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Dark incoherent spatial solitons in logarithmically saturable nonlinear media*

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This paper studies numerically the dark incoherent spatial solitons propagating in logarithmically saturable nonlinear media by using a coherent density approach and a split-step Fourier approach for the first time. Under odd and even initial conditions, a soliton triplet and a doublet are obtained respectively for given parameters. Simultaneously, coherence properties associated with the soliton triplet and doublet are discussed. In addition, if the values of the parameters are properly chosen, five and four splittings from the input dark incoherent spatial solitons can also form. Lastly, the grayness of the soliton triplet and that of the doublet are studied, in detail.

Keywords: incoherent spatial solitons, coherence length, logarithmically nonlinear media
PACC: 4265S, 4225K, 4225B

1. Introduction

Spatially partially incoherent solitons have been attracting considerable attention in the past few years, and to our knowledge, bright incoherent solitons, dark incoherent solitons, and white light incoherent solitons have been experimentally and theoretically investigated.[1−15] As to the theoretical research, three main approaches are traditionally used, namely the coherent density approach,[1] the self-consistent multimode approach,[2] and the geometric optics approach (in the diffractionless limit).[3] In fact, the three approaches are demonstrated to be equivalent to each other in inertial nonlinear media in Ref.[4]. Frankly speaking, incoherent spatial solitons in logarithmically saturable nonlinear media have been researched by means of these approaches by Christodoulides, Coskun *et al*,[5,6] but, their work only discussed bright incoherent solitons. Even though the existence of dark and gray spatial solitons in logarithmically nonlinear media was carefully analyzed in Ref.[7], dark incoherent solitons propagating in this type of media which may be of significant values have not been contacted before.

In this paper, we discuss the theoretical model of dark incoherent spatial solitons and simulate it numerically by using the coherent density approach and split-step Fourier approach for the first time. The study shows that the soliton triplet and the doublet are obtained respectively under odd and even initial conditions in logarithmically saturable nonlinear media. The coherence properties associated with the soliton triplet and doublet are discussed simultaneously. In addition, we find that if values of the parameters are chosen properly, five and four splittings from the input dark incoherent spatial solitons can form. Lastly, we study the grayness of the soliton triplet and the doublet in detail and draw out our conclusions.

2. Theoretical model

In a logarithmically saturable nonlinear medium, the refractive index varies logarithmically with the intensity I, i.e., $n^2(I) = n_0^2 + n_2 \ln(I/I_t)$, where n_0 is the linear refractive index of this material, n_2 is a

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dimensionless coefficient associated with the strength of the nonlinearity,[5,6] and I_t is a threshold intensity. The nonlinearity of the media is focusing with $n_2 > 0$ and defocusing with $n_2 < 0$.[3] The normalized intensity $I_N = I/I_t$ of this incoherent light beam evolves according to the following set of nonlinear integro-differential equations:[5−11]

$$\mathrm{i}\left(\frac{\partial f}{\partial z}+\theta\frac{\partial f}{\partial x}\right)+\frac{1}{2k}\frac{\partial^2 f}{\partial x^2}+\frac{k_0 n_2}{2n_0}\ln\left[I_N\left(x,z\right)\right]f=0, \tag{1}$$

$$I_N\left(x,z\right)=\int_{-\infty}^{\infty}\left|f\left(x,z,\theta\right)\right|^2\mathrm{d}\theta, \tag{2}$$

$$f\left(z=0,x,\theta\right)=\rho^{1/2}G_N^{1/2}\left(\theta\right)\phi_0\left(x\right), \tag{3}$$

where f is the so-called coherent density, θ is an angle with respect to the z-axis, $k = 2\pi n_0/\lambda_0$ in which λ_0 is the free-space wavelength, $G_N(\theta)$ is the normalized angular power spectrum of the incoherent source, ϕ_0 is the input spatial modulation function, and ρ is the normalized intensity of a dark beam at $x \to \pm\infty$, i.e., $\rho = \max(I_N)$. We assume that the normalized angular power spectrum of the incoherent source is Gaussian, i.e., $G_N(\theta) = \exp\left(-\theta^2/\theta_0^2\right)/(\sqrt{\pi}\theta_0)$, where θ_0 is associated with its angular width. We also assume that the modulation function at the input is $\phi_0(x) = \tanh(x/x_0)$ under odd initial conditions and $\phi_0(x) = \left[1-\varepsilon^2\mathrm{sech}^2(x/x_0)\right]^{1/2}$ under even, where x_0 is associated with the intensity full width at half maximum (FWHM) of the beam and ε^2 associated with the beam's grayness. In this paper, we assume $\varepsilon^2 \approx 1$ at $z = 0$.[8,9] Through calculation, the relation between x_0 and FWHM is described by $x_0 = \mathrm{FWHM}/\ln\left(3+2\sqrt{2}\right)$, which is true for both odd initial conditions and even initial conditions.

To describe the coherence properties of these evolving partially incoherent beams, the complex coherent factor μ_{12} and coherence length l_c can be written as[8,10]

$$\mu_{12}\left(x_1,x_2\right)=\frac{1}{\left[I_N\left(x_1,z\right)I_N\left(x_2,z\right)\right]^{1/2}}\times\int_{-\infty}^{\infty}f\left(x_1,z,\theta\right)f^*\left(x_2,z,\theta\right)\times\exp\left[\mathrm{i}k\theta\left(x_1-x_2\right)\right]\mathrm{d}\theta, \tag{4}$$

$$l_c\left(x\right)=\int_{-\infty}^{\infty}\left|\mu_{12}\left(x,y\right)\right|^2\mathrm{d}y. \tag{5}$$

According to the above equations, we can obtain $l_c = \sqrt{2\pi}/(k\theta_0)$ at $z = 0$, which is independent of the spatial modulation function $\phi_0(x)$ but relies on k and θ_0.

3. Numerical simulation and analysis

We first study the case under odd initial conditions. We assume $n_0 = 2$, $n_2 = -5\times10^{-4}$, the input optical wavelength $\lambda_0 = 500$ nm, the angular power spectrum width $\theta_0 = 4.5$ mrad, the intensity FWHM $= 15$ μm and $\rho = 1000$, respectively. And with the parameters mentioned, the coherence length l_c at $z = 0$ is about 22.2 μm through calculation. Figure 1(a) shows the propagation of input dark solitons under odd initial conditions and figure 1(b) presents the coherence length l_c of the solitons shown in Fig.1(a). Through the figures, we can see that the pulse propagates like a gray soliton with a somewhat oscillation.

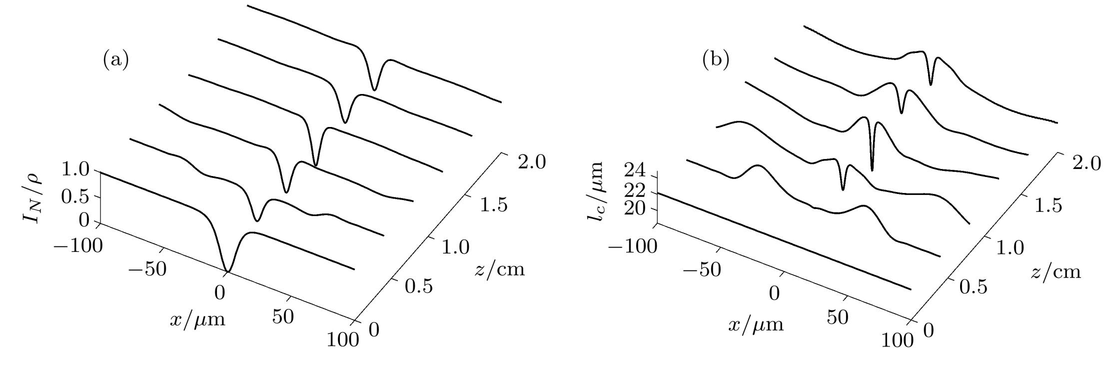

Fig.1. (a) Propagation of an input dark incoherent solitons, (b) the coherence length l_c of this incoherent solitons as a function of distance. The parameters are $n_0 = 2$, $n_2 = -5\times10^{-4}$, FWHM = 15 μm, $\lambda_0 = 500$ nm, $\theta_0 = 4.5$ mrad, $\rho = 1000$.

the coherence length becomes higher within the notch and with a depression and sharp fall at the center, which is similar to the phenomenon when dark incoherent solitons propagate in photorefractive media.[8]

If we let $n_2 = -1 \times 10^{-3}$ and other parameters unchanged, as shown in Fig.2(a) [Fig.2(b)], the input dark solitons change into an incoherent soliton triplet (doublet) under odd (even) initial conditions. The phenomenon can also be obtained when dark incoherent solitons propagate in photorefractive media as shown in Ref.[8]. However, the coherence length l_c of the triplet and that of the doublet have some difference when compared with Ref.[8]. Figure 2(c) shows that the coherence length around the central notch of the triplet increases relatively slowly, and even with a level lower than 22.2 μm. The situation will be better when the solitons propagate far enough. Figure 2(d) exhibits the coherence length of the doublet, and we can find that the depressions at the central notches are relatively more sharp and more obvious.

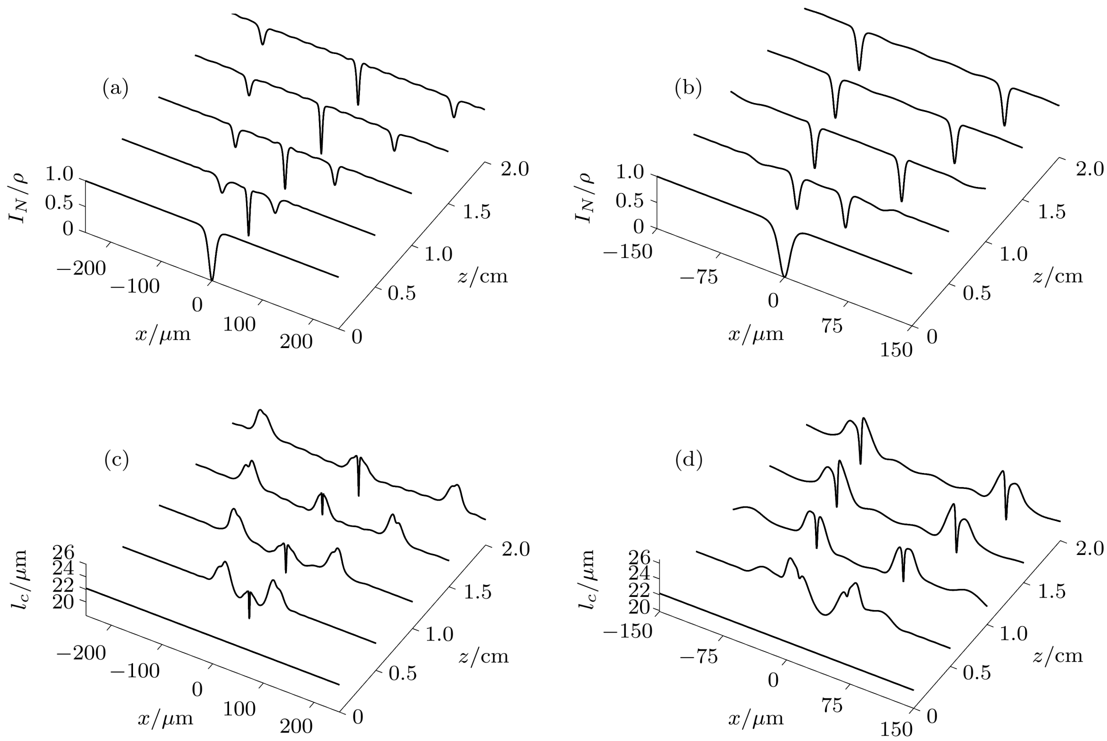

Fig.2. (a) Formation of an incoherent soliton triplet; (b) formation of an incoherent soliton doublet; (c) the coherence length l_c of the triplet as a function of distance; (d) the coherence length l_c of the doublet as a function of distance. The parameters are $n_0 = 2$, $n_2 = -5 \times 10^{-4}$, FWHM = 15 μm, $\lambda_0 = 500$ nm, $\theta_0 = 4.5$ mrad, $\rho = 1000$.

To study it further, we set the input intensity FWHM = 40 μm, $n_2 = -1\times10^{-3}$, and the rest parameters are the same as those used in Fig.1. Obviously, through the top figure of Fig.3(a) [Fig.3(b)] which depicts the case under odd (even) initial conditions, we can see that the input dark solitons split into five (four) soliton stripes during propagation in the media. The bottom figures exhibit the comparison between input dark solitons and output soliton stripes. Notice that there are small fluctuations around the soliton stripes in both of the two cases, we predict that more secondary sets of notches will appear symmetrically on both sides when the values of the parameters are properly changed, that is to say, seven soliton stripes, six soliton stripes and even more higher soliton stripes can also emerge, which is true when dark spatial solitons propagate in biased photorefractive media as shown in Ref.[14].

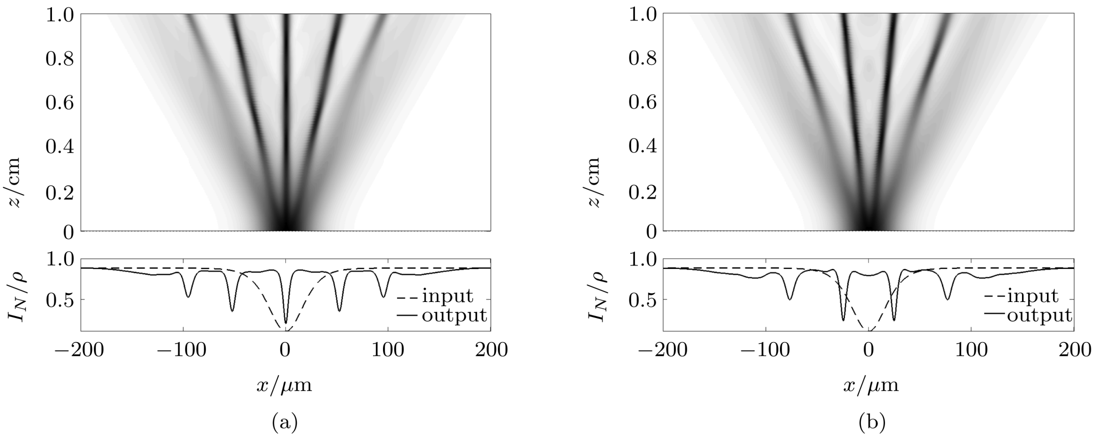

Fig.3. Propagation of an input dark incoherent soliton: (a) under odd initial conditions (top) and the input and the output at $z = 1$ cm (bottom); (b) under even initial conditions (top) and the the input and the output at $z = 1$ cm (bottom). The parameters are $n_0 = 2$, $n_2 = -1 \times 10^{-3}$, FWHM = 40 μm, $\lambda_0 = 500$ nm, $\theta_0 = 4.5$ mrad, $\rho = 1000$.

In order to effectively study the grayness of the triplet and the doublet, we introduce the g parameter ($0 < g < 1$) describing the soliton grayness, and define it as the ratio of the lowest intensity of the beam to the intensity at infinity.[15] Take the triplet and doublet shown in Figs.2(a) and 2(c) for example, figure 4(a) exhibits the grayness of the central notch of the triplet as a function of distance. We can see clearly that the grayness has a sine-like shape with a period of 0.2 cm, and the maximum and minimum are approximately 0.16 and 0.02, respectively. Figure 4(b) shows the grayness of the doublet. Even though it also seems to have a sine-like shape with a relatively stable period (the value is about 0.3 cm) and the difference between the maximum and minimum (the value is approximately 0.13), the maximum and minimum are declining slightly in the process of propagation. We predict that if the propagation distance is long enough, it will become relatively stable as shown in Fig.4(a).

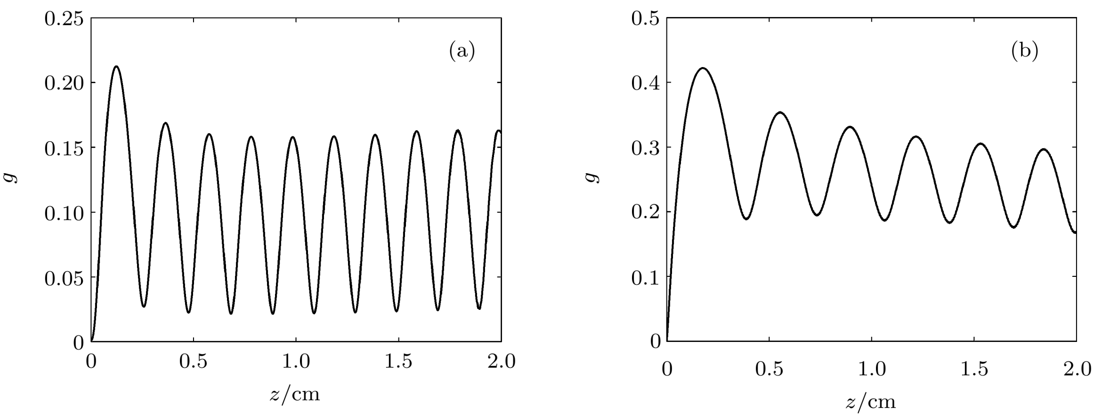

Fig.4. (a) The grayness of the central notch of the triplet as a function of distance; (b) the grayness of the notches of the doublet as a function of distance. The parameters are $n_0 = 2$, $n_2 = -1 \times 10^{-3}$, FWHM = 15 μm, $\lambda_0 = 500$ nm, $\theta_0 = 4.5$ mrad, $\rho = 1000$.

4. Conclusion

In conclusion, we have investigated the propagation of input dark incoherent solitons in logarithmically saturable media through numerical simulation. We find that the soliton triplet (under odd initial conditions) and the doublet (under even initial conditions) can form if we choose proper values for the parameters. And we also find that the input dark incoherent solitons can even split into five or four soliton stripes and even higher soliton stripes under different initial conditions. In order to study the coherence properties of these evolving incoherent solitons, the coherence length associated with each case is discussed in detail. We find that all the phenomena are similar to the case of input gray or dark solitons propagating in photorefractive media, i.e. the phenomena achieved in photorefractive media can be also obtained in logarithmically saturable nonlinear media under proper initial conditions. In the last part, the grayness of the soliton triplet and that of the doublet are numerically studied. The results indicate that the grayness has a sine-like shape which represents the periodically changing grayness.

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