Micro-scale two-phase flow dynamics

Lecture 2 Dimensionless numbers

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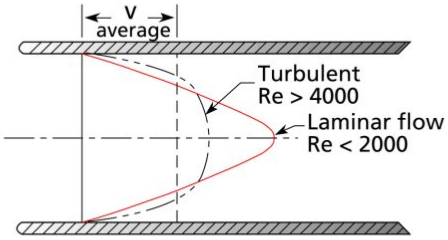
The main goal of physics is to describe a maximum of phenomena with a minimum of variables.

《Dimensionless Physical Quantities in Science and Engineering》
Josef Kuneš
Department of Physics
University of West Bohemia
Czech Republic

Dimensionless number

- Each of the similarity criteria can be expressed in the form of a mutual relation **between,** for example, **two forces, momentums or energies acting in a process**. Therefore, by observing the size of the criterion, an idea can be obtained from the character of the investigated process.

$$Re = \frac{uL}{\nu}$$



http://www.pumpfundamentals.com/pump_glossary.htm

Dimensionless number

- Sometimes, however, it seems as though the transition from dimensional physical quantities to dimensionless ones would obscure the view of the investigated process. In fact, the contrary is true because the reduced number of variable quantities expressed in the dimensionless way enables one to understand the mutual physical contexts in the investigated process more deeply.

- 有时候，无量纲化仿佛会使研究变得晦涩。但实际上，无量纲化会减少变量个数，使我们更加深入地理解物理现象的本质。

Dimensionless number

- The most important group consists of the ***physical similarity criteria*** obtained by some of the similarity theory methods.
- The ***dimensionless physical constants***
- the ***approximate ratio quantities*** can also be included among the dimensionless quantities. They usually come from experimental results and from the experimenter's intuition, i.e. without using any of the similarity theory methods. Usually, the extent of the validity of these quantities is limited only to a certain area.

Dimensionless number in multiphase flow

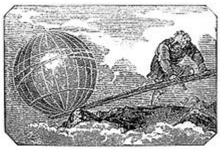
- The criteria concern such things as the solid particle movement in fluid, the dynamics of increasing transfer and collapse of bubbles in fluids, flow accompanied with boiling or condensation and granulation flow. The ***Bond, capillary, Eötvös, Morton, Stokes and Weber*** numbers are among the wide range of similarity criteria applied.

Archimedes number (Ar)

- motion of fluids due to density differences

$$Ar = \frac{gL^3\rho_l(\rho - \rho_l)}{\mu^2}$$

- g = gravitational acceleration (9.81 m/s²),
- ρ_l = density of the fluid, kg/m³
- ρ = density of the body, kg/m³
- μ = dynamic viscosity, kg/ms
- L = characteristic length of body, m



Archimedes is said to have remarked of the lever: *Give me a place to stand on, and I will move the Earth.*

- Archimedes Thoughtful by Fetti (1620)
- 287 BC - 212 BC

Archimedes number (Ar)

- When analyzing potentially mixed convection of a liquid, the Archimedes number parametrizes the **relative strength of free and forced convection.**
- This represents **the ratio of buoyancy and inertial forces**, which stands in for the contribution of natural convection. When Ar >> 1 natural convection dominates, i.e. less dense bodies rise and denser bodies sink, and when Ar << 1 forced convection dominates.

- When the density difference is due to heat transfer

$$\frac{\rho - \rho_l}{\rho_l} = \beta(T - T_l) \qquad Ar = \frac{gL^3\rho_l(\rho - \rho_l)}{\mu^2} = \frac{g\beta(T - T_l)L^3}{\nu^2} = Gr$$

Archimedes number (Ar)

- Entrainment condition

$$Ar = 0.97$$

- Miminum fluidization situation

$$Ar = 88.5$$

- Laminar and turbulent effects are equal

$$Ar = 176,900$$

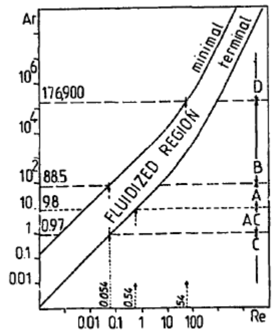


Fig. 2. General classification diagram for fluidized particles.

Powder Technology, 1998, 98(1):48–53

- A: Turbulent effect is negligible at minimum fluidization
- B: Turbulent effects start to influence laminar phenomenon at onset point of fluidization
- D: Turbulent effects predominate from the onset to entrainment

Atwood number (A)

- onset of instabilities in fluid mixtures due to density differences
- a **buoyancy** renormalization to **gravity**

$$A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

- Atwood number is an important parameter in the study of Rayleigh–Taylor instability and Richtmyer–Meshkov instability.
- In Rayleigh–Taylor instability, the penetration distance of heavy fluid bubbles into the light fluid is a function of acceleration time scale

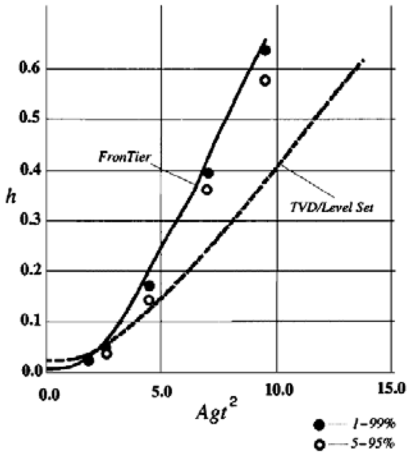
$$Agt^2$$

water tinted with food coloring flows down through ordinary tap water

http://physicscentral.com/explore/pictures/cup.cfm

Atwood number (A)

sun.stanford.edu/~keiji



Journal of Computational Physics
169, 652–677 (2001)

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Bingham number (Bn)

- Ratio of yield stress to viscous stress

$$Bn = \frac{\tau_y L}{\mu_\infty u_\infty}$$

- τ_y: yield stress, Pa
- L: length scale, m
- μ: dynamic viscosity of plastic, Pa·s
- u: plastic flow rate , m/s

- Eugene Cook Bingham
- 1878 – 1948
- American Chemist

Society of Rheology - Bingham Medal

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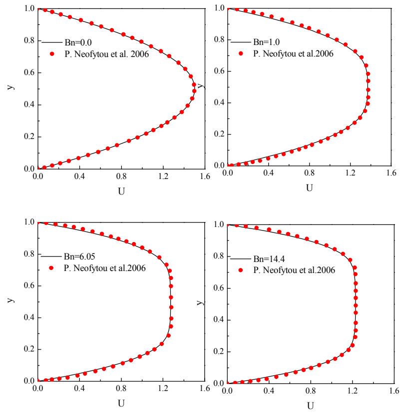
Bingham number (Bn)

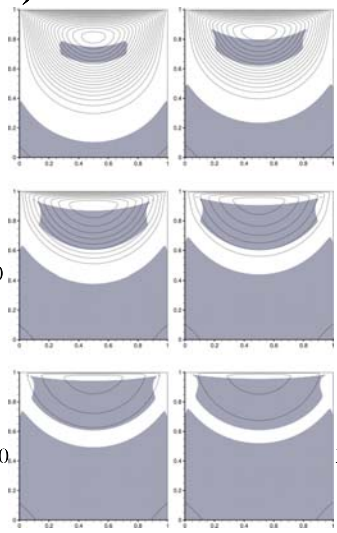


Guoling Zhou, Bin CHEN

Bn=2 Bn=5

Bn=20 Bn=50

Bn=200 Bn=500



Journal of Non-Newtonian Fluid Mechanics 195 (2013) 19–31

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Biot number (Bi)

- surface vs. volume conductivity of solids
- the ratio of the heat transfer resistances inside of and at the surface of a body

$$B_i = \frac{hL}{\kappa}$$

- h: heat transfer coefficient, W/(m^{2}K)
- L: characteristic length, m
- κ: Thermal conductivity, W/(mK)

- Jean-Baptiste Biot
- 1774-1862
- Known for Biot-Savart law

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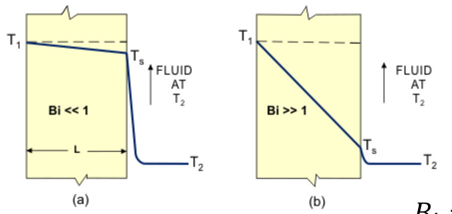
Biot number (Bi)

- Having a Biot number **smaller than 0.1** labels a substance as **thermally thin**, and temperature can be assumed to be constant throughout the materials volume (error < 5%). The opposite is also true: A Biot number **greater than 0.1** (a "**thermally thick**" substance) indicates that one cannot make this assumption, and more complicated heat transfer equations for "transient heat conduction" will be required to describe the time-varying and non-spatially-uniform temperature field within the material body.



Comparison of fully developed temperature profiles in two plates cooled by the same fluid

http://www.thermopedia.com/content/585

$$B_i = \frac{hL}{\kappa}$$

Bond number (Bo)

- The Bond number is a measure of **surface tension** forces compared to **body** forces.

$$B_o = \frac{\rho a L^2}{\sigma}$$

- ρ: density, kg/m^3
- a: acceleration, always gravity, m/s^2
- *L*: length scale, e.g. radius of a drop, m
- σ: surface tension, N/m

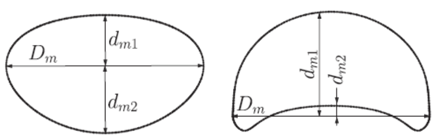
- Wilfrid Noel Bond
- 1897–1937
- English physicist

Bond number (Bo)

$$B_o = \frac{\rho a L^2}{\sigma}$$

- A high Bond number indicates that the system is relatively unaffected by surface tension effects; a low number (typically less than one is the requirement) indicates that surface tension dominates.
- It is also known in a slightly different form as the Eötvös number. The term Eötvös number is more frequently used in Europe, while Bond number is commonly used in other parts of the world.
- This number usually characterizes the shape of liquid bubbles and drops.

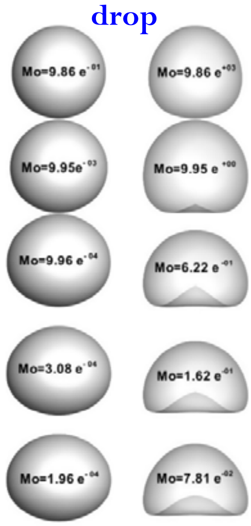
Bond number (Bo)



- convex shape bubble (Ga = 200 and Bo = 0.82)
- concave interface at the bottom (Ga = 20 and Bo = 100)

I. J. Multiphase Flow 51 (2013) 11–21

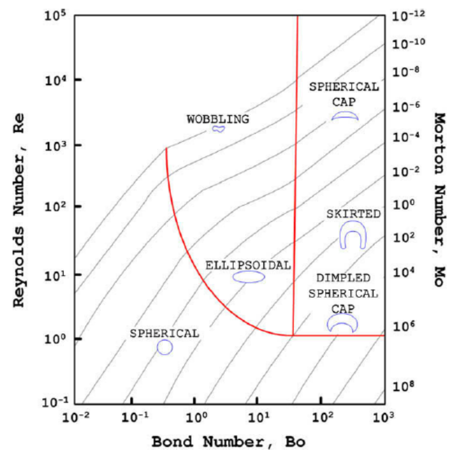
drop



Bo = 1 Bo= 10

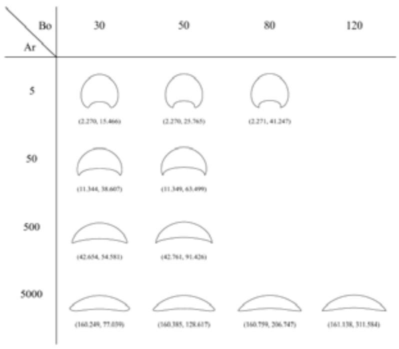
Computers & Fluids 88 (2013) 543–556

Bond number (Bo)



Clift, et al.: Bubbles, drops, and particles, 1978

Computers & Fluids 39 (2010) 1191–1207



J. Non-Newtonian Fluid Mechanics 200 (2013) 34–51

Capillary number (Ca)

- fluid flow influenced by surface tension
- represents the relative effect of **viscous forces** versus **surface tension** acting across an interface between a liquid and a gas, or between two immiscible liquids

$$Ca = \frac{\mu U}{\sigma}$$

- μ: the viscosity of the liquid, Pa·s
- U: characteristic velocity, m/s
- σ: surface or interfacial tension between the two phases, N/m

Capillary number (Ca)

- if Ca >>1, then viscous forces dominate over interfacial forces;
- if Ca <<1, then viscous forces are negligible compared with interfacial forces.

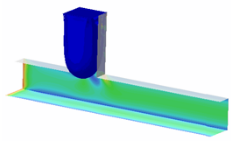
- It is usually denoted NC in the oil field. Capillary numbers are usually large for high-speed flows and low for low-speed flows

- Typically for flow through pores in the reservoir (油藏气孔) NC ~10^{-6}, and for flow in production tubulars (油品管道) NC ~1.

$$Ca = \frac{\mathrm{We}}{\mathrm{Re}}$$

Capillary number (Ca)

➢ Squeezing Regime



Low Ca

Ca=8.75×10^{-4}

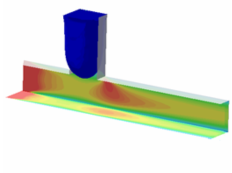
$$Ca = \frac{\mu U_b}{\sigma}$$

➢Shear force

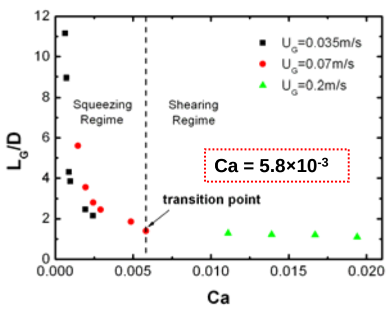
➢Surface tension

High Ca

➢ Shearing Regime

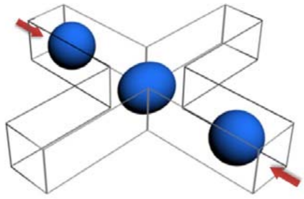


Ca=1.4×10^{-2}

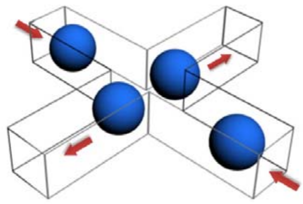


Microgravity - Science and Technology, 2009, 21(S1): 51-58

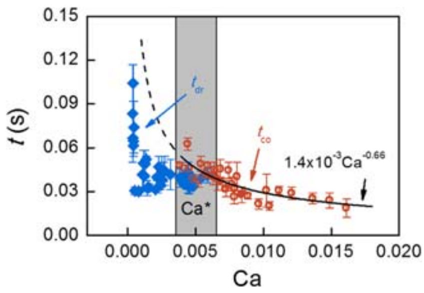
Capillary number (Ca)



Fusion: Ca < 0.005



Separation: Ca > 0.005



- The critical Ca is larger in confined microchannel than in free flowing space.

Chemical Engineering Journal 227 (2013) 90–96

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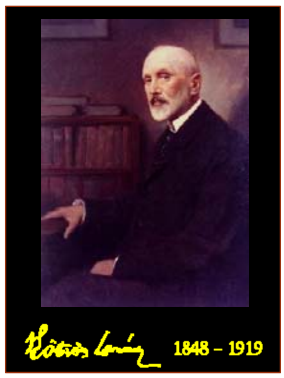
Eötvös number (Eo)

- Together with Morton number it can be used to characterize the shape of bubbles or drops moving in a surrounding fluid. Eötvös number may be regarded as proportional to **buoyancy force** divided by **surface tension force**.

$$E_o = \frac{\Delta \rho g L^2}{\sigma}$$

- $\Delta\rho$: difference in density of the two phases, kg/m^3
- g: gravitational acceleration, m/s^2
- L: characteristic length, m
- σ: surface tension, N/m



- Baron Roland von Eötvös
- 1848 - 1919
- Buda, Hungarian

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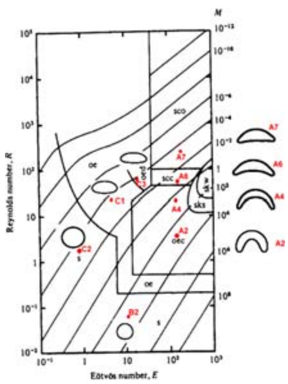
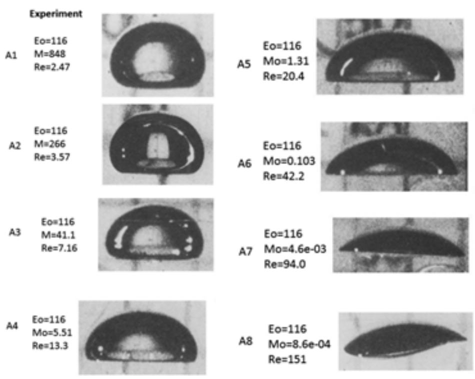
Eötvös number (Eo)

- Eötvös is remembered today for his experimental work on gravity, in particular his study of **the equivalence of gravitational and inertial mass** (the so-called weak equivalence principle) and his study of the gravitational gradient on the Earth's surface.
- The **weak equivalence principle** plays a prominent role in relativity theory and the Eötvös experiment was cited by Albert Einstein in his 1916 paper ***The Foundation of the General Theory of Relativity***.
- From 1886 until his death, Loránd Eötvös researched and taught in the University of Budapest, which in 1950 was renamed after him (Eötvös Loránd University).

Eötvös number (Eo)

- A high number indicates that the system is relatively unaffected by surface tension effects; a low number (typically less than one is the requirement) indicates that surface tension dominates.

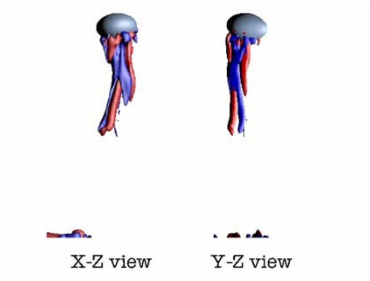




Grace J. Trans Inst Chem Eng 1973;51:116–20

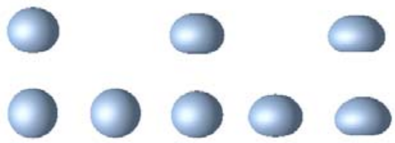
Bhaga D, Weber M. J Fluid Mech 1981;105:61–85

Eötvös number (Eo)



LogM = -2.8

LogM = -4.6



Eo

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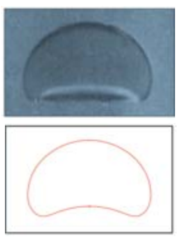
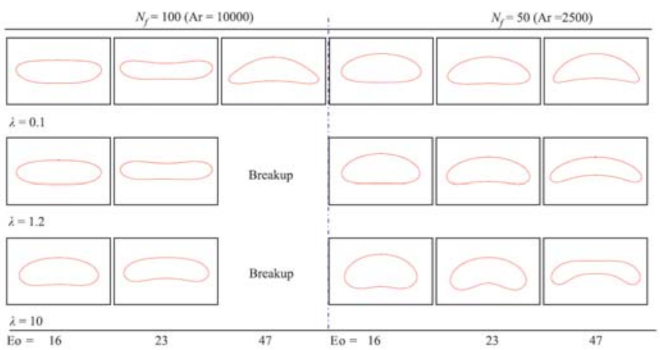
Eötvös number (Eo)

Shapes and terminal velocities of a drop rising in stagnant liquids



Eo = 63.8

Fr = 0.47



Computers & Fluids 81 (2013) 17–25

Wairegi T, Grace JR. The behaviour of large drops in immiscible liquids. Int J Multiphase Flow 1976;3:67–77

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Dean number (De/Dn/Dc)

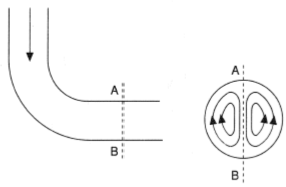
- Dean number occurs in the study of flow in curved pipes and channels, represents vortices in curved ducts

$$De = \frac{\rho U D}{\mu} \cdot \left(\frac{D}{2R}\right)^{1/2}$$

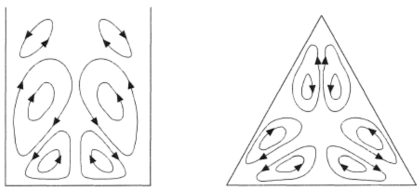
- ρ: density of the fluid, kg/m^3
- μ: dynamic viscosity, Pa·s
- U: axial velocity scale, m/s
- D: diameter, m
- R: the radius of curvature of the path of the channel, m

- William Reginald Dean
- 1896–1973
- British applied mathematician

Dean number (De/Dn/Dc)

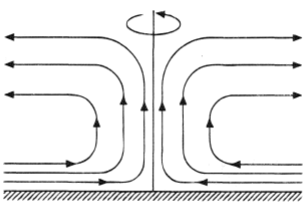


Secondary flow around a bend



Secondary flows in channels

http://www.thermopedia.com/content/1113/?tid=104&sn=1420

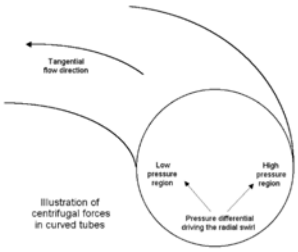


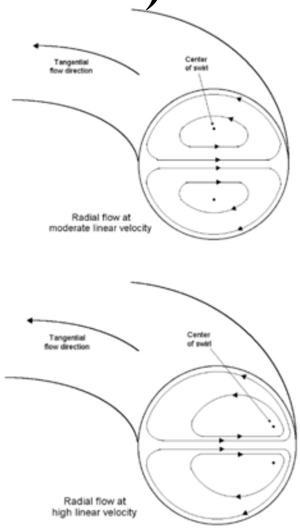
Secondary flow in a cyclone

Secondary flow in a coiled tube

http://www.thermopedia.com/content/639/

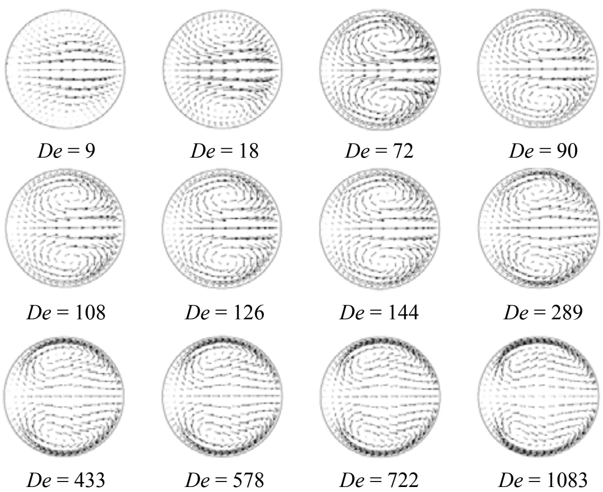
Dean number (De/Dn/Dc)





http://www.magicphil.net/pawel/HPLC97.html

Dean number (De/Dn/Dc)



Journal of Membrane Science 183 (2001) 149–162

Froude number (Fr)

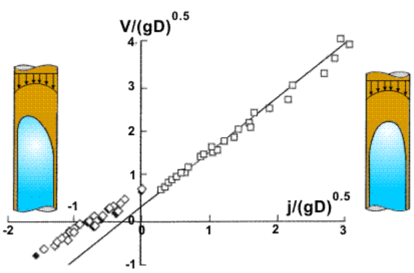
- wave and surface behaviour
- flows in rivers or in the sea are often dominated by the so-called Froude similitude
- the ratio of the **inertial** to **gravity** forces in the flow

$$Fr = \frac{U}{\sqrt{gL}}$$

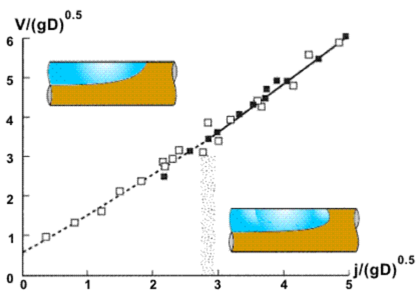
- U: velocity scale, m/s
- g: gravity, m/s^2
- L: length scale, m

- William Froude
- 1810–1879
- English engineer

Froude number (Fr)



Velocity of long bubbles vs. mixture velocity
D=50mm □ (Fréchou, 1986)
140 mm ♦, 100 mm ◊, 26 mm ▲ (Martin)



Velocity of long bubbles vs. mixture velocity
D=146 mm □ (Ferschneider, 1982)
189 mm ■ (Linga)

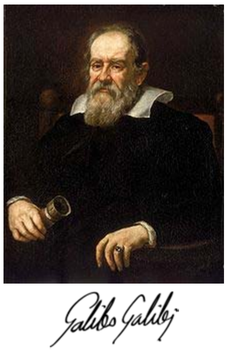
http://www.thermopedia.com/content/38/?tid=104&sn=1297

Galilei number (Ga)

- gravity-driven viscous flow
- as proportional to gravity forces divided by viscous forces. The Galilei number is used in viscous flow and thermal expansion calculations, for example to describe fluid film flow over walls. These flows apply to condensers or chemical columns.

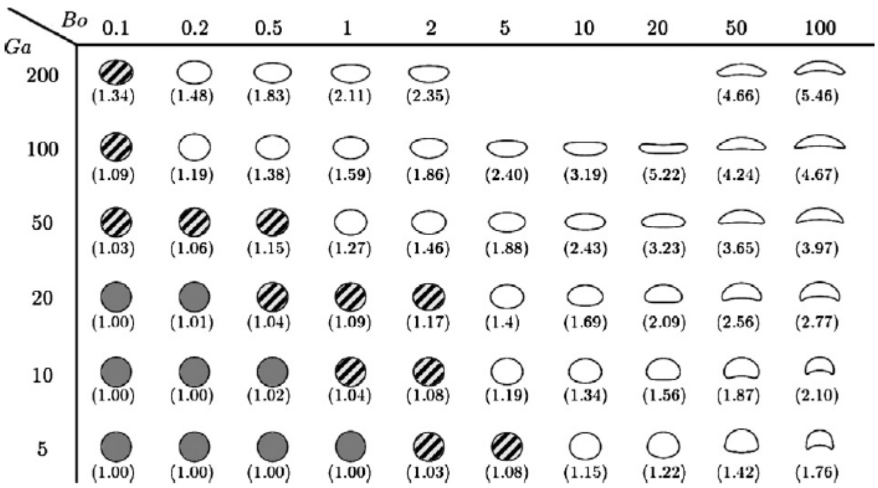
$$Ga = \frac{gL^3}{\nu^2} \quad \text{or} \quad Ga = \frac{\sqrt{gL^3}}{\nu}$$

- g: gravitational acceleration, m/s^2
- L: characteristic length, m
- ν: kinematic viscosity, m^2/s



- Galileo Galilei
- 1564 – 1642
- Italian
- Astronomy, physics and mathematics

Galilei number (Ga)



International Journal of Multiphase Flow 51 (2013) 11–21

Grashof number (Gr)

- approximates the ratio of the **buoyancy** to **viscous force** acting on a fluid. It frequently arises in the study of situations involving natural convection.

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

- g: acceleration due to Earth's gravity, m/s^2
- β: volumetric thermal expansion coefficient (equal to approximately 1/T, for ideal fluids)
- T: absolute temperature, K
- T_s: surface temperature, K
- T_∞: bulk temperature, K
- L: length, m
- ν: kinematic viscosity, m^2/s

- Franz Grashof
- 1826 - 1893
- German Engineer

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Grashof number (Gr)

- Grashof was one of the leaders in founding the Society of German Engineers (Verein Deutscher Ingenieure)
- After Grashof's death, the Association of German Engineers (VDI) honored his memory by instituting the *Grashof Commemorative Medal* as the highest distinction that the society could bestow for merit in the engineering skills.

VDI Verein Deutscher Ingenieure

Westfälischer Bezirksverein e.V.

VDI 创始人员名单纪念碑

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Grashof number (Gr)

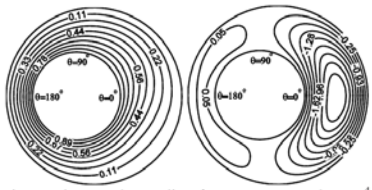


Fig. 3 Isotherms and streamlines for C_2=0.4, α=0° and Gr=10^4

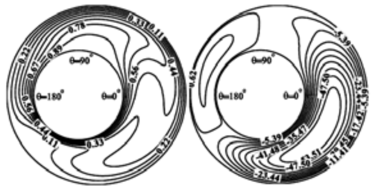


Fig. 5 Isotherms and streamlines for C_2=0.4, α=0° and Gr=10^6

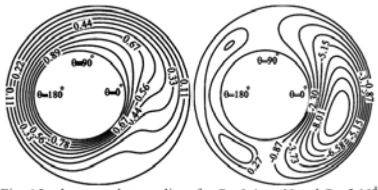


Fig. 4 Isotherms and streamlines for C_2=0.4, α=0° and Gr=5.10^4

- When the Grashof number is equal to 10^4, the heat transfer is essentially conductive
- For Gr = 10^6 the isothermal lines are modified and eventually take the form of a mushroom. The temperature distribution is decreasing in the hot wall towards the cold wall

Energy Procedia 36 (2013) 293 – 302

Knudsen number (Kn)

- continuum approximation in fluids
- The ratio of the mean free path length of the molecules of a fluid to a characteristic length (Knudsen 1909a)

$$kn = \frac{\lambda}{L}$$

- λ: mean free path, m
- L: length scale, m

$$kn = \frac{Ma}{Re}\sqrt{\frac{\gamma\pi}{2}}$$

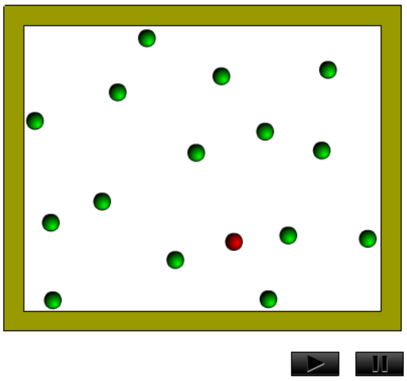
γ is the ratio of specific heats

http://en.wikipedia.org/wiki/Knudsen_number

- Martin Knudsen
- 1871–1949
- Danish physicist

Mean Free Path

$$\bar{\lambda} = \frac{kT}{\sqrt{2}\,\pi d^2 p}$$



dxwl.hubu.edu.cn/JA/12-8.ppt

Pressure	10^5	1
MFP	nm	mm
H_2	12.5	1.25
N_2	67	6.7
O_2	72	7.2
Air	68	6.8

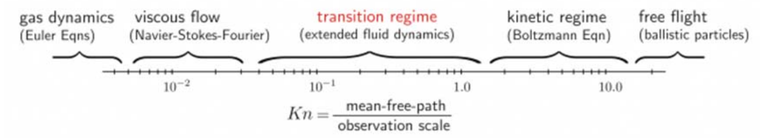
Mean Free Path – for liquids

- For liquids, λ is approximately the intermolecular length L_{mol} (bond length). The volume occupied by one molecule L_{mol}^3 is easily approximated, yielding

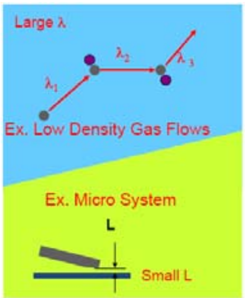
$$L_{mol} = \left(\frac{M_{mol}}{\rho N_A}\right)^{1/3}$$

- N_A: Avogado's number, ~ 6.022×10^{23}
- For water, $\lambda = L_{mol} \approx 0.31nm$

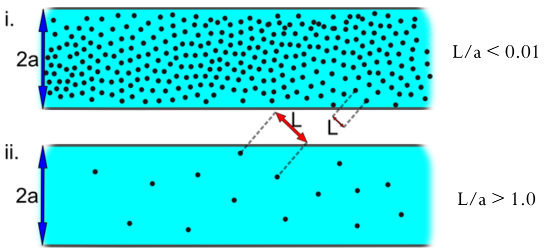
Knudsen number (Kn)



http://www.mathcces.rwth-aachen.de/2research/0mms/0gases/start

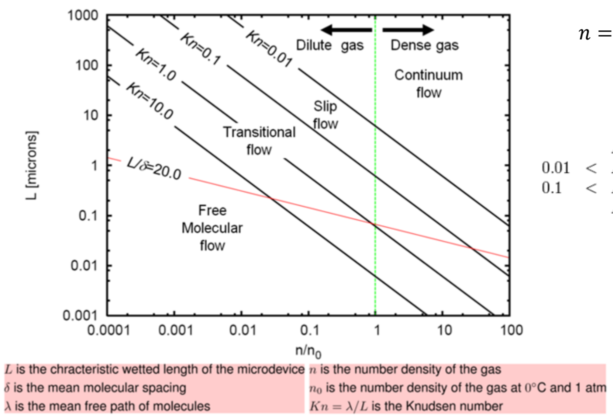


gcoe.mech.nagoya-u.ac.jp/basic/pdf/basic-11.pdf



http://www.tau.ac.il/~phchlab/experiments_new/SemB02_Vacuum/02TheoreticalBackground.html

Knudsen number (Kn)

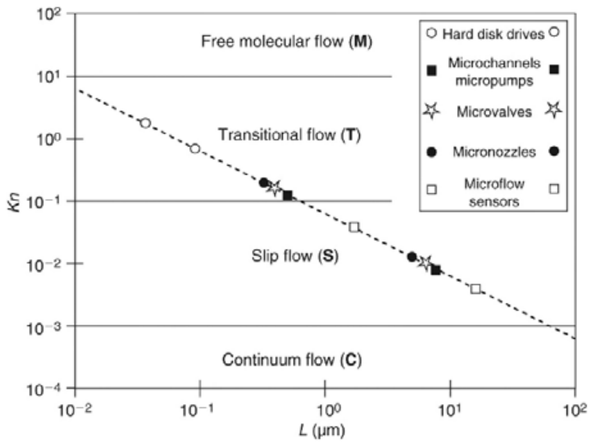


$$n = \frac{\text{particle number}}{\text{Volume}}$$

$$\begin{array}{lllll} & & Kn < 0.01 & & \text{:continuum flow} \\ 0.01 & < & Kn < 0.1 & & \text{:slip flow} \\ 0.1 & < & Kn < 10.0 & & \text{:transitional flow} \\ & & Kn > 10 & & \text{:free molecular flow} \end{array}$$

http://en.wikiversity.org/wiki/Microfluid_Mechanics_Chapter_8._Flow_Phenomena_in_Microflows

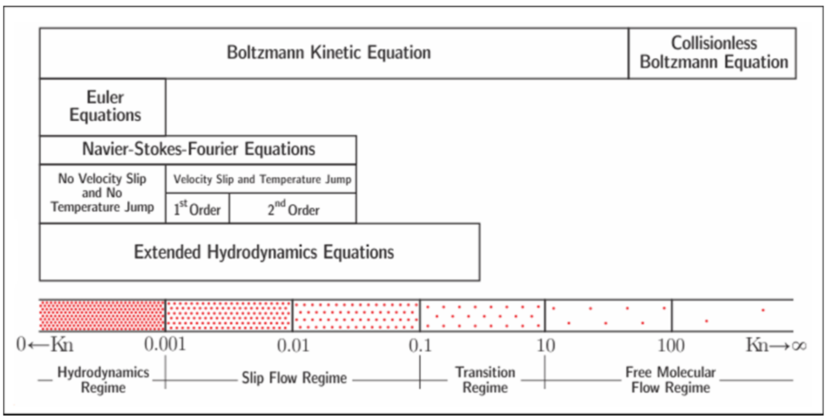
Knudsen number (Kn)



Heat Transfer and fluid flow in minichannels and microchannels .Kandlikar.S. et al.2006

动力工程多相流国家重点实验室 陈斌

Knudsen number (Kn)



http://www.sfu.ca/~ptaherib/research.html

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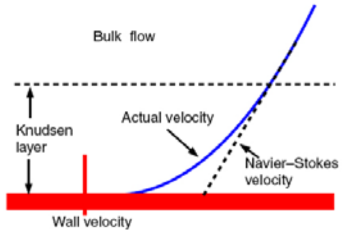
Knudsen layer

- The Knudsen layer (KL) is the kinetic boundary layer as a gas flows over a solid surface, whose **thickness is in the order of the mean free path (MFP) of the gas system**.
- For problems where the KL takes a large portion of the flow domain, such as gas flows in MEMS, it becomes critical to capture the gas motion in the KL.
- For instance, it has been shown that in the planar Poiseuille flow the contribution of the KL to the total mass flow rate increase can be as high as **30%** even as it takes only about **10%** of the channel



http://iopscience.iop.org/0295-5075/80/2/24001/fulltext/

Laplace number (La)

- free convection within immiscible fluids
- It represents a ratio of **surface tension** to the **momentum-transport (especially dissipation)** inside a fluid.
- also known as the Suratman number (Su)

$$La = Su = \frac{\sigma \rho L}{\mu^2} = \frac{Re}{Ca} = \frac{\mathrm{Re}^2}{We} = Oh^{-2}$$

- σ: surface tension, N/m
- ρ: density, kg/m^3
- L: characteristic length, m
- μ: dynamic viscosity, Pa·s

Pierre-Simon Laplace
1749–1827

Laplace, in conjunction with Young, is honored as one of the pioneers in the field of surface tension and capillary phenomena

Suratman number (Su)

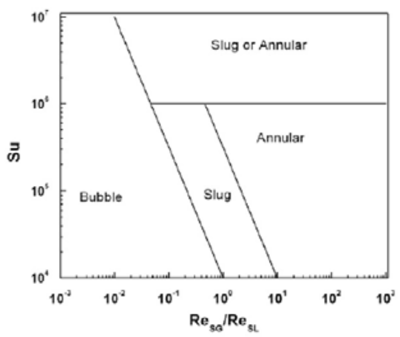
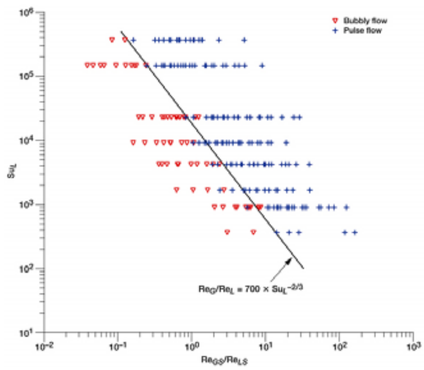


Fig. 3. Microgravity two-phase flow pattern map at $Su < 10^6$.

Acta Astronautica 69 (2011) 365–372



Transition map between bubbly and slug flow for the microgravity environment

http://www.grc.nasa.gov/WWW/RT/2003/6000/6712Bmotil.html

Lewis number (Le)

- defined as the ratio of **thermal diffusivity** to **mass diffusivity**. It is used to characterize fluid flows where there is simultaneous heat and mass transfer by convection.

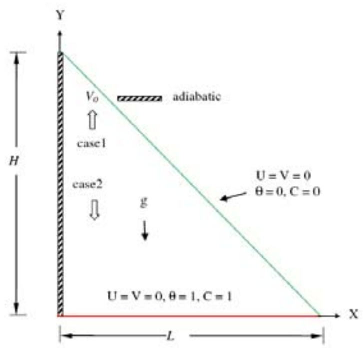
$$Le = \frac{\alpha}{D} = \frac{Sc}{Pr}$$

- α: thermal diffusivity, m^2/s
- D: mass diffusivity, m^2/s

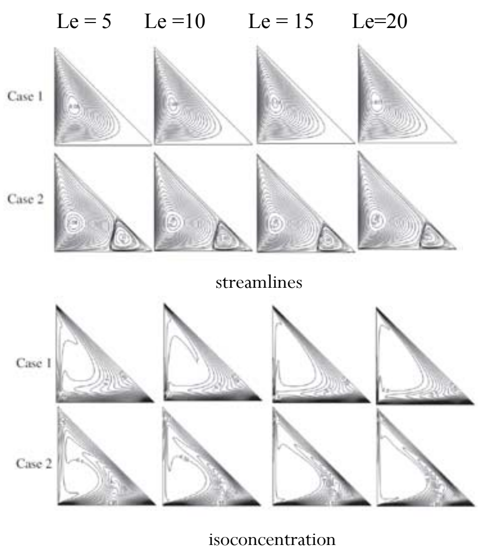
$$Sc = \frac{\nu}{D} \qquad Pr = \frac{\nu}{\alpha}$$

- Warren K. Lewis
- 1882–1975
- first head of the Chemical Engineering Department at MIT

Lewis number (Le)



International Communications in Heat and Mass Transfer 39 (2012) 1213–1219



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Morton number (Mo)

- determination of bubble/drop shape
- The ratio of the **gravitational acceleration** to the **molecular acceleration** of a fluid. It depends on fluid properties only.

$$Mo = \frac{g\mu^4 \Delta\rho}{\rho^2 \sigma^3} = \frac{We^3}{Fr^2 Re^4}$$

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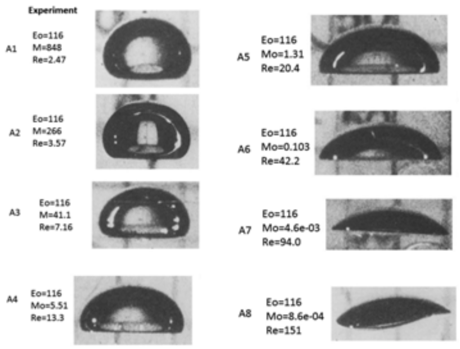
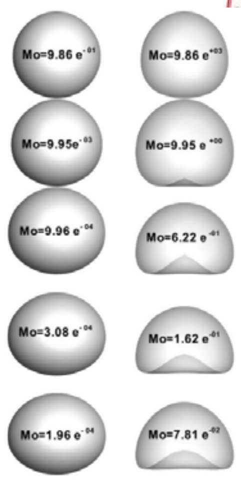
Morton number (Mo)

- used together with the Eötvös number to characterize the shape of bubbles or drops moving in a surrounding fluid or continuous phase



Bhaga D, Weber M. J Fluid Mech 1981;105:61–85



(a) Bo = 1 (b) Bo = 10

Steady state shape of a rising drop at different Reynolds numbers

Computers & Fluids 88 (2013) 543–556

Morton number (Mo)

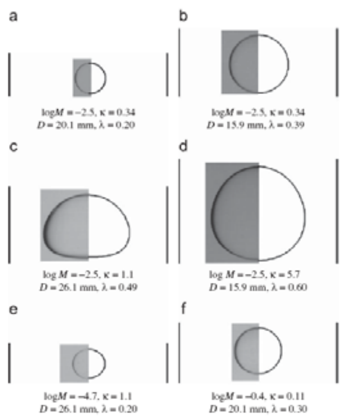
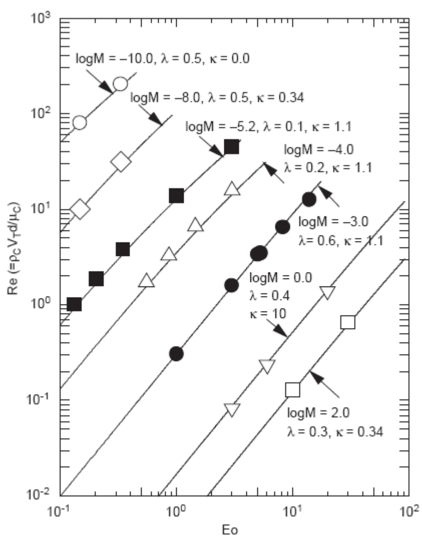


Fig. 9. Measured (left) and predicted (right) shapes of drops in pipes.



Chemical Engineering Science 64 (2009) 3019 -- 3028

Nusselt number (Nu)

- heat transfer with forced convection
- the ratio of **convective** to **conductive** heat transfer across (normal to) the boundary
- 1915, Nusselt published his pioneer paper: "The Basic Laws of Heat Transfer"

$$Nu = \frac{hL}{k_f} = ReStPr = PeSt$$

- h: convective heat transfer coefficient
- L: characteristic length
- K_f: thermal conductivity of the fluid

- Wilhelm Nusselt
- 1882–1957
- German engineer

Nusselt number (Nu)

- A Nusselt number close to one, namely convection and conduction of similar magnitude, is characteristic of laminar flow.
- A larger Nusselt number corresponds to more active convection, with turbulent flow typically in the 100~1000 range.
- For **free convection**, the average Nusselt number is expressed as a function of the Rayleigh number and the Prandtl number, written as: Nu = f(Ra, Pr).
- For **forced convection**, the Nusselt number is generally a function of the Reynolds number and the Prandtl number, or Nu = f(Re, Pr).

Ohnesorge number (Oh)

- atomization of liquids, Marangoni flow
- relates the **viscous forces** to **inertial and surface tension forces**.
- The number was defined by Wolfgang von Ohnesorge in his 1936 doctoral thesis

$$Oh = \frac{\mu}{\sqrt{\rho\sigma L}} = \frac{1}{\sqrt{La}} = \frac{\sqrt{We}}{Re}$$

- μ : liquid viscosity
- ρ: liquid density
- σ: surface tension
- L: characteristic length scale (typically drop diameter)

- Wolfgang von Ohnesorge
- 1901–1976
- German physicist

Ohnesorge number (Oh)

- Broadly speaking, it appears that in the **jet breakup** and **atomization** literature the name of Ohnesorge is more recognized, whereas in the **emulsification** and **drop breakup** literature the Suratman name is more familiar.

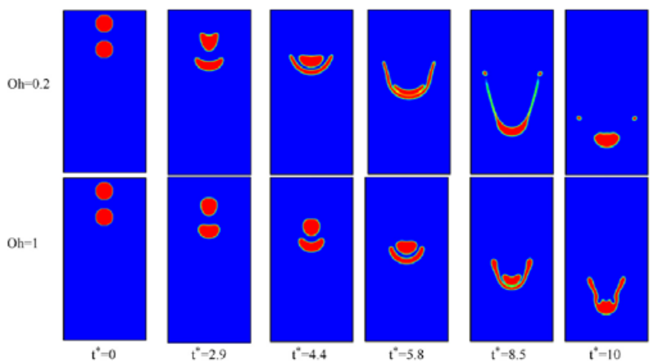


Fig. 7. Deformation and coalescence of two droplets falling in a vertical line for different Ohnesorge number at Eo=43.

Colloids and Surfaces A: Physicochem. Eng. Aspects 424 (2013) 40– 51

Péclet number (Pe)

- advection–diffusion problems
- the ratio of the rate of **advection** of a physical quantity by the flow to the rate of **diffusion** of the same quantity driven by an appropriate gradient

$$Pe = \frac{LU}{h} = ReNu$$

- h: convective heat transfer coefficient
- L: characteristic length
- U: velocity scale

- Jean Claude Eugène Péclet
- 1793–1857
- French physicist

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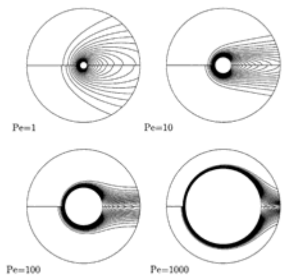
Péclet number (Pe) $Pe = \frac{LU}{h}$

- In the context of particulate motion the Péclet numbers have also been called Brenner numbers, with symbol Br, in honor of Howard Brenner
- In engineering applications the Péclet number is often very large. In such situations, the dependency of the flow upon downstream locations is diminished, and variables in the flow tend to become 'one-way' properties.
- When Pe is small, conduction is much more significant than advection, at least near the cylinder, and, therefore, there is an appreciable conduction against the flow upstream of the cylinder.



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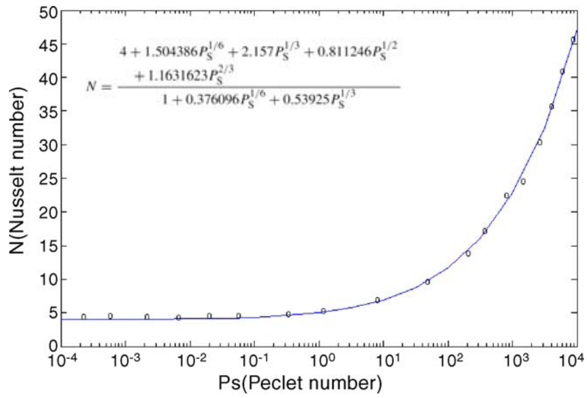
International Journal of Thermal Sciences 43 (2004) 213–220

Péclet number (Pe)

mass transfer rate at microdisc electrodes in a channel flow



Electrochimica Acta 51 (2006) 5407–5411

网格Péclet number (Pe)

- 1976年Roache提出，网格或单元Peclet数可以用来度量某点处φ的对流和扩散的强度比例。用网格尺度作为*Pe*数的长度尺度，即得到网格*Pe*数，写作P_Δ

$$P_\Delta = \frac{\Delta x \cdot U}{h}$$

- P_Δ越大，上游节点φ值对下游节点的影响越大，下游节点对上游节点的影响越小。而当P_Δ=0时，上游节点对下游节点的影响与下游节点对上游节点的影响一样。
- 如果P_Δ>2，一维对流扩散问题的有限体积法离散方程中表示扩散与对流作用的系数将会为负，这样会导致物理上不真实的解。当P_Δ<2时能保证应用中心差分计算有较高的精度。

Courant Number (*CFL*)

- A very important step in numerical simulation is transient time step sizing. It represents a comparison between the particle moving distance during the assumed time step and control volume dimension.

$$Cou = \frac{U\Delta t}{\Delta x}$$

- A low *Cou* value means a small time step size (Δt) and consequently a large simulation time. On the other hand, a high *Cou* value leads to an unstable numeric approach.
- When **Advection** dominates dispersion, designing a model with a **small (<1) Courant number** will decrease oscillations, improve accuracy & decrease numerical dispersion

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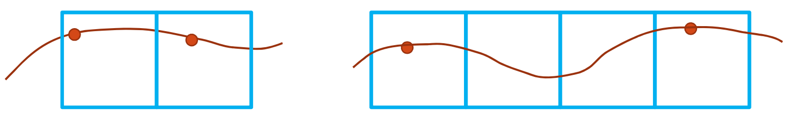
Courant Number (*CFL*)

- numerical solutions of hyperbolic PDEs
- A physical explanation, for a CFD simulation, of the Courant number could be that it tells you something about how fluid is moving through your computational cells. If the Courant number is <=1 fluid particles move from one cell to another within one time step (at most). If it is >1 a fluid particles moves through two or more cells at each time step and this can affect convergence negatively.



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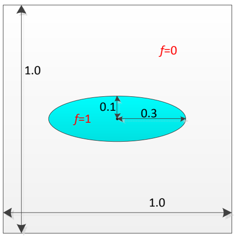
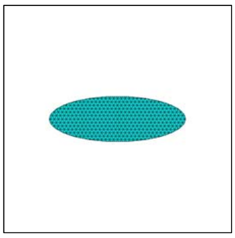
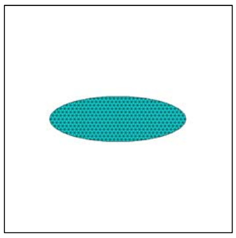
Courant Number (*CFL*)

$$\frac{\partial f}{\partial t}+u\frac{\partial f}{\partial x}+v\frac{\partial f}{\partial y}=0 \qquad \begin{cases} u=-\pi\cdot\cos(\pi x)\cdot\sin(\pi y) \\ v=\pi\cdot\sin(\pi x)\cdot\cos(\pi y) \end{cases}$$





$Cn = 0.1$



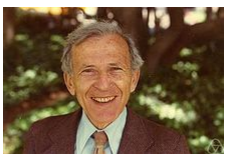
$Cn = 1.1$

Courant–Friedrichs–Lewy condition

Courant, R.; Friedrichs, K.; Lewy, H. (March 1967) [1928], "On the partial difference equations of mathematical physics", IBM Journal of Research and Development 11 (2): 215–234

Richard Courant
1988-1972
German mathematician

Kurt Otto Friedrichs
1901-1982
German American



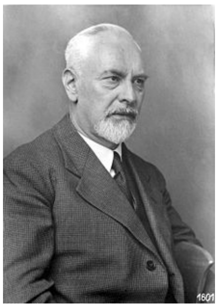
Hans Lewy
1904-1988
USA mathematician

Prandtl number (Pr)

- convection heat transfer (thickness of thermal and momentum boundary layers)
- the ratio of **momentum diffusivity** (kinematic viscosity) to **thermal diffusivity**

$$Pr = \frac{\nu}{\alpha} = \frac{C_p \mu}{k}$$

- ν : kinematic viscosity, m^2/s
- α: thermal diffusivity, m^2/s
- μ: dynamic viscosity, Pa·s
- k: thermal conductivity, W/(m K))
- Cp : specific heat, J/(kg K))



- Ludwig Prandtl.
- 1875–1953
- German physicist

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Prandtl number (Pr)

- The Prandtl number contains no such length scale in its definition and is dependent only on the fluid and the fluid state
- Pr<<1 means thermal diffusivity dominates, Pr>>1 means momentum diffusivity dominates
  - around 0.015 for mercury
  - around 0.16-0.7 for mixtures of noble gases or noble gases with hydrogen
  - around 0.7-0.8 for air and many other gases,
  - between 4 and 5 for R-12 refrigerant
  - around 7 for water (At 20 degrees Celsius)
  - 13.4 and 7.2 for seawater (At 0 degrees Celsius and 20 degrees Celsius respectively)
  - between 100 and 40,000 for engine oil
  - around 1×10^{25} for Earth's mantle (地幔)

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Prandtl

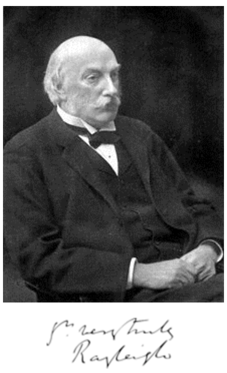
- In 1904 (29 years old) he delivered a **groundbreaking** paper, *Fluid Flow in Very Little Friction*, in which he described the boundary layer flow separation.
- Prandtl and his student Theodor Meyer *developed the first theories of supersonic shock waves and flow* in 1908
- *the Lanchester-Prandtl wing theory* 1918–1919, led to the realization that on any wing of finite length. These tools enabled aircraft designers to make meaningful theoretical studies of their aircraft before they were built.
- he created a method for *designing a supersonic nozzle* with Adolf Busemann in 1929. Today, all supersonic wind tunnels and rocket nozzles are designed using the same method. A full development of supersonics would have to wait for the work of Theodore von Kármán, a student of Prandtl at Göttingen.
- Prandtl's life was marked by overtones of naïveté. At the age of thirty-four, he decided it was time to marry, so he went to his old professor, August Föppl, to ask his daughter's hand in marriage. But Prandtl didn't say which daughter. The professor and his wife had a hurried discussion and wisely decided it should be the older one. That was fine. The marriage was a long and happy one.

Rayleigh number (Ra)

- associated with buoyancy driven flow (also known as free convection or natural convection). When the Rayleigh number is below the critical value for that fluid, heat transfer is primarily in the form of **conduction**; when it exceeds the critical value, heat transfer is primarily in the form of **convection**.
- For free convection near a vertical wall

$$Ra_x = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha} = Gr_x Pr$$

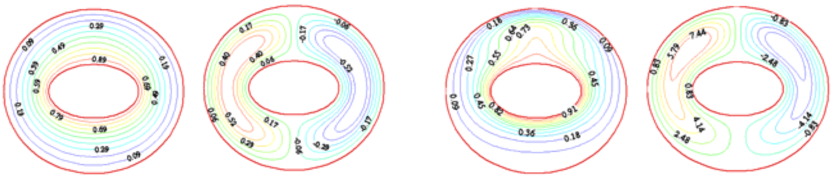
- x = Characteristic length (in this case, the distance from the leading edge)



- Lord Rayleigh
- 1842 - 1919
- UK physicist

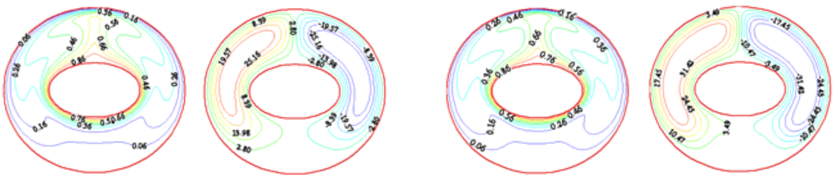
Rayleigh number (Ra)

Natural Convection between two elliptical cylinders



Ra = 50 and Pr=0.702

Ra = 10^3 and Pr=0.702



Ra = 10^4 and Pr=0.702

Ra = 2.10^4 and Pr=0.702

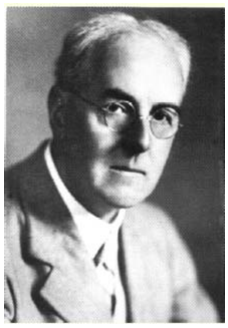
Energy Procedia 36 (2013) 788 – 797

Richardson number (Ri)

- effect of buoyancy on flow stability
- expresses the ratio of **potential** to **kinetic** energy

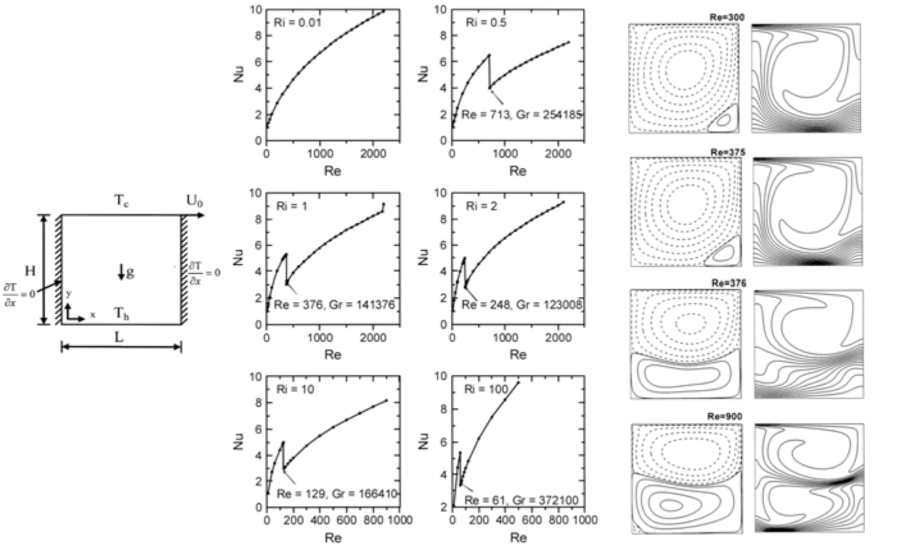
$$Ri = \frac{gh}{u^2} = \frac{1}{Fr^2}$$

- g is the acceleration due to gravity
- h a representative vertical length scale,
- u a representative speed



- Lewis Fry Richardson
- 1881 - 1953
- UK physicist

Richardson number (Ri)



International Journal of Thermal Sciences 50 (2011) 197-205

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Stefan Number (Ste)

- the ratio of **sensible heat** to **latent heat**. It is used in melting and solidification

$$Ste = \frac{c_p \Delta T}{\lambda}$$

- c_p: the specific heat, J/(kg·K)
- ΔT: is the temperature difference between phases, K
- λ: the latent heat, J/kg

- Joseph Stefan
- 1835 - 1893
- Austrian physicist

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Stefan Number (Ste)

Table 2 – Stefan-number for throttling loss, $T_1 = 0\ ^\circ C$, $T_2 = 40\ ^\circ C$.

	Water	Ammonia	R134a	R1234yf	R114
Pseudo Stefan-number, $\tilde{St}_{thr}$	0.05	0.11	0.09	0.12	0.10
Pressure ratio, π	12	3.6	3.4	3.2	3.9
Mechanical throttling number, N_{thr}	2.7×10^{-6}	1.5×10^{-3}	2.6×10^{-3}	4.1×10^{-3}	1.3×10^{-3}
Stefan-number, St_{thr}	0.067	0.14	0.23	0.32	0.29
Throttling loss, x (from diagrams, sources, see Table 1)	0.067	0.15	0.28	0.35	0.30

Int. J. Referigeration 33(2010) 1343-1349

Stokes number (St)

- corresponding to the behavior of particles suspended in a fluid flow (particle dynamics). Stokes number is defined as the ratio of the characteristic time of a particle (or droplet) to a characteristic time of the flow or of an obstacle

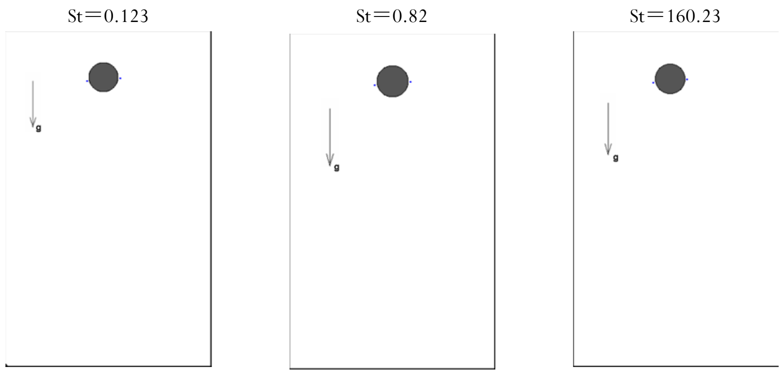
$$St = \frac{\tau U_o}{d_c} = \frac{\rho_d d_d^2}{18\mu_g} \text{ (for stokes flow)}$$

- ρ_d: particle density, kg/m^3
- d_d: particle diameter, m
- μ_g: gas dynamic viscosity, m/s^2

- Sir George Stokes
- 1819 - 1903
- Ireland physicist

Stokes number (St)

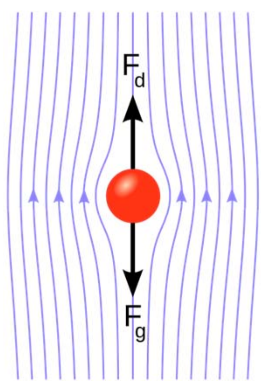
- Particles with low Stokes number follow fluid streamlines (perfect advection) whereas for large Stokes number, the particle's inertia dominates so that the particle will continue along its initial trajectory.
- For PIV, particles follow fluid streamlines closely when St<1. If St<0.1, tracing accuracy errors are below 1%



Chen Bin, Wang Cong, ATE, 2009

Stokes flow

- Stokes flow, also named creeping flow or creeping motion, is a type of fluid flow where advective inertial forces are small compared with viscous forces (Re<<1).
- This is a typical situation in flows where the fluid velocities are very slow, the viscosities are very large, or the length-scales of the flow are very small.



a sphere in Stokes flow
at very low Reynolds number

Strouhal number (St)

- used in studying the vibrations of a body past which a fluid is flowing; it is equal to a characteristic dimension of the body times the frequency of vibrations divided by the fluid velocity relative to the body (oscillating flow)

$$St = \frac{fL}{V}$$

- f is the frequency of vortex shedding
- L is the characteristic length
- V is the velocity of the fluid

- Vincenc Strouhal
- 1850 – 1922
- Czech physicist

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Strouhal number (St)

Optimal Strouhal number for swimming animals

Species	L (cm)	S/h^2	Re	Li	St	U/V	θ_0 (°)
Mammals							
Beluga[a]	364	7.3	8.0×10^6	0.145	0.35	0.48	31
Bottlenose dolphin[a]	258	7.9	1.2×10^7	0.063	0.26	0.52	25
False killer whale[a]	379	7.8	2.1×10^7	0.044	0.26	0.57	28
Florida manatee[b]	334	7.5	4.4×10^6	0.025	0.31	0.66	
Harp seal[c]	153	7.7	1.6×10^6	0.123	0.27	0.45	22
Killer whale[a]	473	7.3	2.6×10^7	0.015	0.28	0.56	29
Ringed seal[c]	106	8.7	1.3×10^6	0.105	0.30	0.44	24
White-sided dolphin[a]	221	6.5	1.3×10^7	0.018	0.24		
Sharks							
Blacktip reef shark[d]	97	7.6	8.3×10^5	0.036	0.25	0.66	
Bonnethead shark[d]	93	5.5	8.0×10^5	0.026	0.27	0.74	
Nurse shark[d]	220	17.8	1.8×10^6	0.072	0.41		
Scalloped hammerhead[e*]	59	4.9	3.8×10^5	0.027	0.37		
Scombrids							
Atlantic mackerel[f]	32	6.7	5.8×10^5	0.034	0.25	0.73	
Chub mackerel[g]	21	10.5	1.6×10^5	0.070	0.25		
Chub mackerel[h*]	21	10.5	1.8×10^5	0.067	0.26	0.63	
Giant bluefin tuna[i]	250	4.8	5.7×10^6	0.015	0.24		
Kawakawa tuna[h*]	21	5.0	1.8×10^5	0.032	0.21	0.60	
Pacific bonito[j]	47	6.2	4.5×10^5	0.033	0.23		
Skipjack tuna[k]	57	5.8	2.2×10^6	0.022	0.27		
Yellowfin tuna[l]	53	5.5	6.1×10^5	0.028	0.29	0.48	

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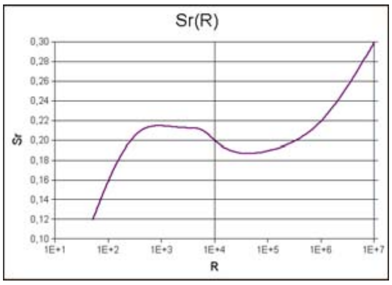
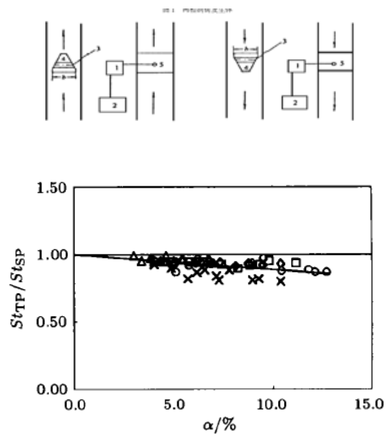
Journal of Fluids and Structures 30 (2012) 205–218

Strouhal number (St)



Strouhal number as a function of the Reynolds number for a long cylinder
http://www.answers.com/topic/strouhal-number



李永光，林宗虎. 1995

Weber number (We)

- It is often useful in analyzing fluid flows where there is an interface between two different fluids, especially for multiphase flows with strongly curved surfaces. It can be thought of as a measure of the relative importance of the fluid's **inertia** compared to its **surface tension**

$$We = \frac{\rho u^2 L}{\sigma} = CaRe = Re^2\left(\frac{Mo}{Eo}\right)^2 = (FrMoRe^4)^{1/3}$$

- ρ: is the density of the fluid (kg/m^3)
- u: is its velocity (m/s)
- L: is its characteristic length (diameter)
- σ: is the surface tension (N/m)

- Moritz Weber
- 1871–1951
- German engineer

Weber number (We)

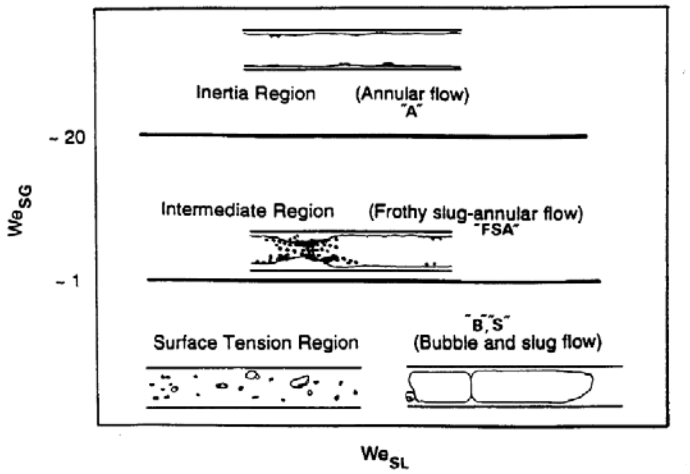


Figure 1. Microgravity two-phase flow pattern map based on the superficial Weber numbers, Zhao & Rezkallah (1993).

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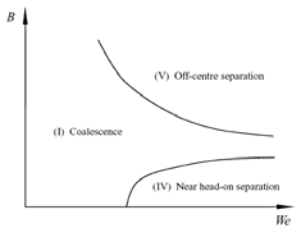
Weber number (We)



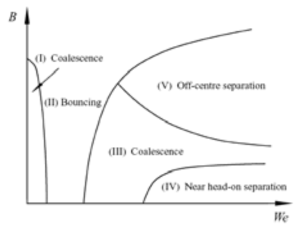
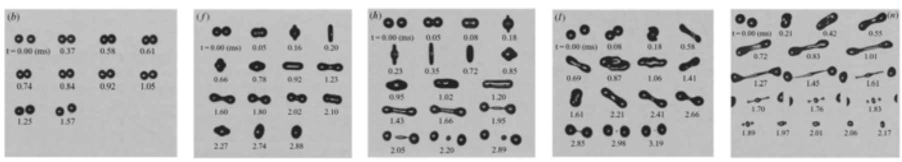


FIGURE 1. Schematic of various collision regimes of water droplets in 1 atm. ai FIGURE 2. Schematic of various collision regimes of hydrocarbon droplets in 1 atm. air.



We=0.5 R=14.8 We=32.8 Re=210.8 We=61.4 Re=296.5 We=48.1 Re=270.1 We=65.1 Re=320.3

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Journal of Computational Physics 253 (2013) 166–188

Weissenberg number (Wi)

- used in the study of viscoelastic flows
- the ratio of the relaxation time of the fluid and a specific process time.
- When the time-scale of a flow t_f is much less than the relaxation time t_r of an elasticoviscous material, elastic effects dominate. When on the other hand t_f is much greater than t_r, elastic effects relax sufficiently for viscous effects to dominate.

$$Wi = \frac{t_f}{t_p}$$

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- Karl Weissenberg
- 1893 – 1976
- Austrian physicist

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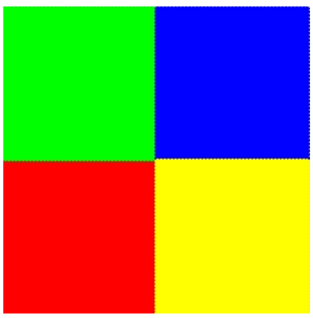
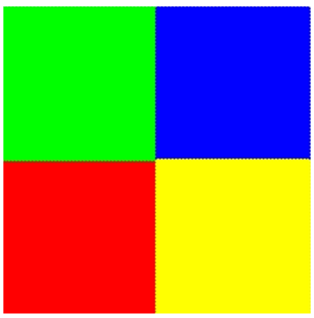
Weissenberg number (Wi)

- For instance, in simple steady shear, it is defined as the **shear rate** times the **relaxation time**

$$Wi = \dot{\gamma}\lambda$$



Wi=0.5



Wi= 10

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http://www.math.nyu.edu/~thomases/oldb.html

Exercise 02

- $Ca = \frac{We}{Re}$ $Mo = \frac{We^3}{Fr^2 Re^4}$ $Oh = \frac{\sqrt{We}}{Re}$
- $We = Re^2 \left(\frac{Mo}{Eo}\right)^2 = (FrMoRe^4)^{1/3}$

Exercise 03 (optional)

- Try to find the origins of Atwood number, Capillary number and Morton number.

To be continued……