Supplemental Document

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Nonlinear higher-order polariton topological insulator: supplement

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The topology of the infinite kagome lattice can be characterized by bulk polarization calculated as

$$p_j = -\frac{1}{S}\iint_{\mathrm{BZ}} A_j \, \mathrm{d}^2 k, \tag{S1}$$

where $A_j = -i\langle u|\, \partial k_j |u\rangle$ is the Berry connection, k_j determine duplicable directions of the Brillouin zone (BZ), u the Bloch function, and S is the area of the first BZ. We use the tight-binding approach (see Fig. 2 in the main text) to calculate the bulk polarization. We introduce $\mathbf{e}_{1,2,3}$ that represent the vectors pointing toward neighboring array sites with $\mathbf{e}_1 = \left(1/2, -\sqrt{3}/2\right)a$, $\mathbf{e}_2 = (-1,0)a$, and $\mathbf{e}_3 = \left(1/2, \sqrt{3}/2\right)a$, where a is the lattice constant. The corresponding tight-binding Hamiltonian of the array can be written as

$$\mathcal{H} = \begin{bmatrix} 0 & v + we^{i\mathbf{k}\cdot\mathbf{e}_1} & v + we^{-i\mathbf{k}\cdot\mathbf{e}_3} \\ v + we^{-i\mathbf{k}\cdot\mathbf{e}_1} & 0 & v + we^{i\mathbf{k}\cdot\mathbf{e}_2} \\ v + we^{i\mathbf{k}\cdot\mathbf{e}_3} & v + we^{-i\mathbf{k}\cdot\mathbf{e}_2} & 0 \end{bmatrix} \tag{S2}$$

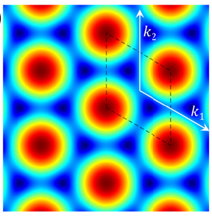
in which $\mathbf{k} = (k_x, k_y)$, intra-cell coupling strength is v and intercell coupling strength is w. For convenience, we set $a = 1$. The first band β_1 of the Hamiltonian in Eq. (S2) is described by

$$\beta_1 = \frac{1}{2}\left(v + w + \sqrt{9v^2 - 6vw + 9w^2 + 8vw\left[\cos(k_x) + 2\cos\left(\frac{k_x}{2}\right)\cos\left(\frac{\sqrt{3}k_y}{2}\right)\right]}\right), \tag{S3}$$

The eigenvector corresponding to β_1 can be written as

$$u = \begin{bmatrix} \dfrac{e^{i(k_x/2-\sqrt{3}k_y/2)}(2w^2-4vw+2v^2+2\beta_1(v+w))+2(1+e^{ik_x})vw+2e^{i(k_x-\sqrt{3}k_y)}vw+e^{-i\sqrt{3}k_y}2vw}{e^{ik_x}(2\beta_1 w-vw-w^2)+e^{i(k_x/2-\sqrt{3}k_y/2)}(2\beta_1 v+2e^{ik_x}vw+2v)+2vw+e^{ik_x}(vw+3w^2)} \\ \dfrac{e^{i(k_x/2-\sqrt{3}k_y/2)}(2\beta_1 v-v^2-vw)+2\beta_1 t_2+2e^{ik_x}vw+2w^2+e^{i(-k_x/2-\sqrt{3}k_y/2)}(2vw+e^{ik_x}(3v^2+vw))}{e^{ik_x}(2\beta_1 w-vw-w^2)+e^{i(k_x/2-\sqrt{3}k_y/2)}(2\beta_1 v+2e^{ik_x}vw+2v)+2vw+e^{ik_x}(vw+3w^2)} \\ 1 \end{bmatrix}. \tag{S4}$$

The dependence $\beta_1(k_x, k_y)$ is displayed in Fig. S1(a), where we also indicate lattice vectors $k_{1,2}$ in the k-space. Since corresponding vectors are not orthogonal, that complicates calculation of the bulk polarization (p_1, p_2) along the lattice vectors k_1 and k_2 in the k-space, we employ coordinate transformation from system (x, y) to (x', y'), where old and new coordinates are related by the expressions $x = x' + y'/2$ and $y = \sqrt{3}y'/2$. This is accompanied by the respective transformation of the Brillouin zone and corresponding lattice vectors $(k_1, k_2) \to (k_x', k_y')$ in the k-space, so that in transformed system the Brillouin zone becomes square, as shown in Fig. S1(b).

(a)


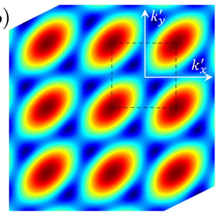
(b)


Fig. S1 Profile of β_1 before (a) and after (b) the transformation with corresponding lattice vectors in the k-space. Dashed rhombus and square represent the first BZ.

Taking the band β_1 as an example, one can easily calculate corresponding bulk polarization components in the transformed coordinate system:

$$\left(p_x', p_y'\right) = \begin{cases} (1/3,1/3), \text{for } v < w \ (d_2 < a/2) \\ (0,0), \text{for } v > w \ (d_2 \geq a/2) \end{cases} \tag{S5}$$

The system is in topological phase when polarization components are nonzero and in trivial phase when polarization components are zero [1-4]. These results are in full agreement with results of Fig. 1 of the manuscript, where for example in truncated triangular array corner states emerge simultaneously in all three corners at $d_2/a < 0.5$ that corresponds to the $v < w$ case.

References

1. A. Hassan, F. Kunst, A. Moritz, G. Andler, E. Bergholtz, and M. Bourennane, "Corner states of light in photonic waveguides," Nat. Photon. **13**, 697-700 (2019).
2. S. Mittal, V. Vikram Orre, G. Zhu, M. A. Gorlach, A. Poddubny, and M. Hafezi, "Photonic quadrupole topological phases," Nat. Photon. **13**, 692-696 (2019).
3. X.-D. Chen, W.-M. Deng, F.-L. Shi, F.-L. Zhao, M. Chen, and J.-W. Dong, "Direct Observation of Corner States in Second-Order Topological Photonic Crystal Slabs," Phys. Rev. Lett. **122**, 233902 (2019).
4. B.-Y. Xie, G.-X. Su, H.-F. Wang, H. Su, X.-P. Shen, P. Zhan, M.-H. Lu, Z.-L. Wang, and Y.-F. Chen, "Visualization of Higher-Order Topological Insulating Phases in Two-Dimensional Dielectric Photonic Crystals," Phys. Rev. Lett. **122**, 233903 (2019).