

## Micro-scale two-phase flow dynamics

### Lecture 03 Macro-to-microscale transition criteria

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## Outline



- Why microchannels
- Macro-to-microscale transition criteria
- Confinement number approach
- Bubble departure diameter approach
- Young-Laplace equation approach



## Why Microchannels

- Ratio of surface to volume of a tube

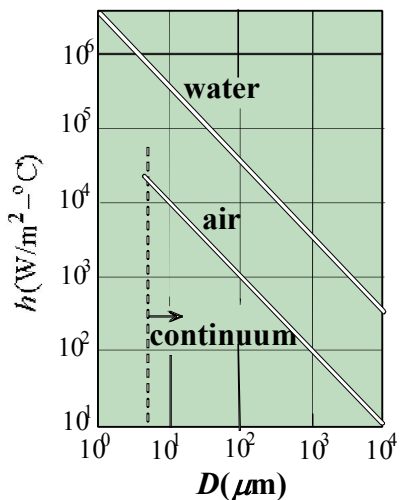
$$\frac{A}{V} = \frac{\pi DL}{\pi D^2 L/4} = \frac{4}{D}$$

- For  $D = 1 \text{ m}$ ,  $A/V = 4 \text{ (m}^{-1}\text{)}$
- For  $D = 1 \mu\text{m}$ ,  $A/V = 4 \times 10^6 \text{ (m}^{-1}\text{)}$

$$Nu = \frac{hD}{k} = 3.66$$

$$h = 3.66 \frac{k}{D}$$

$D \downarrow \quad h \uparrow$

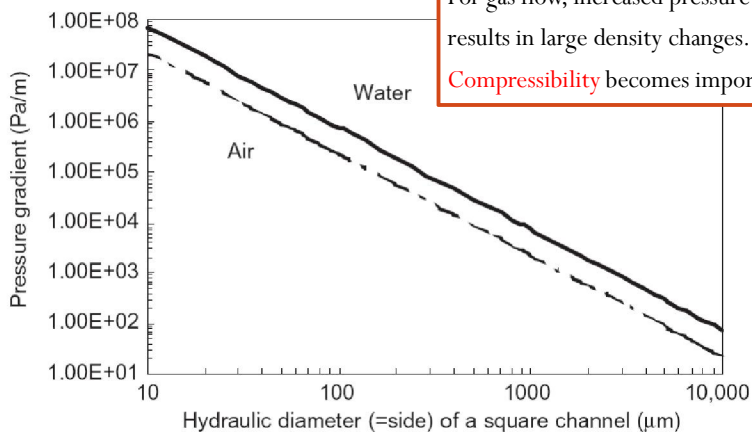


Latif Jiji. Heat Convection

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## Why Microchannels



For gas flow, increased pressure drop results in large density changes.  
**Compressibility** becomes important

Heat Transfer and fluid flow in mini channels and micro channels.

Kandlikar S. et al.2006

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## Micro-scale flow characteristics



	Viscosity / Pa.s	$u/\text{mm.s}^{-1}$	$d$	Re
Water	$1.025 \times 10^{-3}$	1.0	100 nm ~ 100 $\mu\text{m}$	$10^{-1}$ to $10^{-4}$
Oxygen	$20.317 \times 10^{-6}$	1.0	100 nm ~ 100 $\mu\text{m}$	$10^{-5}$ to $10^{-2}$

- Therefore, inertial forces are overwhelmed by **interfacial forces** in microfluidic devices. **Laminar flow** is expected in micro- and nanochannels, and not turbulent or random flow. Consequently, in simple geometries, **parabolic profiles** are expected in pressure-driven flows,

- In the microarea, the relative importance of forces, calculated in a water/oxygen two-phase microflow, is given by the following order:

buoyancy forces < inertial forces < gravitational forces

< viscous forces < interfacial forces

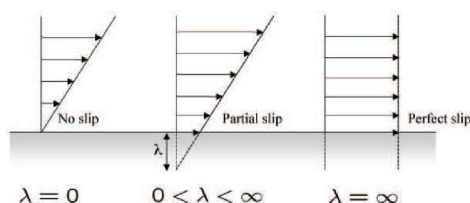


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## Boundary condition



- linear boundary condition (Navier 1823, Maxwell [109]),
  - the component of the fluid velocity tangent to the surface is proportional to the rate of strain (or shear rate) at the surface
  - The velocity component normal to the surface is naturally zero as mass cannot penetrate an impermeable solid surface



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## Boundary condition

- Gas flow in devices with dimensions that are on the order of the mean free path of the gas molecules shows significant slip

$$\frac{\lambda}{l_m} = \frac{2(2-p)}{3p}$$

- slip being important when  $Kn > 0.1$

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## Boundary condition

- Newtonian liquids

Summary of slip results for pressure drop versus flow rate experiments

	Surfaces	Liquids	Wetting	Roughness	Shear rates	Slip length
Schnell [141]	Glass+DDS	Water	—	—	$10^2 - 10^3 \text{ s}^{-1}$	1 – 10 $\mu\text{m}$
Churaev [34]	Quartz+TMS	Water	$70 - 90^\circ$	—	$1 \text{ s}^{-1}$	30 nm
"	"	Mercury	$115 - 130^\circ$	—	$10^3 - 10^4 \text{ s}^{-1}$	70 nm
"	"	$\text{CCl}_4$	Complete	—	—	no-slip
"	"	Benzene	Complete	—	—	no-slip
Kiseleva [88]	Quartz+CTA(+)	CTAB solutions	$70^\circ$	—	$10^2 - 10^3 \text{ s}^{-1}$	10 nm
Cheng [30]	Glass+photoresist	Water	—	5 $\text{\AA}$ (pp)	$10^2 - 10^4 \text{ s}^{-1}$	no-slip
"	"	Hexane	—	"	"	10 nm
"	"	Hexadecane	—	"	"	25 nm
"	"	Decane	—	"	"	15 nm
"	"	Silicon Oil	—	"	"	20 nm
Cheikh [29]	Poly(carbonate)+PVP	SDS solutions	$< 90^\circ$	—	$0 - 10^5 \text{ s}^{-1}$	20 nm
Choi [32]	Silicon	Water	$\approx 0^\circ$	11 $\text{\AA}$ (rms)	$10^3 - 10^5 \text{ s}^{-1}$	0 – 10 nm
	Silicon+OTS	Water	$> 90^\circ$	3 $\text{\AA}$ (rms)	"	5 – 35 nm

Microfluidics: The No-Slip Boundary Condition, 2005

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## Outline



- Why microchannels
- **Macro-to-microscale transition criteria**
- Confinement number approach
- Bubble departure diameter approach
- Young-Laplace equation approach

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## Macro-to-microscale transition criteria



- What happens in micro-channels in two-phase flows can be quite different from single-phase flows.
- Macroscale methods for **single-phase flow** work well down to at least diameters of **5~10 micros** and hence channels larger than this size can still be considered to macroscale.
- Macroscale **two-phase flow** methods usually do not work well at all when compared to data for channels below about **3mm diameter**. It's very risky to extrapolate macroscale two-phase **flow pattern maps**, flow boiling heat transfer methods and two-phase pressure drop models to microchannels, except for specific documented cases.

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## Macro-to-micro transition criteria

- **lower boundary of the macroscale:** A well-proven criterion is required for predicting the limiting diameter **above** which **macroscale** methods and theory are applicable.
- **upper boundary of the microscale:** A well-proven criterion for predicting the limiting diameter **below** which **microscale** methods and theory are reliably applicable.
- **Mesoccale:** Methods are also required for predicting flow and thermal performance in channel sizes falling in between ().
- A further **nanoscale** two-phase flow and heat transfer domain may emerge as well for even smaller sizes of channels where yet other two-phase mechanisms control the process.

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## Macro-to-micro transition criteria

Gas	Channel dimensions ( $\mu\text{m}$ )			
	Continuum flow	Slip flow	Transition flow	Free molecular flow
Air	>67	0.67–67	0.0067–0.67	<0.0067
Helium	>194	1.94–194	0.0194–1.94	<0.0194
Hydrogen	>123	1.23–123	0.0123–1.23	<0.0123

Heat Transfer and fluid flow in mini channels and micro channels.  
Kandlikar S. et al. 2006

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## Macro-to-micro transition criteria



- As of today, there appears to be **no exact definition or proven criterion available** for definitively distinguishing the transition between macroscale and microscale for two-phase flows and heat transfer.
- International Conference on Microchannels and Minichannels:

{	<i>macrochannels</i>	$> 3mm$	Kandlikar, 2001
	<i>minichannels</i>	$0.6 \sim 3.0mm$	
	<i>microchannels</i>	$< 0.6mm$	

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## Macro-to-micro transition criteria



{	<i>macrochannels</i>	$> 6mm$	Mehendale and Jacobi, 2000
	<i>compact</i>	$1 \sim 6mm$	
	<i>mesoscale</i>	$100\mu m \sim 1mm$	
	<i>microscale</i>	$1\mu m \sim 100\mu m$	
{	<i>conventional channels</i>	$> 3mm$	Kandlikar and Grande, 2003
	<i>minichannels</i>	$200\mu m \sim 3mm$	
	<i>microchannels</i>	$10\mu m \sim 200\mu m$	
	<i>nanoscale</i>	$< 10\mu m$	

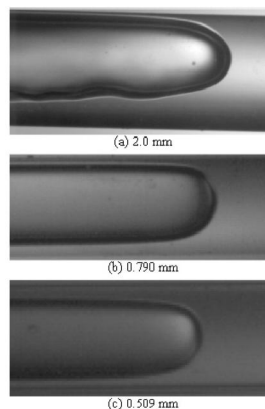
based merely on **dimensions** and not on **physical behavior**

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## Evidence – Flow pattern



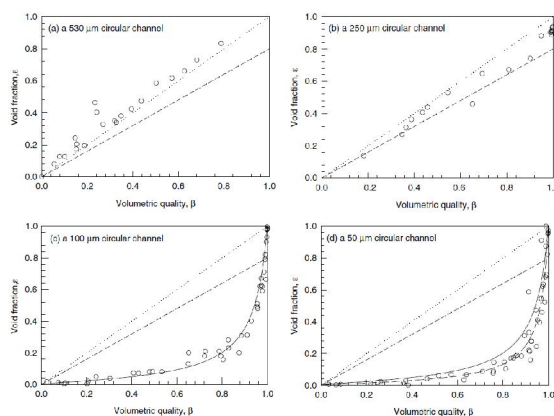
- In the 2.0mm channel, **no stratified flow** was observed while the difference in **film thickness** at the top compared to that at the bottom is quite noticeable.
- Similarly, the **film thickness** in the 0.79mm channel is still not uniform above and below the bubble.
- In contrast, in the 0.509mm channel, **the film is quite uniform**.
- **Stratified-wavy and fully stratified flows disappear** in small horizontal channels. This transition is perhaps an indication of the **lower boundary** of macroscopic two-phase flow (>2mm), while the **upper boundary** of microscale two-phase flow may be interpreted as the point in which the **effect of gravity becomes insignificant**.



The influence of channel size on the buoyancy effect on the elongated bubble with R134a at 30°C (Latif Jiji. Heat Convection)

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## Evidence – Void fraction



Chung & Kawaji, International Journal of Multiphase Flow  
30 (2004) 735–761

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$$\varepsilon = \frac{C_1 \beta^{0.5}}{1 - C_2 \beta^{0.5}}$$

The physical reason why the relationship between  $\varepsilon$  and  $\beta$  in a microchannel deviates from the conventional linear relationship for a minichannel may be explained as follows. In a microchannel, between  $\beta = 0$  and 0.8, the void fraction data fell below traditional correlations developed for minichannels and conventional channels. At these volumetric qualities, short gas slugs and long liquid slugs appeared. The long liquid slugs would increase the pressure gradient which would in turn cause the short gas slugs to accelerate through the liquid evading capture by the video camera and decreasing their residence time in the microchannel, which results in a low time-averaged void fraction.





## Outline

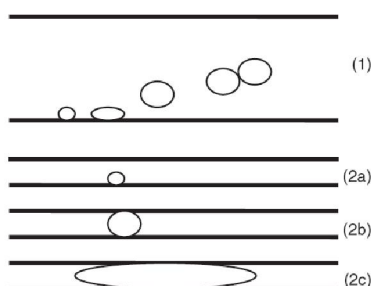
- Why microchannels
- Macro-to-microscale transition criteria
- **Confinement number approach**
- Bubble departure diameter approach
- Young-Laplace equation approach

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## Macro-to-micro transition criteria

- **Confinement number approach**
- The bubble confinement approach is based on the confined growth of a bubble in small channels (Ong & Thome, 2011).



Heat Transfer and fluid flow in mini channels and micro channels.  
Kandlikar S. et al. 2006

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## Macro-to-micro transition criteria



- **Confinement number approach**
- In microchannels of the order of **a few  $\mu\text{m}$  to a few tens  $\mu\text{m}$** , two-phase flow is believed to be influenced mainly by **surface tension, viscosity and inertia forces**. (Serizawa et al. 2002)
- Suo and Griffith (1964) derived the following criterion for tube diameter D:

$$Co = \frac{\lambda}{D} = \frac{1}{D} \sqrt{\frac{\sigma}{(\rho_l - \rho_g)g}} \geq 3.3$$

$\lambda$ : Laplace constant: 2.7 mm for air–water flow at 0.1 Mpa (D = 818 $\mu\text{m}$ )

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## Macro-to-micro transition criteria



- **Confinement number approach**
- In the microscale, the influence of gravity is surpassed by that of surface tension, i.e., no stratified flow exists if the tube diameter is sufficiently small.

$$B_o = \frac{\rho a L^2}{\sigma} = \frac{(\rho_l - \rho_g)g d^2}{\sigma} < 4 \quad d_h < 2 \left( \frac{\sigma}{(\rho_l - \rho_g)g} \right)^{1/2}$$

Kew and Connwell, 1997

- This is apparently the **first criterion** proposed for the threshold to confined bubble flow and perhaps is thus **the first macro-to-microscale two-phase flow transition criterion**. Notably, this criterion also does not contain any flow forces, i.e., no superficial velocities nor shear effects on the bubble or annular films.

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## Macro-to-micro transition criteria



- **Confinement number approach**
- Kew and Connwell (1997) in fact proposed **bubble growth confinement within a channel as the parameter** to distinguish the macro-to-microscale transition in the form of the Confinement number:

$$Co = \frac{1}{D} \sqrt{\frac{\sigma}{(\rho_l - \rho_g)g}} \begin{cases} < 0.5 & \text{macro scale} \\ > 0.5 & \text{microscale} \end{cases}$$

- based on the flow boiling experiments for single channels with internal diameters of 1.10, 1.80, 2.80 and 3.60 mm and for a square channel of  $2.0 \times 2.0$  mm.
- As an example, for R134a at  $T_{sat} = 0^\circ\text{C}$ , the threshold diameter is given by this criterion to be 1.92mm and becomes smaller as the saturation pressure increases.

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## Macro-to-micro transition criteria



**Table 6.2** Microchannel Transition Criteria

	Transition Diameter (mm) $T_{sat} = 50^\circ\text{C}$	
	$Co > 0.5$	$L > D_H$
R-134a	1.39	0.69
R-404A	0.91	0.46
Propane	2.01	1.01
Ammonia	3.26	1.63
Water	5.30	2.65

Heat Transfer and fluid flow in mini channels and micro channels.  
Kandlikar S. et al.2006

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## Macro-to-micro transition criteria



- Thus, the practical value of such categorizations is somewhat limited—even a change in operating temperature for the same fluid could affect whether the channel should be treated as a microchannel **through the influence of temperature on surface tension and other properties**.
- What is more important is to recognize and account for the respective flow phenomena in the development of **pressure drop** and **heat transfer** models.
- For example, substantial differences due to channel size have been documented by several researchers at hydraulic diameters of a few millimeters, as noted by Palm (2001).

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## Macro-to-micro transition criteria



- **Confinement number approach**
- Kawaji and Chung (2003) reviewed the characteristics of two-phase flow and the available data and came up the following recommendation as a threshold criterion:

$$B_o = \frac{(\rho_l - \rho_g)gd^2}{\sigma} \ll 4 \quad Ca_L = \frac{\mu_L U_L}{\sigma} \ll 1$$

$$We_{SL} = \frac{\rho_l U_L^2 d}{\sigma} \ll 1 \quad Re_{SL} = \frac{\rho_l U_L d}{\mu_L} \ll 2000$$

$$We_{SG} = \frac{\rho_g U_G^2 d}{\sigma} \ll 1 \quad Re_{SG} = \frac{\rho_g U_G d}{\mu_G} \ll 2000$$

- where  $U_L$ ,  $U_G$  are superficial velocities

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## Macro-to-micro transition criteria



- **Confinement number approach**
- Ullmann and Brauner (2006) suggested the threshold to occur at  $E_o = 0.2$  (for air-water this corresponds to a diameter of about 0.35mm):

$$E_o = \frac{(\rho_l - \rho_g)gd^2}{8\sigma} < 0.2 \quad E_o = \frac{Bo}{8} = \frac{1}{8Co^2} \quad E_o \approx 0.2 \sim Co \approx 0.79$$

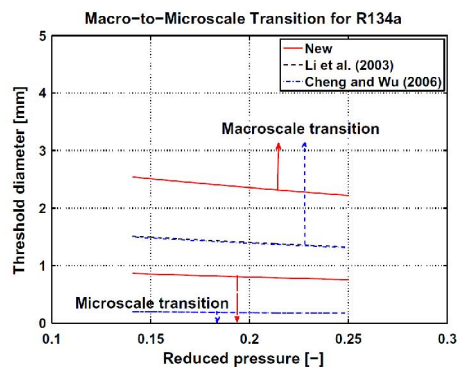
- $E_o$  number play an important role in flow pattern transitions and determining the characteristics length in **dispersed two-phase flows** in wall wetting effects in separated flows.
- Perhaps the earliest such transition criterion might be that of Bretherton (1961), who suggested a transition at  $E_o < 0.84$  as the threshold at which a Taylor bubble would no longer rise under only the influence of gravity in a vertical, water filled capillary tube.

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## Macro-to-micro transition criteria



- **Confinement number approach**
- Ong & Thome (2011) observed that the two-phase flow patterns and transitions is a function of channel confinement, mass velocity, saturation temperature and flow pattern.
- Fluid properties such as surface tension, phase densities, and viscosity were found to affect the flow pattern transitions.
- lower boundary of macroscale ( $Co < 0.3-0.4$ )
- Upper boundary of symmetric microscale flow ( $Co > 1.0$ )



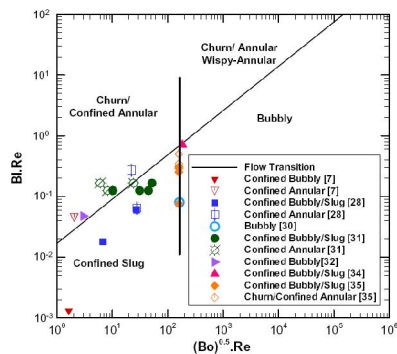
西安交通大学动力工程多相流国家重点实验室 陈斌 L. Ong, J.R. Thome, Experimental Thermal and Fluid Science 35 (2011) 37-47



## Macro-to-micro transition criteria

- **Confinement number approach**
- Harirchian & Garimella (2010) proposed a new criterion in term of **convective confinement number** that incorporates the effects of **mass flux**, as well as channel cross-sectional area and fluid properties:

$$\begin{aligned}
 & Bo^{0.5} \times Re \\
 &= \left( \frac{(\rho_l - \rho_g)gd^2}{\sigma} \right)^{0.5} \frac{Gd}{\mu_f} \\
 &= \left( \frac{(\rho_l - \rho_g)g}{\sigma} \right)^{0.5} \frac{Gd^2}{\mu_f} \leq 160
 \end{aligned}$$



西安交通大学动力工程多相流国家重点实验室 陈斌 T. Harirchian, S.V. Garimella / International Journal of Heat and Mass Transfer 53 (2010) 2694–2702



## Macro-to-micro transition criteria

- **Bubble departure diameter approach**

$$d_{bub} = 0.0208\beta \left( \frac{\sigma}{(\rho_l - \rho_g)g} \right)^{1/2} \quad \text{Fritz, 1935}$$

( $\beta$  is contact angle in  $^\circ$ )

$$d_{bub} = \left[ 0.12 + 0.08 \left( \frac{c_{pL}T_{sat}}{h_{LG}} \right)^{2/3} \right] \left( \frac{\sigma}{(\rho_l - \rho_g)g} \right)^{1/2} \quad \text{Nishikawa et al., 1976}$$

$$d_{bub} = [0.25(1 + 10^5 K_1)^{1/2}] \left( \frac{\sigma}{(\rho_l - \rho_g)g} \right)^{1/2} \quad \text{Kutateladze and Gogonin, 1979}$$

$$K_1 = \left( \frac{\rho_L c_{pL} (T_w - T_{sat})}{\rho_G h_{LG} Pr_L} \right)^2 \left[ \frac{\mu_L^2 [(\rho_l - \rho_g)g]^{1/2}}{\rho_L \sigma^{2/3}} \right]$$

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## Macro-to-micro transition criteria



- **Bubble departure diameter approach**
- It's interesting to notice the role played by the **Bond number** in all these relations, which are only strictly valid for **pool boiling**.
- The flow in microchannel will tend to **promote the detachment of the bubble** before it completely spans the channel. Thus, a macro-to-microscale criterion set equal to the passive bubble detachment diameter will probably **overestimate the value of the channel diameter** of the macro-to-microscale threshold.

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## Macro-to-micro transition criteria



- **Young-Laplace equation approach**

$$d_{crit} = 0.224L_{cap} \quad L_{cap} = \left( \frac{\sigma}{(\rho_l - \rho_g)g} \right)^{1/2} \quad \text{Li and Wang, 2003}$$

$$d_h = 1.75L_{cap}$$

{	<i>Effect of gravity is insignificant</i>	$d \leq d_{crit} (Co > 4.46)$
	<i>Effect of gravity is visible but surface tension is dominated</i>	$d_{crit} \leq d \leq d_{th}$ ( $0.57 < Co < 4.46$ )
	<i>Effect of surface tension will be small with respect to gravity</i>	$d \geq d_{th} (Co < 0.57)$

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# Young-Laplace equation



Thomas Young  
1773 – 1829

English polymath

who developed the qualitative theory of surface tension in 1805



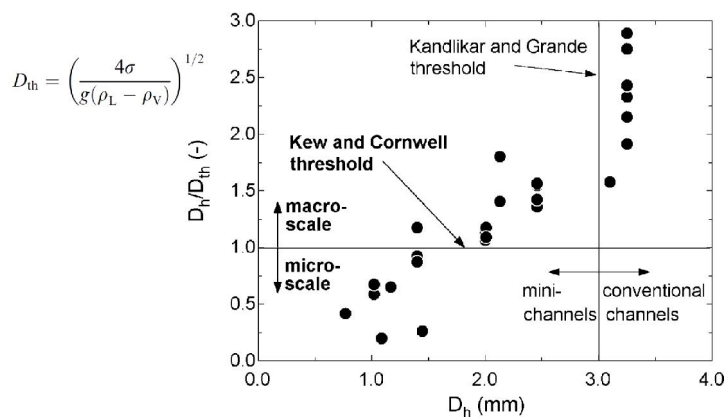
Pierre-Simon Laplace  
1749 – 1827

French mathematician and astronomer

who completed the mathematical description in the following year

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# Macro-to-micro transition criteria



Comparison between the experimental hydraulic diameters,  $D_h$ , included in the present pressure drop database and the threshold diameter criteria of Kew and Cornwell [1997], given by  $D_h/D_{th} = 1$ , and of Kandlikar and Grande [2003], given by  $D_h = 3$  mm. (Experimental Thermal and Fluid Science 31 (2006) 1–19)

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## Macro-to-micro transition criteria



Table 1. The different criteria for small tubes

Parameters	Air / water	R-134a		
		0.60	1.00	1.40
Pressure (MPa)	0.10	0.60	1.00	1.40
Temperature (°C)	25.0	21.6	39.4	52.5
	Critical Diameter (mm)			
Criterion based on $E\delta=1$	17.1	5.3	4.7	4.3
Criterion based on $Co=0.5$	5.4	1.7	1.5	1.4
Criterion based on $E\delta=100$	1.71	0.53	0.47	0.43
Criterion based on $Bo=0.3$	0.81	0.25	0.23	0.20

L. Chen, Y. S. Tian, and T. G. Karayiannis.

The Effect of Tube Diameter on Vertical Two-phase Flow Regimes in Small Tubes

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## Exercise 04



- Compare the transition diameter for air-water two-phase flow in 25°C under different macro-to-micro transition:
- $\sigma = 0.072\text{N/m}$ ,  $\rho_g = 1.293\text{kg/m}^3$

$$Co = \frac{1}{D} \sqrt{\frac{\sigma}{(\rho_l - \rho_g)g}} \begin{cases} < 0.5 & \text{macro scale} \\ > 0.5 & \text{microscale} \end{cases}$$

$$Bo = \frac{\rho_l a L^2}{\sigma} = \frac{(\rho_l - \rho_g)g d^2}{\sigma} < 4$$

$$Eo = \frac{(\rho_l - \rho_g)g d^2}{8\sigma} < 0.2$$

$$Bo^{0.5} \times Re \leq 160$$

$$d_{crit} = 0.224 L_{cap}, \text{ where } L_{cap} = \left( \frac{\sigma}{(\rho_l - \rho_g)g} \right)^{1/2}$$

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**To be continued.....**

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