in phase with the velocity $\dot{x}$.

阻尼的特性

- Linear velocity damping

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = F(t)$$

- 最大动位移与静位移之比

$$\frac{|x|}{x_s} = \frac{1}{\sqrt{(1-\lambda^2)^2 + (2\zeta\lambda)^2}}, \quad \lambda = \frac{\omega}{\omega_n}$$

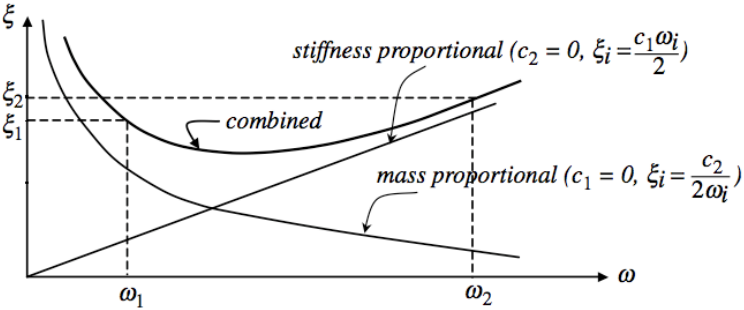
- Structural damping

$$\ddot{x} + \omega_n^2(1 + i\,\eta)x = F(t)$$

- 最大动位移与静位移之比

$$\frac{|x|}{x_s} = \frac{1}{\sqrt{(1-\lambda^2)^2 + \eta^2}}$$

阻尼的特性



比例阻尼的特性 ($c_2 = \alpha$, $c_1 = \beta$)

各种材料的阻尼比

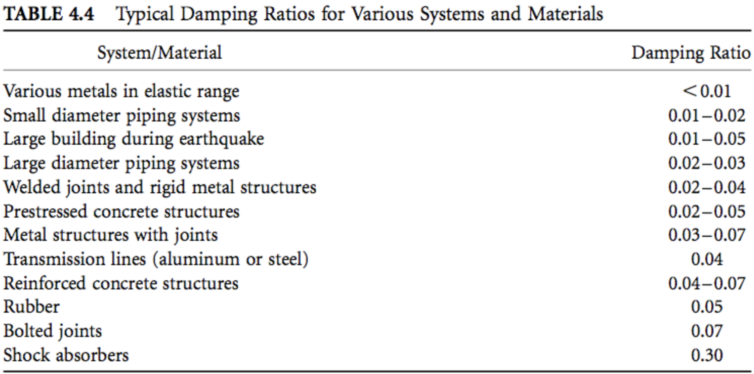
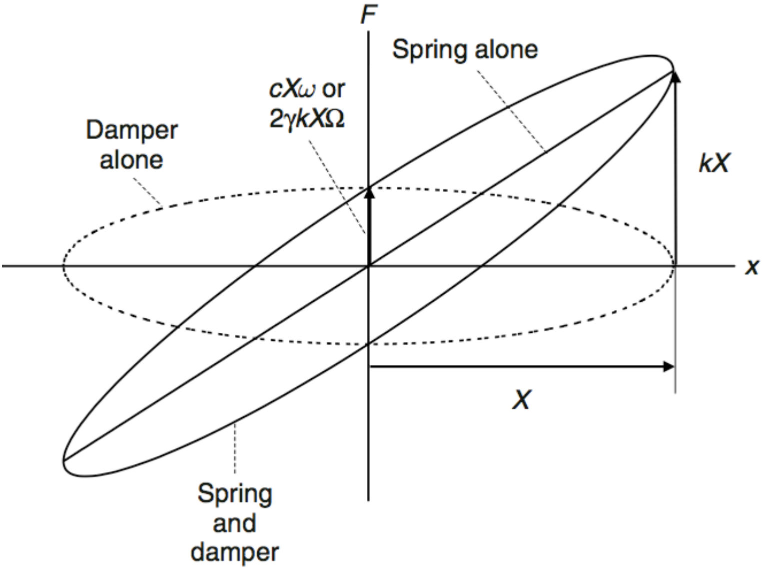


TABLE 4.4 Typical Damping Ratios for Various Systems and Materials

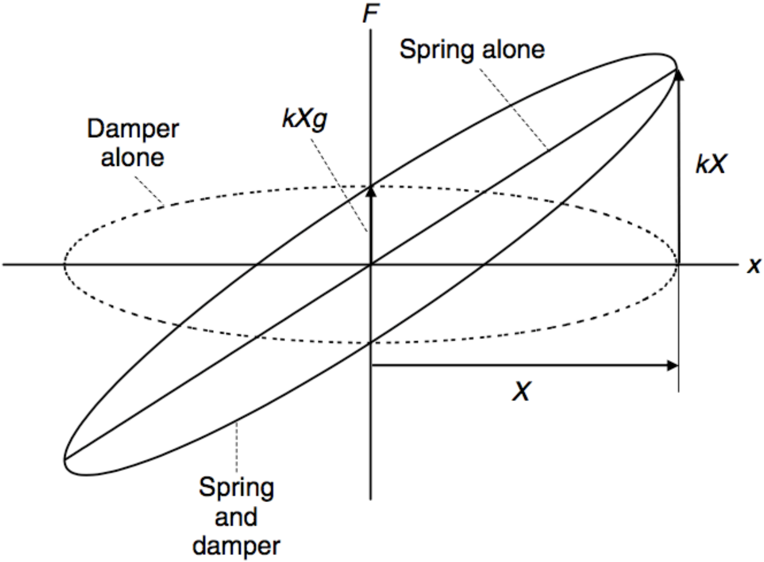
System/Material	Damping Ratio
Various metals in elastic range	< 0.01
Small diameter piping systems	0.01–0.02
Large building during earthquake	0.01–0.05
Large diameter piping systems	0.02–0.03
Welded joints and rigid metal structures	0.02–0.04
Prestressed concrete structures	0.02–0.05
Metal structures with joints	0.03–0.07
Transmission lines (aluminum or steel)	0.04
Reinforced concrete structures	0.04–0.07
Rubber	0.05
Bolted joints	0.07
Shock absorbers	0.30

(参见 Adams and Askenazi, 1999)

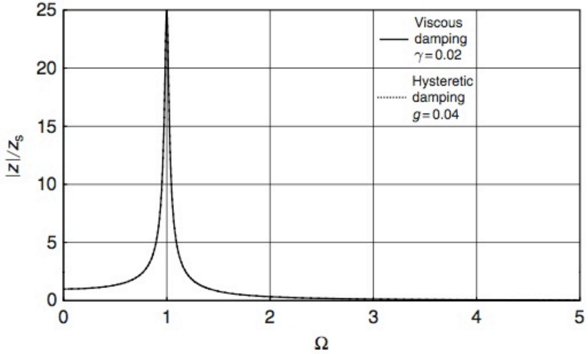
粘性阻尼的耗能特性


F
Spring alone
cXω or
2γkXΩ
Damper
alone
kX
x
X
Spring
and
damper

结构阻尼的耗能特性

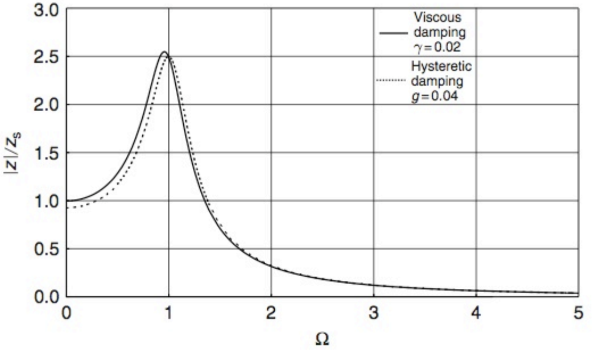


粘性阻尼与结构阻尼的对比



Comparison of the frequency response of a single-DOF system with: (1) a viscous damping coefficient of 0.02 and (2) a hysteretic damping coefficient of 0.04.

粘性阻尼与结构阻尼的对比



Comparison of the frequency response of a single-DOF system with: (1) a viscous damping coefficient of 0.20 and (2), a hysteretic damping coefficient of 0.40.

复模态的应用

尽管复模态方法较为复杂，但在下列情况下还是很有用处的。

- Rotating systems such as helicopter rotors, where centrifugal and Coriolis forces can lead to damping coupling between the modes.
- Aircraft flutter analysis and response calculations using the p method, where large cross-damping terms can arise from aerodynamic forces. The k method was devised to avoid the use of the complex eigenvalues and eigenvectors.
- Systems such as vehicle suspensions where discrete hydraulic dampers may couple the modes.
- Some buildings and off-shore structures where the soil or sea-bed damping can be large in relation to that of the structure itself.

复模态的应用

在工程实际中，特别是对于结构系统，常常将复模态进行简化为实模态，只是因为：

- In a structure, the damping terms tend to be small, and ill-defined, and although the damping may, in theory, couple the modes, the eigenvectors are "nearly real" and the undamped modes are a good approximation to the actual modes.
- The damping can often be considered to be of the proportional or Rayleigh type.

谢谢!