# CONTROLLABLE ACCELERATION AND DECELERATION OF AIRY BEAMS VIA INITIAL VELOCITY 

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#### Abstract

Acceleration and deceleration of one-dimensional and two-dimensional obliquely incident Airy beams are investigated theoretically and numerically. Onedimensional Airy beams with given initial velocities propagate along parabolic trajectories. However, the beams will undergo deceleration and then acceleration if the initial velocity is in the opposite direction to the acceleration. In the opposite case, the beams will only accelerate during propagation. Two-dimensional tilted Airy beams are treated as products of two 1D Airy beams with initial velocities. Such accelerating and decelerating properties due to initial velocities can be effectively controlled by small angles of incidence.


Key words: obliquely incident Airy beam, acceleration and deceleration.

## 1. INTRODUCTION

Self-accelerating nondiffracting optical beams have attracted much attention from research community all over the world [1-10]. Such beams can find potential applications in tweezing [11, 12], the generation of plasma channels [13, 14], material modifications [15], light bullet production [5, 16, 17], particle clearing [18], and manipulation of dielectric microparticles [19]. Special attention has been focused on the Bessel [20, 21] and Airy function beams [22]. Analyses have been mostly confined to linear media, for the reason of wanting to observe minimally diffracting beams in linear optics. In the paraxial approximation, the beam in the form of Airy function evolves according to the linear Schrödinger equation; it can propagate practically without diffraction and accelerate along a parabolic trajectory [23, 24]. However, in different media the acceleration is not necessarily along the parabolic curve. By introducing a linear index potential, the Airy beam can accelerate along
any predefined path [25]. Recently, three new types of accelerating beams have been reported, the accelerating trajectories of which also do not proceed along parabolic curves [26].

Even though the analysis of Airy beam propagation has become quite involved, almost all research in the last decade has been focused on the cases without initial velocity. Therefore, it is of interest to investigate what will happen when an Airy beam is launched into a medium with an initial velocity. The question is, how does the beam accelerate in this case? Having this question in mind, we present here a thorough investigation of the propagation of both one-dimensional (1D) and twodimensional (2D) incident Airy beams with nonzero initial velocities that includes both theoretical and numerical aspects of the problem. Nonzero initial velocity is most easily obtained by launching an Airy beam obliquely. Although the obliquely incident Airy beams exhibiting ballistic properties during propagation have been discussed previously [27, 28], we address the problem of Airy beams with initial velocities using classical Newton's dynamics.

The organization of this article is as follows: In Sec. 2, we introduce the theoretical model and exhibit numerical results. We discuss the 1D case first, and then the 2D case. In the end we briefly mention the finite energy Airy beams. With Sec. 3 , we conclude the article.

## 2. MODELING AND NUMERICAL SIMULATION

### 2.1. ONE-DIMENSIONAL CASE

In the paraxial approximation, the normalized equation for the evolution of a slowly-varying envelope $\psi$ of the beam's electric field has the form:

$$
\begin{equation*}
i \frac{\partial \psi}{\partial z}+\frac{1}{2} \frac{\partial^{2} \psi}{\partial x^{2}}=0 \tag{1}
\end{equation*}
$$

where $x$ and $z$ are the dimensionless transverse coordinate and the propagation distance, respectively, presented in units of $x_{0}$ and $k x_{0}^{2}$. Here, $x_{0}$ is some convenient transverse scale and $k x_{0}^{2}$ is the corresponding Rayleigh range. As usual, $k=2 \pi n / \lambda_{0}$ is the wavenumber, $n$ is the refractive index, and $\lambda_{0}$ the wavelength. Typical values of the parameters can be taken as $x_{0}=100 \mu \mathrm{~m}, n=1.45$, and $\lambda_{0}=600 \mathrm{~nm}$ [1, 7]. Obviously, Eq. (1) is just the Schrödinger equation without potential; as a linear 2nd order PDE, it possesses infinitely many solutions. One of the accelerating solutions of Eq. (1) is the well-known Airy function

$$
\begin{equation*}
\psi(x, z)=\operatorname{Ai}\left(x-z^{2} / 4\right) \exp \left[i\left(6 x z-z^{3}\right) / 12\right] . \tag{2}
\end{equation*}
$$

The trajectory of this accelerating beam can be directly obtained from Eq. (2), which is $x=z^{2} / 4$. In analogy to classical mechanics, the velocity of the beam during prop-
agation is $\mathbf{v}=z \hat{x} / 2$ and the acceleration $\mathbf{g}=\hat{x} / 2$, where $\hat{x}$ is the unit vector; the motion is akin to a projectile in the $(z, x)$ plane.

If one launches an Airy beam into the medium with an angle, the transverse velocity vector of the beams becomes relevant. The launched Airy beam can be written as

$$
\begin{equation*}
\psi_{\mathrm{ob}}(x, z=0)=\operatorname{Ai}(x) \exp \left(i v_{0} x\right) \tag{3}
\end{equation*}
$$

where $v_{0}$ is the magnitude of the velocity vector. Different from Eq. (2), the beam in Eq. (3) has an initial velocity $\mathbf{v}_{\text {in }}=v_{0} \hat{x}$. Therefore, the total velocity of the tilted beam during propagation should be written as

$$
\begin{equation*}
\mathbf{v}_{\text {total }}=\mathbf{v}+\mathbf{v}_{\text {in }}=\left(v_{0}+z / 2\right) \hat{x} \tag{4}
\end{equation*}
$$

Integrating Eq. (4) in the range $[0 z]$, one obtains

$$
\begin{equation*}
x=v_{0} z+z^{2} / 4 \tag{5}
\end{equation*}
$$

which is the trajectory of the tilted Airy beam. It is clear that Eq. (5) is still a parabolic curve, the symmetry axis of which is $z_{\mathrm{sym}}=-2 v_{0}$.


Fig. 1 - Acceleration of Airy beams with $v_{0}=0$ (a), $v_{0}=-3$ (b), and $v_{0}=3$ (c), respectively. The dashed curves, according to Eq. (5), represent the trajectories of the main lobe.

In Figs. 1(a)-1(c), we display three examples of accelerating Airy beams with $v_{0}=0, v_{0}=-3$, and $v_{0}=3$, respectively. Figure 1(a) is just the case without initial transverse velocity [1]. When the Airy beam is incident obliquely with a negative incidence angle ( $v_{0}<0$ ), the symmetry axis of the accelerating parabolic curve is positive, as shown in Fig. 1(b). In this case, the direction of the initial velocity is opposite to that of the acceleration, so the absolute value of acceleration decreases first, and then increases. Therefore, the Airy beam undergoes deceleration and then acceleration during propagation. If the incident angle is positive $\left(v_{0}>0\right)$, the initial velocity is in the same direction as the acceleration, so the beam will only accelerate during propagation. The dashed curves in Fig. 1 are the theoretical accelerating
trajectories obtained from Eq. (5), so we can see that the theory is in accordance with numerical simulations.

In order to analyze the accelerating process more clearly, we display the accelerating trajectories of the main lobes of oblique incident Airy beams in Fig. 2. Along the curved arrow, the values of $v_{0}$ are negative and increase for all the cases. The thick solid blue curve is for the vertical incidence; the cases $v_{0}<0$ are above the horizontal and the cases $v_{0}>0$ are bellow the horizontal axis. The shaded region in Fig. 2 represents the cases where the tilted beams propagate along the negative $z$ direction. The usual projectile trajectories are seen if one rotates the graph for $\pi / 2$. Combining the cases along positive and negative $z$ directions, one obtains the whole spectrum of parabolic trajectories of the main lobes. From Fig. 2 one can also see that the symmetry axis of the trajectory is positive for $v_{0}<0$ and negative for $v_{0}>0$.


Fig. 2 - Trajectories of the main lobes. Along the arrow, the values of $v_{0}$ are $-7,-5,-3,-1,0,1,3$, 5 , and 7 , respectively. The shaded region represents the propagation along the negative $z$ direction.

The initial velocity of the Airy beams is related to the incident angle $\theta$, so that the accelerating properties of tilted beams can be controlled by $\theta$, which can be written as

$$
\begin{equation*}
\theta=\arcsin \frac{v_{0}}{x_{0} k} . \tag{6}
\end{equation*}
$$

Based on the parameters mentioned above, one can calculate incident angles $\theta$ belonging to different $v_{0}$, as shown in Table 1 that corresponds to the cases exhibited in Fig. 2. From the values shown in Table 1, we find that the accelerating properties can be adjusted by small incident angles.

Table 1
Incident angle $\theta$ corresponding to $v_{0}$

| $v_{0}$ | $\pm 7$ | $\pm 5$ | $\pm 3$ | $\pm 1$ | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\theta$ | $\pm 0.2641^{\circ}$ | $\pm 0.1887^{\circ}$ | $\pm 0.1132^{\circ}$ | $\pm 0.0377^{\circ}$ | 0 |

### 2.2. TWO-DIMENSIONAL CASE

Applying the variable separation technique to the 2D linear Schrödinger equation leads to the oblique incident 2D Airy beam solution

$$
\begin{equation*}
\psi_{\mathrm{ob}}(x, y, z=0)=\operatorname{Ai}(x) \operatorname{Ai}(y) \exp \left[i\left(v_{x} x+v_{y} y\right)\right] \tag{7}
\end{equation*}
$$

where $v_{x}$ and $v_{y}$ are the components of the transverse velocity vector along $x$ and $y$ directions. The trajectory of the accelerating 2D tilted Airy beam can be determined by writing the $x$ and $y$ components parametrically in terms of the $z$ coordinate, which turned out to be

$$
\begin{equation*}
x=v_{x} z+z^{2} / 4, y=v_{y} z+z^{2} / 4, z=z \tag{8}
\end{equation*}
$$

in Cartesian coordinates, or

$$
\begin{equation*}
r=\frac{z}{2 \sqrt{2}} \sqrt{\left[z+2\left(v_{x}+v_{y}\right)\right]^{2}+4\left(v_{x}-v_{y}\right)^{2}} \tag{9}
\end{equation*}
$$

in the cylindrical coordinates. Thus, Eq. (9) does not describe a parabolic trajectory, although the $x$ and $y$ beam components are accelerated along parabolic curves, according to Eq. (8). However, for a specific choice $v_{x}=v_{y}=v_{0}$, Eq. (9) can be rewritten as

$$
\begin{equation*}
r=\frac{\sqrt{2}}{4}\left(4 v_{0} z+z^{2}\right) \tag{10}
\end{equation*}
$$

which is a parabolic function.
We display accelerating beams with different $v_{x}$ and $v_{y}$ (the values are provided in the caption) in Fig. 3. These beams are solutions to the 2D linear Schrödinger equation

$$
\begin{equation*}
i \partial \psi / \partial z+\left(\partial^{2} / \partial x^{2}+\partial^{2} / \partial y^{2}\right) \psi / 2=0 \tag{11}
\end{equation*}
$$

The bottom three panels in Fig. 3 present the projections of the trajectories onto $x 0 z$ plane, $y 0 z$ plane, and $x 0 y$ plane, respectively. Note that Fig. 3(e) represents the vertical incidence case. Different from the straight beam in Fig. 3(e), the accelerating trajectories of oblique incident 2D Airy beams greatly change. For the case in Fig. 3(a), both $x$ and $y$ components undergo deceleration first and then acceleration during propagation, since $v_{x}=v_{y}=-3<0$. On the other hand, for the cases in Figs. 3(b) and 3(c) [resp. Figs. 3(d) and 3(g)] only the $y$ (resp. $x$ ) component undergoes deceleration first and then acceleration, while the other component just accelerates. In one word, the concrete accelerating properties are determined by the signs of $v_{x}$ and $v_{y}$ that reflect the obliqueness of the incident 2D Airy beam. The dashed curves in Fig. 3 are the theoretical accelerating trajectories of the main lobes obtained from Eq. (8). Clearly, the theoretically predicted trajectories agree with the numerics very well.

Similarly to the 1D case, one can also obtain the incident angle of a tilted 2D Airy beam as

$$
\begin{equation*}
\theta=\arcsin \frac{\sqrt{v_{x}^{2}+v_{y}^{2}}}{x_{0} k}, \tag{1}
\end{equation*}
$$

in which the signs of $v_{x}$ and $v_{y}$ determine the quadrant of the transverse projection of the total wavevector located in the transverse $x 0 y$ plane. For parameters in Fig. 3, the maximum $\theta$ is approximately $0.3735^{\circ}$. Therefore, the accelerating properties of 2D oblique Airy beams with given initial velocities can also be controlled by small incident angles.


Fig. 3 - Accelerating obliquely incident Airy beams. (a)-(i) Values of $\left(v_{x}, v_{y}\right)$ are $(-3,-3),(0,-3)$, $(3,-3),(-3,0),(0,0),(3,0),(-3,3),(0,3)$, and $(3,3)$, respectively. Dashed curves are the trajectories of accelerating main lobes, according to Eq. (8). Bottom three panels (from left to right) are the projections of accelerating beams onto $x 0 z, y 0 z$, and $x 0 y$ planes, respectively.

### 2.3. FINITE ENERGY AIRY BEAMS

It is well known that Airy beams possess infinite energy, and as such are not very realistic physical quantities. The finite energy Airy beams are produced by an
appropriate apodization, for example, by introducing transverse exponentially decaying factors $\exp (a x)$ for the 1D case and $\exp (a x+a y)$ for the 2D case, with $a \geq$ being the decay constant [1]. For the parameters used in this paper, the Rayleigh length is about 15.18 cm , and the apodized Airy beam with $a=0.1$ can propagate invariantly for about 6 Rayleigh lengths [1]. Taking the case $v_{0}=-5$ as an example, the symmetry axis is $z_{\text {sym }}=10$ which is bigger than 6 , so the corresponding finite energy beam will mainly decelerate during propagation.


Fig. 4 - Propagation of oblique 1D Airy beams with finite energy. (a)-(c) Corresponding to Figs. 1(a)-1(c), respectively. Inset shows the inputs and outputs for the three cases.

In Fig. 4 we display the propagation of oblique incident 1D Airy beams with finite energy. Figures 4(a)-4(c) correspond to Figs. 1(a)-1(c), respectively. The inset displays the intensity of the corresponding input and output beams. It is clear that the cases displayed in Figs. 4(a) and 4(c) accelerate, whereas the case displayed in Fig. 4(b) decelerates during propagation. One can also predict that the cases in between Figs. 4(a) and 4(b) will at first decelerate and then accelerate.

Finally, we should mention that one can use the same procedure to treat the propagation of nonlinear Airy beams, that is, the ones propagating according to the nonlinear Schrödinger equation. In this case, a similar kind of controllability can be achieved for the oblique incidence of beams.

## 3. CONCLUSION

In summary, we have theoretically and numerically investigated the propagation of both 1D and 2D Airy beams with oblique incidence. We find that the Airy beams with initial velocities still propagate along parabolic trajectories, but may undergo deceleration before acceleration. The initial velocity can be determined by the incident angles and directions. We also find that small incident angles can adjust the accelerating process effectively. Our research may broaden the potential applications of Airy beams.

Since Airy beams can be used to produce light bullets [5, 16, 17], which is quite popular research these days [29-32], the research reported here may help others generate accelerating as well as decelerating light bullets.

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