



Interactions of two humps of dipoles in anisotropic nonlinear media

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ABSTRACT

Propagation of dipoles and superposed dipoles in anisotropic nonlinear medium is investigated. The two humps of the dipoles will repel each other if attraction between them is smaller than repulsion, while if the former is larger than the latter, they attract. If there is phase gradient on the input dipoles or the energy distribution of the dipoles does not along the boundary directions of the medium, the dipoles will exhibit incomplete rotation during propagation.

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1. Introduction

Anisotropic nonlinearity that can be observed in photorefractive crystal [1–5] has been studied almost exclusively in the last two decades. As a nonlocal nonlinearity, solitons and their dynamics in such a kind of nonlinear medium have also been greatly investigated [6–10]. The previous researches predict that the waveguides induced by solitons and their anomalous interactions in anisotropic media can be used to produce optical logical devices in the future [5,6]. In recent years, solitons in photonic lattices supported by anisotropy were reported [11–15], which indicates that anisotropic nonlinearity is still a research hot that attracts scientists.

Among a quiverful of the previous literatures on solitons in anisotropic nonlinear media, dipole mode vector solitons [7,10] and solitons composed by two mutually incoherent beams [5] were discussed. Owing to the anisotropic nonlinearity, the reported solitons would exhibit incomplete rotation, repulsion, attraction and other properties. Seemingly similar to the previous work, we investigate the propagation properties of coherent dipoles and superposed of them in anisotropic nonlinear media in this article. The dipoles used in the article are constructed (not dipole solutions), which or superposed of which will be the input when we execute the propagations. And the structure is organized as: in Section 2, we firstly introduce the theoretical model that will be used briefly, secondly construct the dipoles, and lastly display the induced refractive indices by the dipoles and superposed dipoles; in Section 3, we investigate the propagation

properties of the structures mentioned in Section 2 in detail, and meanwhile compare the results with those in the previous ones; in Section 4, we conclude the article.

2. Theoretical model

We begin our analysis by considering light wave propagates in anisotropic nonlocal media, which is described by the set of coupled equations, i.e., the so-called Zozulya–Anderson model [2,3,5]

$$\left[\frac{\partial}{\partial z} - \frac{i}{2} \nabla^2 \right] f(\vec{r}, z) = i \frac{\partial \varphi}{\partial x} f(\vec{r}, z), \quad (1a)$$

$$\nabla^2 \varphi + \nabla \varphi \cdot \nabla \ln(1 + |f|^2) = \frac{\partial}{\partial x} \ln(1 + |f|^2), \quad (1b)$$

where $\nabla = \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y)$ and φ is the dimensionless electrostatic potential induced by the light beam with the boundary conditions $\nabla \varphi(\vec{r} \rightarrow \infty) \rightarrow 0$. Eq. (1a) without analytical solutions is highly anisotropic. For a incident beam, it will induce a potential in the medium according to Eq. (1b), and then the refractive index change determined by the derivative of x of the potential in Eq. (1a) will control the propagation properties of the incident beam. Both two processes that guide beam propagation in the anisotropic nonlocal self-focusing medium happen simultaneously.

To form a incident beam, we construct two orthogonal dipoles

$$\begin{aligned} D_1(\vec{r}) &= Ax \cdot \operatorname{sech}\left(\frac{1}{B} \sqrt{x^2 + y^2}\right), \\ D_2(\vec{r}) &= Ay \cdot \operatorname{sech}\left(\frac{1}{B} \sqrt{x^2 + y^2}\right), \end{aligned} \quad (2)$$

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where A is the amplitude of the dipoles, B relates with the full width of half maximum (FWHM) of the intensity of dipoles, and $A = \sqrt{2.3}$, $B = 2$ throughout this paper. By superposing the dipoles [16], as exhibited in Eq. (3), we can get a variety of beam structures, which will be the input for Eq. (1a). Now let us go back to Eq. (1a), in which the nonlinear refractive index Δn induced by the beam is proportional to $\partial\varphi/\partial x$, i.e. $\Delta n \sim \partial\varphi/\partial x$. Based on Eq. (1b), the refractive index distributions corresponding to $f_{1-8}(\vec{r}, z=0)$ are exhibited in Fig. 1(a–g), respectively. The insets are the profiles of the refractive indices at places where marked by white solid and dashed lines. For the dipoles, each hump is surrounded by regions of both positive and negative Δn

$$f_{7,8} = \sqrt{\frac{\iint |D_1|^2 d^2\vec{r}}{\iint |D_1 \pm D_2|^2 d^2\vec{r}}}(D_1 \pm D_2). \tag{3}$$

In Fig. 1(a), the refractive index induced between the two humps (f_1) is negative along the x -axis, which means that the two humps will repel each other during propagation. But in Fig. 1(b), which is induced by f_2 , even though the Δn is always positive along the y -axis, two humps may attract or repel each other, that is because humps are out of phase which will lead repulsion between them internally. Fig. 1(c), which depicts Δn induced by a vortex (f_3 or f_4), indicates that the vortex will be divided into two parts because of the focusing effect brought by the two maximums [3]. Fig. 1(d)–(g) is the Δn distribution induced by f_{5-8} , respectively.

3. Numerical simulations and analysis

To present the trajectories of the dipoles (f_1 or f_2) it is sufficient to record the transverse intensity distribution along the x - or y -axis during propagation as shown in Fig. 2, because the dipole will stay on the (x,z) or (y,z) plane without rotation. While

$$f_{1,2} = D_{1,2},$$

$$f_{3,4} = \sqrt{\frac{\iint |D_1|^2 d^2\vec{r}}{\iint |D_{1,2} + iD_{2,1}|^2 d^2\vec{r}}}(D_{1,2} + iD_{2,1}),$$

$$f_{5,6} = \sqrt{\frac{\iint |D_1|^2 d^2\vec{r}}{\iint |D_{1,2} + iD_{2,1}/2|^2 d^2\vec{r}}}\left(D_{1,2} + \frac{i}{2}D_{2,1}\right),$$

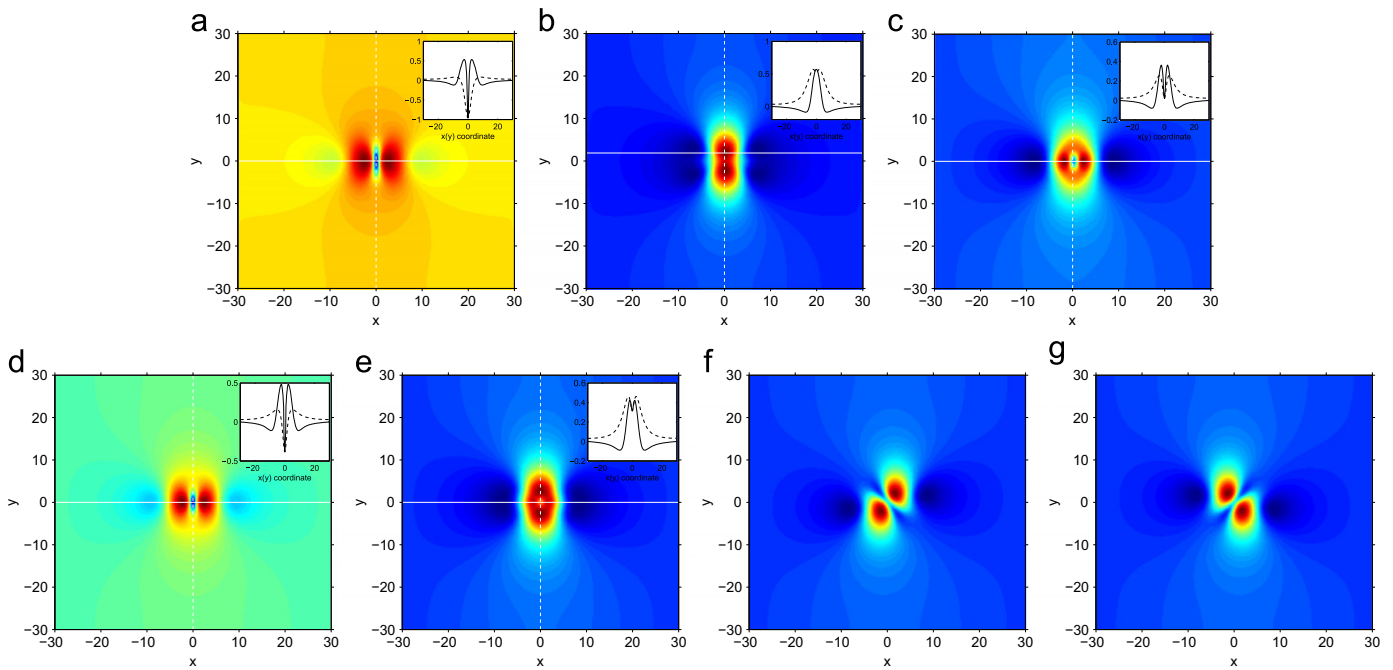


Fig. 1. The refractive indices induced by f_1 (a), f_2 (b), f_3 (or f_4) (c), f_5 (d), f_6 (e), f_7 (f), and f_8 (g), respectively. The insets in (a)–(e) show the refractive index distribution along the white solid and dashed lines. The parameters are fixed: $A = \sqrt{2.3}$ and $B = 2$.

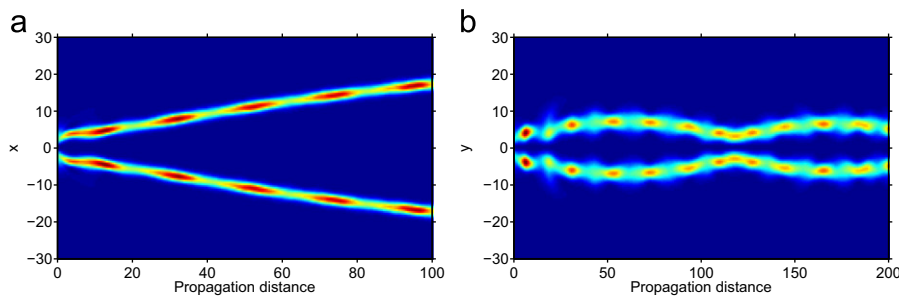


Fig. 2. (a) The transverse intensity profile of f_1 along the x -axis with the propagation distance. (b) The transverse intensity profile of f_2 along the y -axis with the propagation distance.

considering the superposed dipoles which will rotate during propagation, we present the corresponding 3D iso-surface plots to record their trajectories (as shown in Figs. 3–5).

Fig. 2(a) is the propagation of f_1 in the anisotropic nonlinear medium, in which the two humps repel each other and the gap between them increases linearly with the propagation distance. Fig. 2(b) depicts the propagation of f_2 ; the two humps first repel each other and then attract, and during propagation the process repeats to form a breath-like phenomenon. The explanation is that the incident is a dipole, so that the two humps are out of phase, and this will bring a repulsion between them. Even though the refractive index shown in Fig. 1(b) is positive, the attraction between the two humps is smaller than the repulsion at the very beginning. With the increasing of the gap between them during propagation, the repulsion becomes weaker and weaker, and the attraction will play a main role. In a word, it is the existence of attraction and repulsion simultaneously between the two humps that leads the breath-like behavior of f_2 . This is quite different from the case shown in Fig. 2(c) displayed in Ref. [5], where the incident is composed by two mutually incoherent beams, which just attract each other during propagation.

Composed by superposition of two orthogonal dipoles, f_3 and f_4 are vortices, and their propagation are shown in Fig. 3. They cannot maintain their radial symmetric shapes during propagation, because the refractive indices induced by them are asymmetric, and this will

lead to the split of them. The formation way of the vortices introduces phase gradient to them, so that the splitting induced dipoles exhibit rotation properties during propagation. And the rotations of the dipoles (f_5 and f_6) shown in Fig. 4 are also because of the same reason. In addition to the rotation, we can also see that the two humps in Figs. 3 and 4 always repel each other during propagation.

In comparison with Figs. 3–5 which displays the propagation of f_7 and f_8 also exhibits the rotation of the two humps. However, the reason is quite different—the rotation shown in Fig. 5 is driven by the focusing regions of the refractive indices induced by the two humps, the energy of which distributes along the diagonal directions (see the inputs shown in Fig. 5). According to Fig. 1(f) and (g), the focusing regions of the refractive indices do not parallel with the diagonal direction that they would pull the humps to rotate. Thus, the humps will rotate during propagation if the input has a phase gradient or the distribution of the energy does not parallel with the x - or y -axis.

4. Conclusion

The propagation of dipoles in anisotropic nonlinear media is investigated. There are both attraction and repulsion between the two humps, if the attraction is bigger they will attract each other, while if the attraction is smaller they repel. Phase gradient and

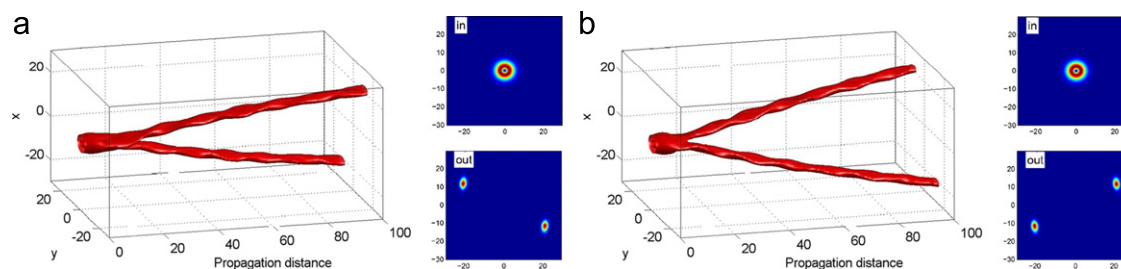


Fig. 3. The 3D iso-surface plots of the intensity of f_3 (a) and f_4 (b) with the propagation distance. The right two panels next to f_3 and f_4 are the intensity distributions corresponding to input (up) and output (down) beam, respectively.

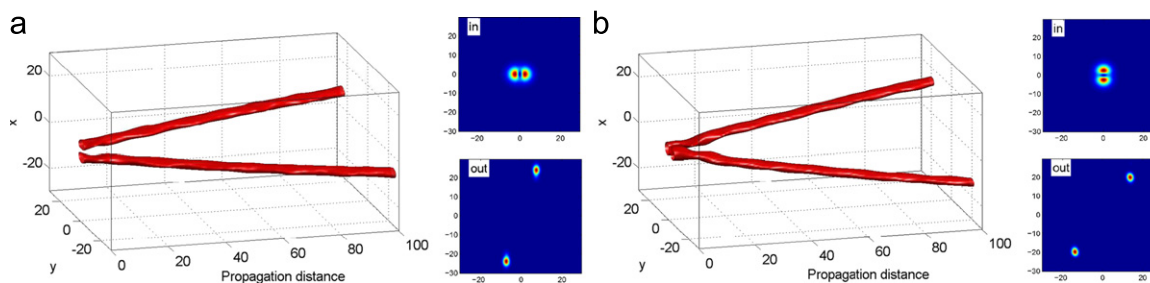


Fig. 4. The 3D iso-surface plots of the intensity of f_5 (a) and f_6 (b) with the propagation distance. The right two panels next to f_5 and f_6 are the intensity distributions corresponding to input (up) and output (down) beam, respectively.

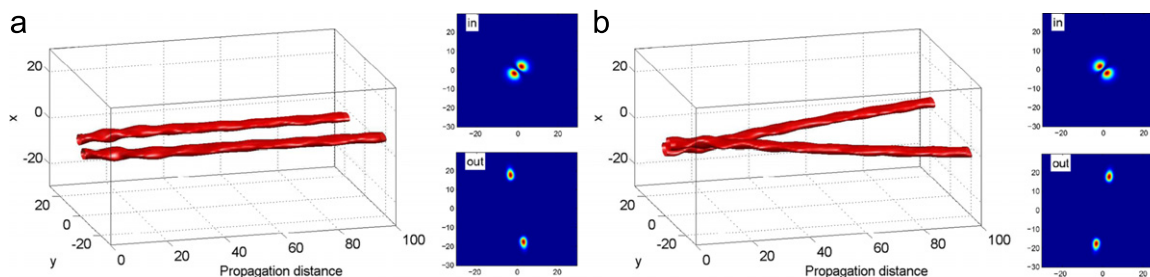


Fig. 5. The 3D iso-surface plots of the intensity of f_7 (a) and f_8 (b) with the propagation distance. The right two panels next to f_7 and f_8 are the intensity distributions corresponding to input (up) and output (down) beam, respectively.

energy distribution not parallel with the boundaries would drive the dipoles to rotate during propagation.

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