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Optical Bloch Oscillations of a Dual Airy Beam

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We investigate the propagation of a dual Airy beam in Hermitian and non-Hermitian waveguides, theoretically and numerically. Optical Bloch oscillations (OBOs) of the beam are demonstrated during propagation in both types of waveguides, and the numerical OBO period is found to be in accordance with the theoretical predictions. The two branches of the dual Airy beam do not display translational symmetry — the peaks will form in one branch only, due to the desynchronized Bragg reflection of the lobes. In the non-Hermitian waveguides, the dual Airy beam will be damped or amplified during propagation — depending on the imaginary part of the complex potential, which may provide loss or gain to the beam. In the $\mathcal{PT}$-symmetric-like potential, the dual Airy beam may undergo amplification during propagation, but the total power will exhibit a stair-like behavior. The non-reciprocity is also exhibited by the dual Airy beam in such a potential. We believe that our research not only provides a new geometry for optical switches but also deepens the understanding of OBO in dual Airy beams.

1. Introduction

In quantum mechanics, electrons in a periodic lattice perform Bloch oscillations if a dc electric field is applied to the system. Bloch oscillations were predicted in 1929[1] and experimentally observed in a semiconductor superlattice[2] some 60 years later. Due to the formal equivalence between the paraxial wave equation in photonics and the Schrödinger equation in quantum mechanics, Bloch oscillations were also reported in optics, where they are refered to as the optical Bloch oscillations (OBOs). Bloch oscillations as well as OBOs are reported in but not limited to cold atoms,[3–7] optical waveguides,[8–11] photonic lattices,[12–17] integrated photonic circuits,[18,19] and non-Hermitian systems.[20,21] It is worth noting that the Bloch oscillations are different from the Rabi oscillations,[22–27] where an ac field is introduced to resonantly induce transitions between Bloch states of different bands. As an example of quantum-optical analogies,[28] OBO maps the temporal evolution of wavefunctions in coherent quantum phenomena onto the spatial propagation of optical fields in photonic devices, and displays applicative potential for producing beam combiners, splitters, and interferometers. For a thorough survey of research progress in OBO, we suggest review papers.[28–31]

On the other hand, Airy beams[32–38] are quite popular in optics community, due to their self-accelerating, nondiffracting, and self-healing properties.[39–41] It is interesting to note that linear potentials produced by the changes in the index of refraction can control well the behavior of Airy beams during propagation[42–45] and thereby enrich their applicative potential. Even more interestingly, if an Airy beam is launched into a combined periodic and linear potential, the resulting system may exhibit truly exotic properties, and one of the most charming examples is the OBO of Airy beams.[46–48]

In addition, the propagation dynamics of Airy beams in non-Hermitian systems is still inadequately understood and explored. Although non-Hermitian systems are widely discussed in a myriad of models with $\mathcal{PT}$-symmetric potentials, work remains to be done in adequately addressing the extension of non-Hermitian quantum mechanics to the world of nondiffracting accelerating optical beams. To this end, we present here research on OBO of Airy beams in non-Hermitian systems. In order to study Airy beams with opposite accelerating directions as a convenient optical system, we utilize the dual Airy beams, which display several distinguishing features, such as symmetric transverse intensity patterns and improved self-regeneration property.[49] The two branches of the dual Airy beam exhibit opposite acceleration directions in propagation, thereby breaking this symmetry and producing an unexpected dynamical behavior. In addition, by changing the distance between the branches, one can study the interaction between the two Airy beams, caused by the non-Hermitian medium.

Thus, in this paper we investigate the OBO of dual Airy beams in non-Hermitian waveguides, in one transverse dimension. For comparison, we first discuss the OBO of a dual Airy beam in

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a Hermitian waveguide, and then extend the analysis to non-Hermitian waveguides. Our analysis focuses on the three cases: a lossy waveguide, a gainy waveguide, and a $\mathcal{PT}$-symmetric-like waveguide. The goal is to enhance understanding of dual Airy beam propagation in various waveguides and to explore the applicative potential of such propagation. We believe that our research will not only enrich the OBO family of phenomena, but also broaden the potential for applications of dual Airy beams, with specific applications to optical switches.

The organization of the paper is as follows. In Sec. 2 we briefly outline the theory on which our investigation is based; in Sec. 3 the numerical simulation of dual Airy beams in both Hermitian and non-Hermitian systems is carried out; and in Sec. 4 the paper is concluded.

2. Basic Theory

The governing Schrödinger-like equation for the scalar optical field is of the form:

$$i\frac{\partial\psi(X,Z)}{\partial Z}=-\frac{1}{2k}\frac{\partial^2\psi(X,Z)}{\partial X^2}+V(X)\psi(X,Z)+\alpha X\psi(X,Z), \tag{1}$$

in which $k=2\pi/\lambda$, with λ being the wavelength, $V(X)$ is the external potential (proportional to the refractive index change in the system), and α determines the strength of the transverse linear potential, modeling an external dc force. If the characteristic beam width of the input beam is x_0, then by using the variable substitutions $X=x_0x$ and $Z=kx_0^2z$, Eq. (1) can be recast into a dimensionless form:

$$i\frac{\partial\psi(x,z)}{\partial z}=-\frac{1}{2}\frac{\partial^2\psi(x,z)}{\partial x^2}+[V(x)+\alpha x]\psi(x,z), \tag{2}$$

where the coefficients kx_0^2 and kx_0^3 are absorbed into V and α. In a real physical system, we may choose $\lambda=600$ nm and $x_0=10\,\mu$m, which leads to $kx_0^2\approx 1$ cm. To observe the desired OBO phenomena, one has to specify the potential V.

We assume that the potential can be written as a series of equidistant transversely displaced Gaussians,

$$V(x)=V_0\sum_n\left\{1+i\gamma\left[\beta+(-1)^n\right]\right\}\exp\left(-\frac{(x-nd)^2}{\omega^2}\right), \tag{3}$$

such that each exponential function defines a waveguide channel. Here, V_0 is the amplitude of the potential, n goes over integers, γ accounts for the loss or gain in each waveguide, β is a control parameter that can be 0 or 1, d is the interval between two adjacent waveguides, and ω is responsible for the width of each waveguide. From Eq. (3), one can clearly see that:

(I) The system is Hermitian if $\gamma=0$.
(II) The system is gainy if $\gamma>0$ and $\beta=1$, while it is lossy if $\gamma<0$ and $\beta=1$.
(III) The potential meets the condition $V(x)=V^*(-x)$ if $\beta=0$, which is a requirement for the $\mathcal{PT}$ symmetry.

Without much loss of generality, we set $V_0=10$, $\alpha=0.05$, $d=\pi/2$ and $\omega=0.5$ throughout the paper. Since the potential in Eq. (3) is periodic and there is a transverse potential gradient in Eq. (2), light beams which are launched into the medium that is described by Eq. (3) will exhibit OBO behavior during propagation, and the corresponding oscillation period is

$$\mathcal{D}=\frac{2\pi}{\alpha d}. \tag{4}$$

This number happens to be 80, for the parameter values mentioned above.

Here, we are interested in the propagation of a dual Airy beam in the assumed potential, so we set as the input beam[49,50]

$$\psi(x,0)=10\mathrm{Ai}\left[\pm 0.4\left(|x|+x_{\mathrm{gap}}\right)\right]\exp\left[\pm a\left(|x|+x_{\mathrm{gap}}\right)\right], \tag{5}$$

where a is the decay constant and x_{gap} is the transverse displacement of each branch of this dual beam. We set throughout $a=0.1$, but vary x_{gap}. It is worth mentioning that the dual Airy beam is a solution of the fractional Schrödinger equation.[51] In **Figure 1**, we display intensity distributions of the dual Airy beam with different parameters. It is similar to the intensity of the circular Airy beam[45,52,53] in the radial cross section, the difference being that the circular Airy beam is two-dimensional (2D), whereas the dual Airy is 1D. In the following, we discuss propagation dynamics of dual Airy beams in different variants of the system. We only consider the middle case of the first row in Figure 1 as the input beam; other cases can be analyzed by the same method, and the results do not show significant differences.

3. Numerical Simulations

3.1. Hermitian System

To start with, we consider the Hermitian system. As aforementioned, the system is Hermitian when $\gamma=0$. The potential with a transverse gradient is shown in **Figure 2**a, in which the real and imaginary parts are indicated by the blue and orange curves, respectively. Evidently, the imaginary part is zero. The propagation of the dual Airy beam across 4 periods is shown in Figure 2b, in which the OBO of the beam is clearly visible. One finds that the OBOs of the two branches are not the same — there are peaks in the left branch during propagation which are absent from the right branch. The reason for the formation of the peaks is that the secondary lobes of the left branch reach the Bragg reflection point earlier than the main lobe, so the reflected secondary lobes interfere with the main lobe, making the power distribution of the left branch relatively focused. The right branch does not display such a phenomenon because all the lobes reach the Bragg reflection point at the same time. If the sign of α is changed, the peaks will appear in the right branch, because the beam will deflect along the positive x direction first. During propagation, we also record the maximum intensity of the dual Airy beam and the corresponding total power $P=\int|\psi(x,z)|^2dx$; the results are displayed in Figure 2c. One finds that the maximum intensity roughly exhibits a periodic behavior, with the two sets of

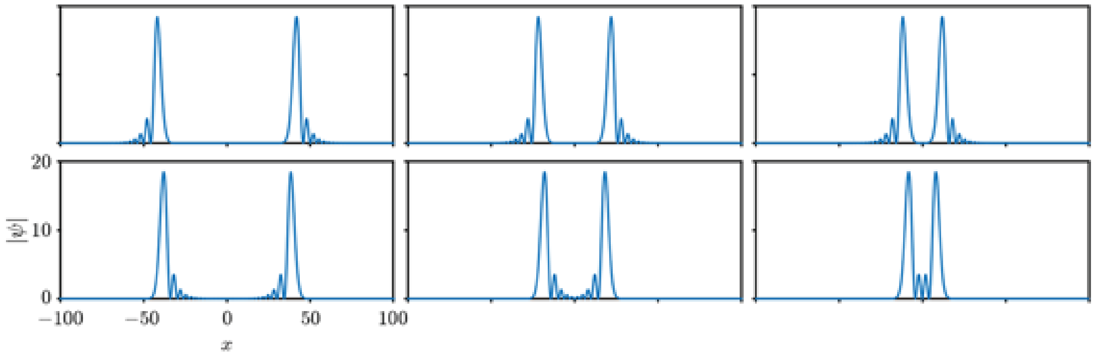

Figure 1. Intensity of the dual Airy beam. Upper row: With the negative sign in Eq. (3); Lower row: With the positive sign. Columns: $x_{gap} = -40$, -20, and -10, from left to right.

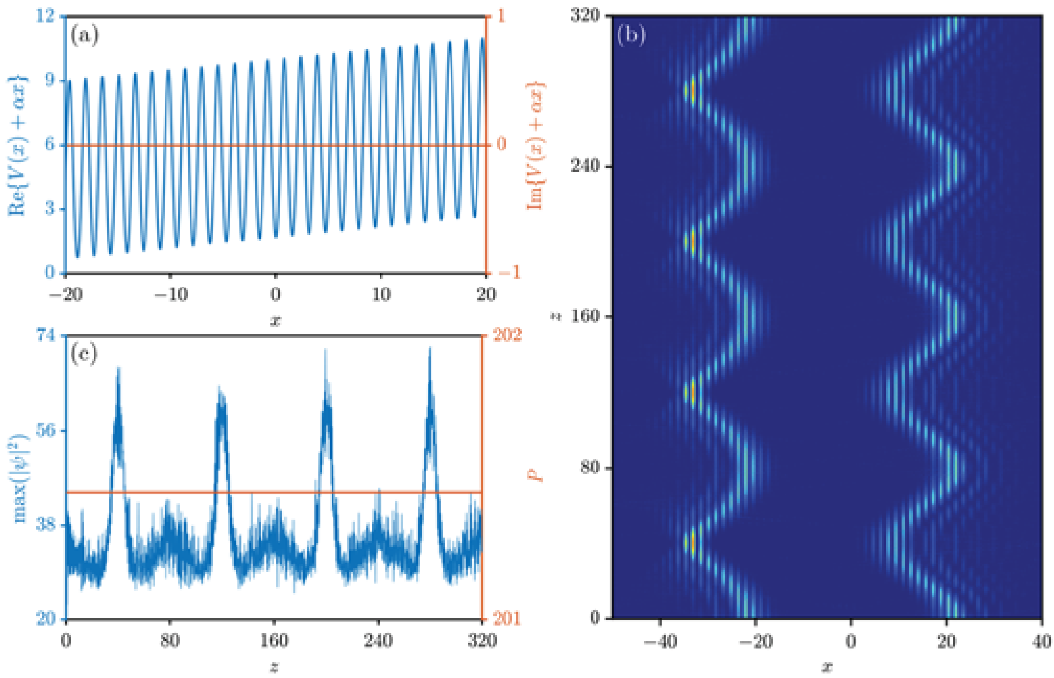

Figure 2. a) Profile of the potential with $\gamma = 0$. b) OBO of the dual Airy beam with $x_{gap} = -20$. c) Maximum intensity and the total power of the beam during propagation. In a) and c), the blue and orange curves refer to the left and right y axes.

maxima. The higher maxima originate from the pronounced peaks in the left branch. The total power remains constant.

In comparison with the result for a single Gaussian input, one finds that the OBO is feasible in the dual Airy beam without change in the OBO period $\mathcal{D}$. However, the accelerating property to one side (as well as the self-healing property) disappears if no further treatment is imposed on the transverse periodic potential with a gradient. This means that the OBO effect is strong for the chosen set of parameters, so that it disrupts the usual internal accelerating properties of the Airy beam. As described in [46], one has to weaken the OBO phenomenon by adjusting the parameters (e.g. α, d, V_0, etc.) to recover the usual properties of the Airy beam. As each coin has two sides, OBO helps to completely recover the Airy beam at $z = m\mathcal{D}$, in which m is a positive integer. In addition, the peaks accompanied with the OBO of dual Airy beams provide the possibility for making an optical switch, which is not observed for the Gaussian input.

3.2. Non-Hermitian System

If $\gamma \neq 0$, the system becomes non-Hermitian. We consider the gainy system first, then the lossy system, and at the end the special $\mathcal{PT}$-symmetric-like system. The potential profile is shown in **Figure 3**a, in which the imaginary part allows for the gain to appear in every second channel. The propagation dynamics is shown in Figure 3b. One finds that the dual Airy beam still exhibits OBO during propagation, and the OBO period is the same as in the Hermitian system. A different aspect is that the beam

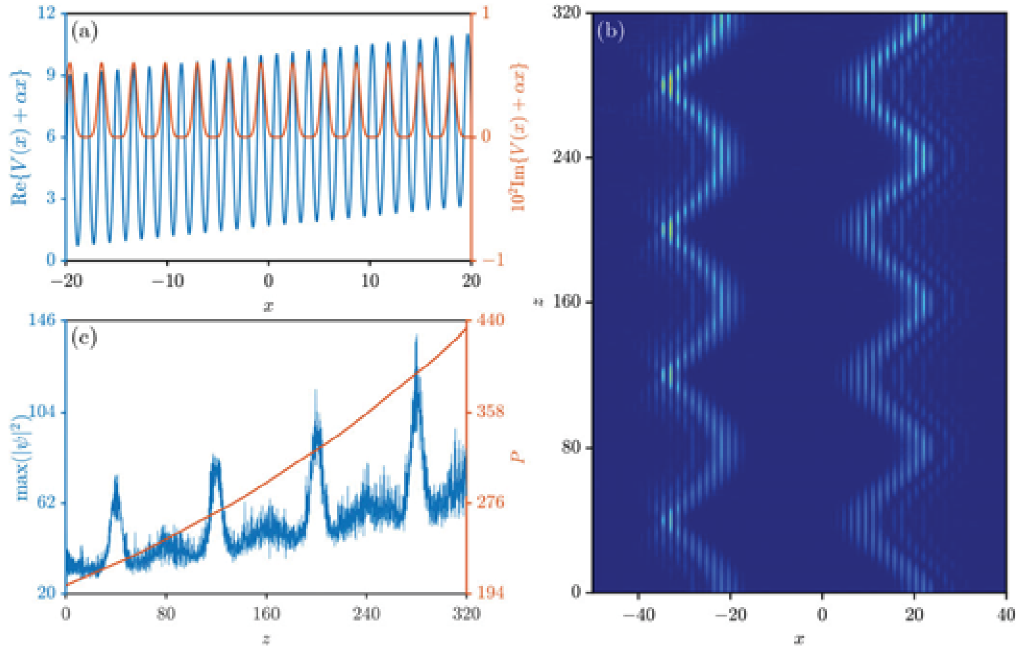

Figure 3. Figure setup is as in Figure 2, but with $\gamma = 3 \times 10^{-4}$ and $\beta = 1$, which corresponds to a gainy medium.

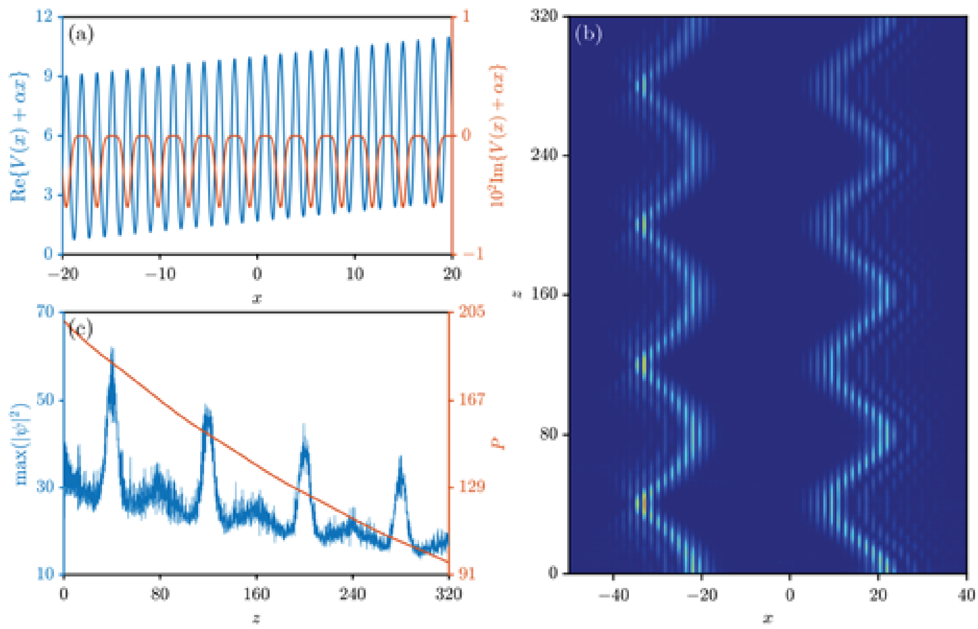

Figure 4. Figure setup is as in Figure 2, but with $\gamma = -3 \times 10^{-4}$ and $\beta = 1$, which corresponds to a lossy medium.

intensity increases with the propagation distance, which reflects the gainy nature of the system. In Figure 3c, the maximum intensity and the total power are shown. Clearly, both quantities increase with the propagation distance.

The propagation of the dual Airy beam in the lossy system is shown in **Figure 4**. After comparison, one finds that the phenomena in Figures 3 and 4 are exactly the opposite. Hence, there is no need to discuss this case in more detail.

In the third case of the non-Hermitian system, we take $\gamma \neq 0$ and $\beta = 0$, so that if there is gain in one channel, there will be loss in its neighboring channel, as shown in **Figure 5**a. Such a non-Hermitian system is called the binary lattice,[20,21] and the threshold value for the $\mathcal{PT}$ symmetry breaking is $\gamma_{\text{th}} = 0$. This is also the reason why we call such a non-Hermitian system the $\mathcal{PT}$-symmetric-like system — the eigenvalues are completely complex for all the values of $\gamma \neq 0$. In other words, the beam propagation will always be damped or amplified during propagation. However, in [20], it is demonstrated that the transverse gradient may lead to a discrete real band structure, which results in neither the damped nor amplified OBO. But this is not an automatic

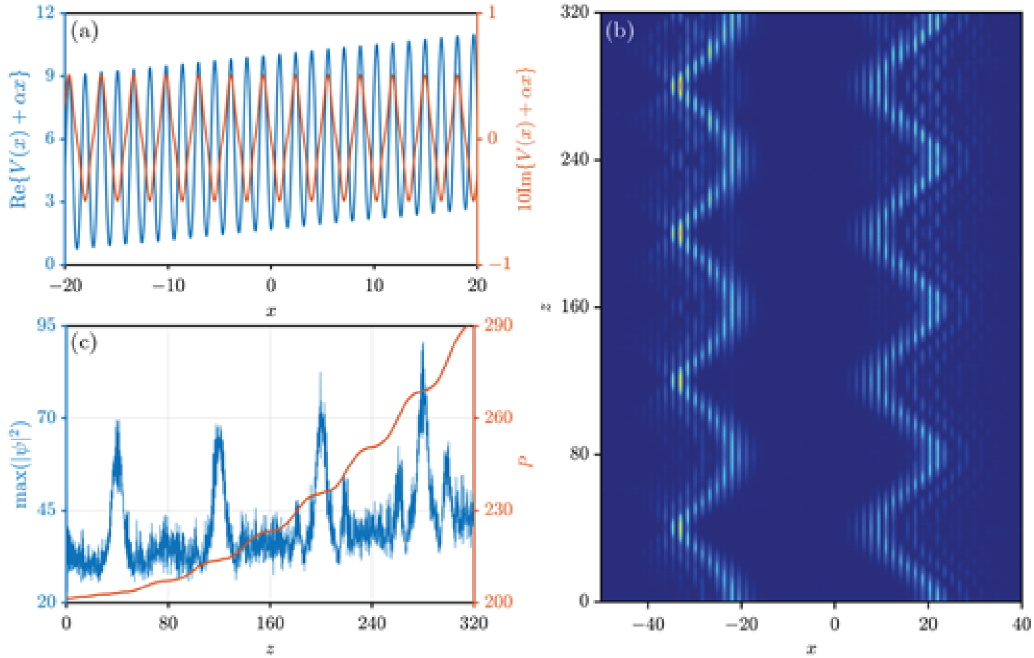

Figure 5. Figure setup is as Figure 2, but with $\gamma = 5 \times 10^{-3}$ and $\beta = 0$. This is the system with a special $\mathcal{PT}$ symmetry.

outcome, one has to make sure that the parameters are properly chosen. In this paper, we do not consider such a situation. Instead, we choose $\alpha = 0.05$, $\gamma = 5 \times 10^{-3}$ and $\beta = 0$, and display the propagation of the dual Airy beam in Figure 5b. As expected, the beam undergoes amplification during propagation. We also display the maximum intensity and the total power of the beam in Figure 5c. An interesting phenomenon is that the total power does not increase exponentially — it displays a step-wise increase with the propagation distance. The reason is that the Wannier-Stark states are non-orthogonal in the complex potential.[20] More numerical simulations show that the results are not simply mirrored for $\alpha = -0.05$, and this indicates the non-reciprocal nature of the system.

As is well known, when the system meets the criteria for $\mathcal{PT}$ symmetry, there will be no gain or loss during propagation. However, the existence of the transverse linear potential breaks the $\mathcal{PT}$ symmetry, even though the criteria are satisfied. As a consequence, amplification or damping always accompanies Bloch oscillations during propagation.

4. Conclusion and Outlook

In summary, we have investigated the OBO of the dual Airy beam in both Hermitian and non-Hermitian systems. We find that the two branches of the dual Airy beam do not exhibit the same behavior during propagation. Due to the desynchronous Bragg reflection of the lobes in one branch, the local intensity will focus and form peaks. In non-Hermitian systems, the phenomena are preserved, except for the appearance of damping or amplification of the beam. In addition, in the $\mathcal{PT}$-symmetric-like system, the total power of the dual Airy beam exhibits a stair-like behavior during propagation. By flipping the transverse gradient of the waveguides, the non-reciprocity of the system can be demonstrated.

Our research possesses applicative potential for optical switches and provides a new method for manipulating Airy beams. In comparison with the OBO of Gaussian beams, the advantage of research performed here lies in that the dual Airy beam may involve the properties of Airy beams which display broad potential for applications in different areas of photonics.[39,41]

As demonstrated previously, if a sufficiently strong transverse force that acts on the beam is introduced, Zener tunneling will happen. This is still true for the dual Airy beam in both Hermitian and non-Hermitian systems. Typical parameters for observing the optical Zener tunneling for our case are $d = \pi/3$ and $\alpha = 0.5$ (results are not shown). Note that an increment of α indicates a higher index gradient and a decrement of d corresponds to a larger OBO period. Considering the equivalence between curved waveguide arrays and straight waveguide arrays with a transverse gradient, the OBO and Zener tunneling of dual Airy beams can also be observed in curved waveguide arrays that can be prepared by the femtosecond laser writing technique[21] or the two beam interference method.[7]

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

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