Micro-scale two-phase flow dynamics

Lecture 05 Theoretical models for gas-liquid two-phase flow

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How to model multiphase flow

- **Choose a model**
  - What detail of the flow is required?
  - How many phases require modelling?
  - What is the geometry of the flow?
  - What is the relative motion between the different phases like?

How to model multiphase flow

- **Write down conservation equations**
  - Mass
  - Momentum- what are the significant forces on the flow components? Is the flow unsteady?
  - Energy- Required when there are significant variation in temperature or if there are important temperature related phenomena such as phase change.

How to model multiphase flow

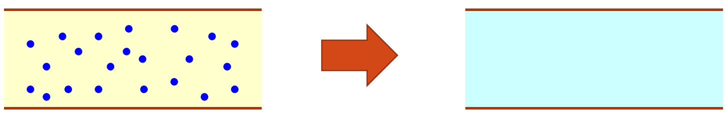
- **Determine the constitutive relations**
  - These specify how the components of a flow behave and interact with one another.
  - Can be very difficult to achieve and often they are flaky (不可靠).
  - Most difficult to describe are terms for friction and the interaction between the different phases.

Outline

- Homogeneous flow model
- Separated flow model
- Drift-flux model
- Two-fluid model
- CFD

The homogeneous flow model

- In the homogeneous model, both liquid and vapor phases move at the same velocity (slip ratio = 1). Consequently, the homogeneous model has also been called the zero slip model. The homogeneous model considers the two-phase flow as a single phase flow having average fluid properties depending on quality.



- The central assumption of this model is that the two phases travel at equal velocities and mix well. Therefore, they can be treated as if there is only one phase.

The homogeneous flow model

- The slip between the phases must be small i.e. $u_l \sim u_g$ or the slip ratio $u_g/u_l \sim 1$. Often true when $\rho_f/\rho_d > 10$ or G≥2000kg/m^{2}s, so that the flow regime is either bubbly or misty flow (adiabatic). This model also works better for two-phase flow near the critical point, where the differences between the properties of the liquid and vapor are insignificant.

- Using the homogeneous modeling approach, the frictional pressure gradient can be calculated using formulas from single-phase flow theory using mixture properties. For flow in pipes and channels, it can be obtained using the familiar equations:

$$f = \frac{d}{2\rho U^2}\left(\frac{dp}{dz}\right)_f$$

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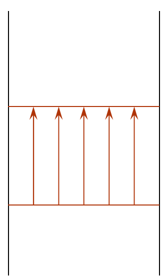
The homogeneous flow model

- New combined 'mixture' properties (e.g. ρ, μ) have to be defined

$$x = \frac{\dot{m}_g}{\dot{m}}$$

$$\dot{Q} = \frac{\dot{m}}{\dot{\rho}} = \frac{\dot{m}_g}{\rho_g} + \frac{\dot{m}_l}{\rho_l} \qquad \frac{1}{\rho} = \frac{x}{\rho_g} + \frac{1-x}{\rho_l}$$

$$\alpha = \frac{V_g}{V_g + V_l} = \frac{\dot{m}_g/\rho_g}{\dot{m}_g/\rho_g + \dot{m}_l/\rho_l} = \frac{x}{x + (1-x)\,\rho_g/\rho_l}$$



一维一速度模型

$$\dot{m}" = \frac{\dot{m}_l + \dot{m}_g}{A} = \frac{\rho_l \langle w \rangle A_l + \rho_g \langle w \rangle A_g}{A} = \rho \langle w \rangle$$

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Darcy friction factor

$f = \tau_w / \frac{\rho u^2}{2}$

- In fluid dynamics, the Darcy–Weisbach equation is a phenomenological equation, which relates the head loss — or pressure loss — due to friction along a given length of pipe to the average velocity of the fluid flow. The equation is named after Henry Darcy and Julius Weisbach.
- The Darcy–Weisbach equation contains a dimensionless friction factor, known as the Darcy friction factor:

$$\Delta p = f_D \cdot \frac{L}{D} \cdot \frac{\rho u^2}{2}$$

  - *L* is the length of the pipe
  - *D* is the hydraulic diameter of the pipe
  - u is the mean velocity

Henry Philibert Gaspard Darcy
1803 –1858
French engineer

Julius Weisbach
1806 –1871
German mathematician and engineer

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Fanning friction factor

- The Fanning friction factor, named after John Thomas Fanning (1837–1911), is a dimensionless number used in fluid flow calculations.

$$\Delta p = f \cdot \frac{L}{D} \cdot 2\rho u^2$$

- The Darcy friction factor is four times the Fanning friction factor

$$f_D = 4f$$

- f_D is more commonly used by civil and mechanical engineers, and the Fanning factor, *f*, by chemical engineers

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Simple friction models

- *Hagen-Poiseuille model* (White, 2005)

$$fRe_{dh} = \begin{cases} 16 & circular \\ 24 & parallel\ plates \\ 14.23 & square \end{cases} \qquad Re_{dh} < 2300$$

- *Blasius model*
- For turbulent flow, the value of the Fanning friction factor cannot be predicted from the theory alone, but it must be determined experimentally. Dimensional analysis shows that the Fanning friction factor is a function of the Reynolds number (Re_{dh}) and relative roughness (ε / dh).
- For turbulent flow in smooth pipes (Blasius, 1912):

$$f = 0.079 Re_{dh}^{-0.25} \qquad 3000 < Re_{dh} < 10{,}000$$

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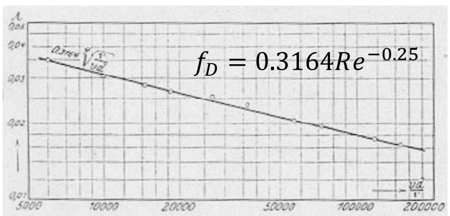
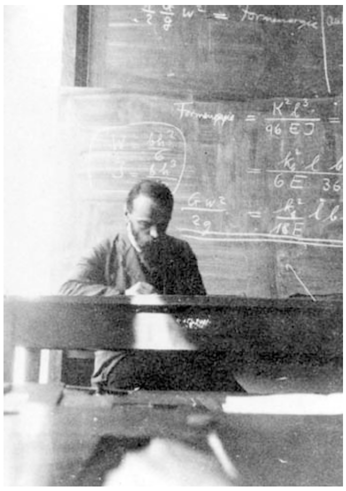
Paul Richard Heinrich Blasius (1883–1970)

- German fluid dynamics physicist, first Ph.D. student of Ludwig Prandtl



- Nobody would have assumed anything so simple, especially after the numerous and complex proposals made in the 19th century. Blasius was again the first to realize the significance of hydraulic similitude.



Blasius at Hamburg engineering college (1920)

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Simple friction models

- *Drew et al. model* (Drew et al.,1932)
- *Petukhov model* (Petukhov,1970)
- *Glielinski model* (Glielinski,1976)
- *Swamee and Jain model* (Swamee and Jain, 1976)
- ***Churchill model* (Churchill, 1977)**
- *Phillips model* (Phillips, 1987)
- *García et al. model* (García et al., 2003)
- *Fang et al. model* (Fang et al., 2011)

$$f = 2\left[\left(\frac{8}{Re_{dh}}\right)^{12} + \frac{1}{(a+b)^{3/2}}\right]^{1/12}$$

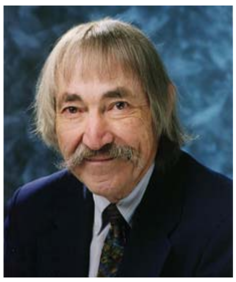
$$a = \left[2.457 \ln \frac{1}{\left(7/Re_{dg}\right)^{0.9} + \left(0.27\,\varepsilon/d_h\right)}\right]^{16}$$

$$b = \left(\frac{37530}{Re_{dh}}\right)^{16}$$

ε: roughness of the inner surface of the pipe (dimension of length)

Stuart W. Churchill

- president of AIChE, 1966
- Fellow - American Institute of Chemical Engineers (1971)
- National Academy of Engineering (1974)
- Founders Award - American Institute of Chemical Engineers (1980)
- Founders Award of National Academy of Engineering – 2002
- December 2, 2013, was honored at the AIChE Annual meeting



Stuart W. Churchill
1920-
Professor Emeritus
Chemical and Biomolecular Engineering

Penn
UNIVERSITY of PENNSYLVANIA

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Effective density models

- For the homogeneous model,

$$\rho_m = \alpha\rho_g + (1-\alpha)\rho_l = \left(\frac{x}{\rho_g} + \frac{1-x}{\rho_l}\right)^{-1}$$

- Dukler et al. (1964)

$$\rho_m = \rho_l \frac{(1-\alpha)^2}{H_l}\alpha + \rho_g \frac{\alpha^2}{1-H_l}$$

- Oliemans (1976)

$$\rho_m = \frac{\rho_l(1-\alpha) + \rho_g(1-H_l)}{(1-\alpha) + (1-H_l)}$$

- Ouyang (1998)

$$\rho_m = \rho_l H_l + \rho_g(1-H_l)$$

Effective viscosity models

- In the homogeneous model, the **mixture viscosity** for two-phase flows (μm) has received much attention in literature. The expressions available for the two-phase gas-liquid viscosity are mostly of an empirical nature as a function of mass quality *(x)*.
- important limiting conditions:

$$\begin{cases} \mu_m = \mu_l & x = 0 \\ \mu_m = \mu_g & x = 1 \end{cases}$$

- In the two-phase homogeneous model, the selection of a suitable definition of two-phase viscosity is inevitable as the Reynolds number would require this as an input to calculate the friction factor.

Researcher	Model
Arrhenius (1887)	$\mu_m = \mu_l^{1-\alpha_m} \mu_g^{\alpha_m}$
Bingham (1906)	$\mu_m = \left(\frac{1-\alpha_m}{\mu_l} + \frac{\alpha_m}{\mu_g} \right)^{-1}$
MacAdams et al. (1942)	$\mu_m = \left(\frac{x}{\mu_g} + \frac{1-x}{\mu_l} \right)^{-1}$
Davidson et al. (1943)	$\mu_m = \mu_l \left[1 + x \left(\frac{\rho_l}{\rho_g} - 1 \right) \right]$
Vermeulen et al. (1955)	$\mu_m = \frac{\mu_l}{\alpha_m} \left[1 + \left(\frac{1.5\mu_g (1-\alpha_m)}{\mu_l + \mu_g} \right) \right]$
Akers et al. (1959)	$\mu_m = \mu_l \left[(1-x) + x \left(\frac{\rho_l}{\rho_g} \right)^{0.5} \right]^{-1}$
Hoogendoorn (1959)	$\mu_m = \mu_l^{H_l} \mu_g^{1-H_l}$
Cicchitti et al. (1960)	$\mu_m = x\mu_g + (1-x)\mu_l$
Bankoff (1960)	$\mu_m = H_l \mu_l + (1-H_l)\mu_g$

Owen (1961)	$\mu_m = \mu_l$
Dukler et al. (1964)	$\mu_m = \rho_m \left[x \frac{\mu_g}{\rho_g} + (1-x) \frac{\mu_l}{\rho_l} \right]$
Oliemans (1976)	$\mu_m = \frac{\mu_l (1-\alpha_m) + \mu_g (1-H_l)}{(1-\alpha_m) + (1-H_l)}$
Beattie and Whalley (1982)	$\mu_m = \mu_l (1-\alpha_m)(1+2.5\alpha_m) + \mu_g \alpha_m$ $= \mu_l - 2.5\mu_l \left(\frac{x\rho_l}{x\rho_l + (1-x)\rho_g} \right)^2 + \left(\frac{x\rho_l (1.5\mu_l + \mu_g)}{x\rho_l + (1-x)\rho_g} \right)$
Lin et al. (1991)	$\mu_m = \frac{\mu_l \mu_g}{\mu_g + x^{1.4}(\mu_l - \mu_g)}$
Fourar and Bories (1995)	$\mu_m = \rho_m \left(\sqrt{x \frac{\mu_g}{\rho_m}} + \sqrt{(1-x) \frac{\mu_l}{\rho_l}} \right)^2$
García et al. (2003, 2007)	$\mu_m = \mu_l \left(\frac{\rho_m}{\rho_l} \right) = \frac{\mu_l \rho_g}{x\rho_l + (1-x)\rho_g}$

Researcher	Model
Awad and Muzychka (2008) Definition 1	$\mu_m = \mu_l \frac{2\mu_l + \mu_g - 2(\mu_l - \mu_g)x}{2\mu_l + \mu_g + (\mu_l - \mu_g)x}$
Awad and Muzychka (2008) Definition 2	$\mu_m = \mu_g \frac{2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-x)}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-x)}$
Awad and Muzychka (2008) Definition 3	$(1-x)\frac{\mu_l - \mu_m}{\mu_l + 2\mu_m} + x\frac{\mu_g - \mu_m}{\mu_g + 2\mu_m} = 0$
Awad and Muzychka (2008) Definition 4	$\mu_m = \left[\frac{\mu_l}{2}\frac{2\mu_l + \mu_g - 2(\mu_l - \mu_g)x}{2\mu_l + \mu_g + (\mu_l - \mu_g)x} + \frac{\mu_g}{2}\frac{2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-x)}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-x)}\right]$
Muzychka et al. (2011) Definition 1	$\mu_m = \left[\mu_l \frac{2\mu_l + \mu_g - 2(\mu_l - \mu_g)x}{2\mu_l + \mu_g + (\mu_l - \mu_g)x} * \mu_g \frac{2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-x)}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-x)}\right]^{0.5}$
Muzychka et al. (2011) Definition 2	$\mu_m = \left[2\mu_l \frac{2\mu_l + \mu_g - 2(\mu_l - \mu_g)x}{2\mu_l + \mu_g + (\mu_l - \mu_g)x} * \mu_g \frac{2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-x)}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-x)}\right] / \left[\mu_l \frac{2\mu_l + \mu_g - 2(\mu_l - \mu_g)x}{2\mu_l + \mu_g + (\mu_l - \mu_g)x} + \mu_g \frac{2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-x)}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-x)}\right]$

Effective viscosity models

$$\mu_h = x\mu_g + (1-x)\mu_l$$ Cicchitti's equation

$$\mu_h = \beta\mu_g + (1-\beta)\mu_l$$ Dukler's equation

$$\frac{1}{\mu_h} = \frac{x}{\mu_g} + \frac{1-x}{\mu_l}$$ Isbin's equation

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Effective viscosity models

- In liquid-liquid two-phase flows, Taylor (1932) presented the effective viscosity for a dilute emulsion of two immiscible incompressible Newtonian fluids by

$$\mu_m = \mu_c\left[1 + 2.5\alpha\frac{\mu_d + 0.4\mu_c}{\mu_d + \mu_c}\right] = \mu_c\left[1 + \alpha\frac{1 + 2.5(\mu_d/\mu_c)}{1 + (\mu_d/\mu_c)}\right]$$

$$\begin{cases} \mu_m = \mu_c(1+\alpha) & \mu_d/\mu_c \ll 1 \\ \mu_m = \mu_c(1+2.5\alpha) & \mu_d/\mu_c \gg 1 \end{cases}$$

the well known Einstein model (1906, 1911). It is frequently used in prediction of nano fluid viscosity.

Effective viscosity models

researcher	formuler
A. Einstein (1905)	$\mu_m = \mu_c\ (1 + 2.5\alpha)$
Brinkman (1952)	$\mu_m = \mu_c\ (1 - \phi)^{2.5}$
Lundgren (1972)	$\mu_m = \mu_c\ (1 + 2.5\phi + 6.5\phi^2)$
Wang et l. (1999)	$\mu_m = \mu_c\ (1 + 7.3\phi + 123\phi^2)$
Tseng and Lin (2003): TiO_2/water	$\mu_m = \mu_c \times 13.47e^{35.98\phi}$
Chen et al. (2007)	$\mu_m = \mu_c\ (1 + 10.6\phi + (10.6\phi)^2)$
Nguyen et al. (2007) 47nm Al_2O_3 / water	$\mu_m = \mu_c \times 0.904e^{0.1482\phi}$
Masoumi et al. (2009) 13 and 28nm Al_2O_3 / water	$\mu_m = \mu_c + \frac{\rho P V_B d_p^2}{72C\delta}$ δ is the distance between the nanoparticles

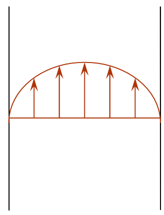
Latest developments on the viscosity of nanofluids. International Journal of Heat and Mass Transfer 55 (2012) 874–885

Bankoff variable density model

- It was at time when boiling-water nuclear reactors were just being developed, and the science of two-phase flow and heat transfer was still in its infancy (初级阶段). Mechanistic models were needed to describe the fact that in an upward or horizontal steam-water flow the steam flows faster than the water.
- My model, taking into account radial distributional effects, was the simplest that could be derived which quantified the steam-water velocity ratio, and at the same time enabled the designer to predict the frictional pressure drop.
- It is interesting, however, that shortly afterward, it led directly, by way of minor modifications, to a more famous model, due principally to Zuber, called the 'drift flux model. … it has been enshrined (铭记) in the two-phase flow literature, and is today probably the most important single concept in two-phase flow modeling.

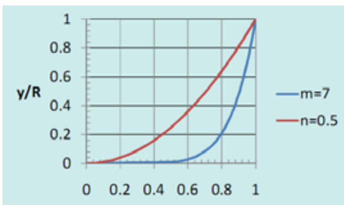
Bankoff variable density model

- 在垂直向上的较低含气率泡状流动中，气泡有聚集到流道中心区的趋势，流通截面上的空泡份额分布呈现为流道中心大、沿径向向外单调减小
- Bankoff 模型假设气相与液相间无滑移。流道截面中心区的速度快些、且气体密集，相平均速度高于液相平均速度，两相流体可视为密度是径向位置函数的单相流体，故称为变密度模型：



二维一速度模型

$$\frac{u}{u_{CL}} = \left(\frac{y}{R}\right)^{\frac{1}{m}} \qquad \frac{\alpha}{\alpha_{CL}} = \left(\frac{y}{R}\right)^{\frac{1}{n}}$$



Bankoff variable density model

$$Q_g = \int_0^R \alpha u 2\pi r dr = \int_0^R \alpha_{CL} u_{CL} \left(\frac{y}{R}\right)^{\frac{1}{m}} \left(\frac{y}{R}\right)^{\frac{1}{n}} 2\pi (R-y) dy$$

$$Q_g = \frac{2\pi R^2 \alpha_{CL} u_{CL} m^2 n^2}{(m+n+mn)(m+n+2mn)}$$

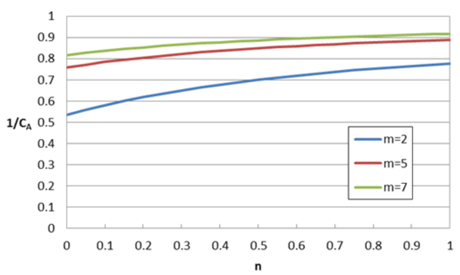
$$Q = \int_0^R u 2\pi r dr = \frac{2\pi R^2 u_{CL} m^2 n^2}{(1+m)(1+2m)}$$

$$\beta = \frac{\alpha_{CL}(1+m)(1+2m)n^2}{(m+n+mn)(m+n+2mn)}$$

Bankoff variable density model

$$\bar{\alpha} = \frac{1}{\pi R^2}\int_0^R \alpha 2\pi r dr = \int_0^R \alpha_{CL} \left(\frac{y}{R}\right)^{\frac{1}{n}} 2\pi r dr = \frac{2\alpha_{CL} n^2}{(1+n)(1+2n)}$$

$$\frac{\alpha}{\beta} = \frac{2(m+n+mn)(m+n+2mn)}{(1+m)(1+2m)(1+n)(1+2n)} = C_A$$

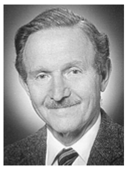


- for turbulent profiles, C_A hovers (徘徊) between 0.8 to 0.9
- Thus variable velocity and density profile is able to explain C_A being less than 1

$$\frac{\alpha}{\beta} = C_A = 0.833$$

S. George Bankoff

- a member of the National Academy of Engineering (NAE) : "For contributions to the field of two-phase flow and heat transfer and its application to nuclear-reactor thermohydraulics."
- won numerous awards, including the Ernest W. Thiele, Robert E. Wilson, Donald Q. Kern, and Heat Transfer and Energy Division awards from the American Institute of Chemical Engineers.
- "*This paper was written in the course of a summer appointment at Argonne National Laboratory, and I certainly never thought that it would be regarded as a classic piece of work.*"



S. George Bankoff
1921-2011
Professor Emeritus
Chemical Engineering

NORTHWESTERN UNIVERSITY

The homogeneous flow model

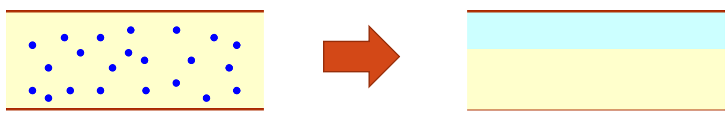
- The homogeneous flow model provides an easier approach to determining flow properties and behaviors, but it **underestimates the pressure drop**, particularly in a moderate pressure range.
- Furthermore, the homogeneous flow model is **less accurate** when velocity and flow conditions for both phases are **more dispersed**.

Outline

- Homogeneous flow model
- Separated flow model
- Drift-flux model
- Two-fluid model
- CFD

The separated flow model

- Two-phase flow is considered to be divided into liquid and vapor streams. Hence, the separated model has been referred to as the **slip flow model**.



- The phases are modelled separately with a set of mass, momentum, and energy equations for each phase.
- Terms are required to describe the interaction between the phases, i.e., the exchange of mass, momentum and energy over the boundaries.
- Geometry (the structure of the flow) is still lost.

The separated flow model

- The separated model was originated from the classical work of Lockhart and Martinelli (1949) that was followed by Martinelli and Nelson (1948).

- The Lockhart-Martinelli method is one of the best and simplest procedures for calculating two-phase flow pressure drop and hold up. One of the biggest advantages of the **Lockhart-Martinelli method is that it can be used for all flow patterns**. However, relatively **low accuracy** must be accepted for this **flexibility**.

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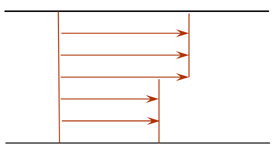
The separated flow model

- The homogeneous model lumps （结块） both phases together to provide a homogeneous flow. In the separated flow model, however, the flow of each phases is determined independently and the effects of the two phases are then summed.
- It allows two phases to have **different properties and one-dimensional velocities**, while the conservation equations are written for the combined flow.



一维二速度模型

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The separated flow model

- The separated model is popular in the power plant industry. Also, the separated model is relevant for the prediction of pressure drop in heat pump systems and evaporators in refrigeration. The success of the separated model is due to the basic assumptions in the model are closely met by the flow patterns observed in the major portion of the evaporators.
- For two-phase flow modeling in **microchannels and minichannels**, it should be noted that the literature review on this topic can be found in tabular form in a number of textbooks such as Celata (2004), Kandlikar et al. (2006), Crowe (2006), Ghiaasiaan (2008), and Yarin et al. (2009).

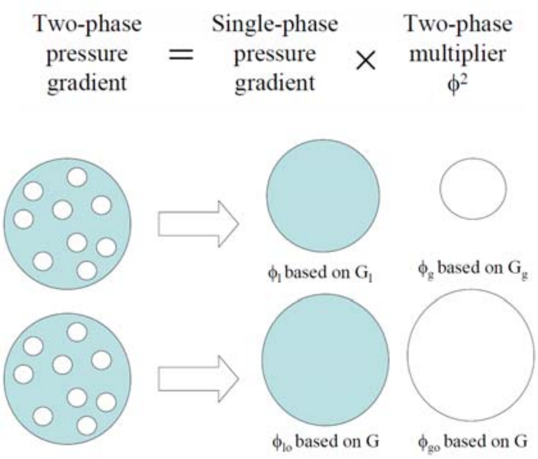
Lockhart-Martinelli model

- Lockhart and Martinelli (1949) presented data for the simultaneous flow of air and liquids including benzene (苯), kerosene (煤油), water, and different types of oils in pipes varying in diameter from 0.0586 in. to 1.017 in.

Liquid	Turbulent	Turbulent	Viscous	Viscous
gas	Turbulent	Viscous	Turbulent	Viscous

- The data used by Lockhart and Martinelli consisted of experimental results obtained from a number of sources as detailed in their original paper and covered 810 data sets including 191 data sets that are for inclined and vertical pipes and 619 data sets for horizontal flow (Cui and Chen (2010))

Lockhart-Martinelli model



Different sorts of two-phase multipliers

Lockhart-Martinelli model

- Lockhart and Martinelli (1949) correlated the pressure drop resulting from these different flow mechanisms by means of the Lockhart-Martinelli parameter *(X)*:

$$\chi^2 = \frac{(dp/dz)_{f,l}}{(dp/dz)_{f,g}}$$

- In addition, they expressed the two-phase frictional pressure drop in terms of factors, which multiplied single-phase drops. These multipliers

$$\phi_l^2 = \frac{(dp/dz)_f}{(dp/dz)_{f,l}} \qquad \phi_g^2 = \frac{(dp/dz)_f}{(dp/dz)_{f,g}}$$

Lockhart-Martinelli model

Flow Condition	X
turbulent-turbulent	$X_{tt}^2=\left(\frac{1-x}{x}\right)^{1.8}\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right)^{0.2}$
laminar-turbulent	$X_{lt}^2=\mathrm{Re}_g^{-0.8}\left(\frac{C_l}{C_g}\right)\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right)$
turbulent- laminar	$X_{tl}^2=\mathrm{Re}_l^{0.8}\left(\frac{C_l}{C_g}\right)\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right)$
laminar-laminar	$X_{ll}^2=\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right)$

Lockhart-Martinelli model

- Although the Lockhart-Martinelli correlation related to the adiabatic flow of low pressure air-liquid mixtures, they purposely presented the information in a generalized form to enable the application of the model to single component systems, and, in particular, to **steam-water mixtures**. Their empirical correlations were shown to be as reliable as any annular flow pressure drop correlation (Collier and Thome, 1994).
- The disadvantage of this method was its limit to **small-diameter pipes** and **low pressures** because many applications of two-phase flow fell beyond these limits.
- Since Lockhart and Martinelli published their paper on two-phase or two-component flows in 1949 to define the methodology for presenting two-phase flow data in non-boiling and boiling flows, their paper has received nearly **1000 citations in journal papers** alone is a testament (确证) to its contribution to the field of two-phase flow.

Turner model

- In his Ph. D. thesis, Turner (1966) developed the **separate-cylinder model** by assuming that the **two-phase flow, without interaction, in two horizontal separate cylinders** and that that the areas of the cross sections of these cylinders added up to the cross-sectional area of the actual pipe.
- The liquid and gas phases flow at the same flow rate through separate cylinders. **The pressure gradient in each of the imagined cylinders was assumed to be equal**, and its value was taken to be equal to the two-phase frictional pressure gradient in the actual flow. For this reason, the separate-cylinder model was **not valid for gas-liquid slug flow**, which gave rise to large pressure fluctuations.
- The pressure gradient was due to frictional effects only, and was calculated from single-phase flow theory.
- The separate cylinder model resembled Lockhart and Martinelli correlation (1949) but had the advantage that it could be pursued to an analytical conclusion.
- The method is still widely accepted because of its simplicity.

Turner model

$$\left(\frac{1}{\phi_l^2}\right)^{\frac{1}{n}} + \left(\frac{1}{\phi_g^2}\right)^{\frac{1}{n}} = 1$$

Flow Type	*n*
Laminar Flow	2
Turbulent Flow (analyzed on a basis of friction factor)	2.375~2.5
Turbulent Flows (calculated on a mixing-length basis)	2.5~3.5
Turbulent-Turbulent Regime	4
All Flow Regimes	3.5

$$\begin{cases} f = 0.079/Re^{0.25} & n = 2.375 \\ f = 0.046/Re^{0.20} & n = 2.4 \\ f = const & n = 2.5 \end{cases}$$

Chisholm model

- Chisholm (1967) proposed a **more rigorous analysis** that was an extension of the Lockhart-Martinelli model, except that a **semi-empirical closure** was adopted. Chisholm's rationale (原理、论据) for his study was the fact that the Lockhart-Martinelli model failed to produce suitable equations for predicting the two-phase frictional pressure gradient, given that the empirical curves were only presented in graphical and tabular form.

$$\Phi^2 = 1 + \frac{C}{X} + \frac{1}{X^2}$$

$$X = \left[\frac{(dp/dz)_L}{(dp/dz)_G}\right]^{1/2}$$

$$\Delta p_{frict} = \Delta p_L \cdot \Phi^2$$

Liquid	Gas	C
Turbulent	Turbulent	20
Laminar	Turbulent	12
Turbulent	Laminar	10
Laminar	Laminar	5

Chisholm model

$$\phi_l^2 = \left(\frac{dp}{dz}\right)_{f,tp} = \left(\frac{dp}{dz}\right)_{f,l} + C\left[\left(\frac{dp}{dz}\right)_{f,l}\left(\frac{dp}{dz}\right)_{f,g}\right]^{0.5} + \left(\frac{dp}{dz}\right)_{f,g}$$

- The physical meaning is that the two-phase frictional pressure gradient is the sum of three components: **the frictional pressure of liquid-phase alone**, **the interfacial contribution to the total two-phase frictional pressure gradient**, and **the frictional pressure of gas-phase alone**.

Chisholm model

$$\left(\frac{dp}{dz}\right)_i = C\left[\left(\frac{dp}{dz}\right)_{f,l}\left(\frac{dp}{dz}\right)_{f,g}\right]^{0.5}$$

- Constant C in Chisholm's model can be viewed as a weighting factor for the geometric mean of the single-phase liquid and gas only pressure gradients.
- The Chisholm parameter (C) is a **measure of two-phase interactions**. The larger the value, the greater the interaction, hence the Lockhart-Martinelli parameter (X) can involve ll, tl, lt, and tt regimes. It just causes the data to shift outwards on the Lockhart-Martinelli plot.

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Chisholm model

- The Chisholm constant (C) can be derived analytically for a number of special cases. For instance, Whalley (1996) obtained **for a homogeneous flow** having constant friction factor:

$$C = \left[\left(\frac{\rho_l}{\rho_g}\right)^{0.5} + \left(\frac{\rho_l}{\rho_g}\right)^{0.5}\right]$$

- that **for an air-water combination gives $C = 28.6$** that is in good agreement with Chisholm's value for turbulent-turbulent flows.
- Whalley (1996) shows that **for laminar and turbulent flows with no interaction between phases the values of $C = 2$ and $C = 3.66$** are obtained, respectively.

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Chisholm model

- Awad and Muzychka (2007, 2010b) mentioned that a value of $C = 0$ can be used as **a lower bound for two-phase frictional pressure gradient in minichannels and microchannels**.

$$\left(\frac{dp}{dz}\right)_{f,tp} = \left(\frac{dp}{dz}\right)_{f,l} + C\left[\left(\frac{dp}{dz}\right)_{f,l}\left(\frac{dp}{dz}\right)_{f,g}\right]^{0.5} + \left(\frac{dp}{dz}\right)_{f,g}$$

- which means there is no contribution to the pressure gradient through phase interaction.
- Using the homogeneous model with the Dukler et al. (1964) definition of two-phase viscosity for laminar-laminar flow leads to the same result.

Hemeida-Sumait model

- The Lockhart-Martinelli (1949) correlation in its present form cannot be used to study a large set of data because **it requires the use of charts and hence cannot be simulated numerically**.
- As a result, Hemeida and Sumait (1988) developed a correlation between Lockhart and Martinelli parameters ϕ and X for a two-phase pressure drop in pipelines using the Statistical Analysis System (SAS).

$$\phi = exp\begin{bmatrix} 2.303a + bLn(\chi) \\ + \frac{c}{2.30}\left(Ln(\chi)\right)^2 \end{bmatrix}$$

Parameter	a	b	c
$\phi_{g,ll}$	0.4625	0.5058	0.1551
$\phi_{g,lt}$	0.5673	0.4874	0.1312
$\phi_{g,tl}$	0.5694	0.4982	0.1255
$\phi_{g,tt}$	0.6354	0.4810	0.1135
$\phi_{l,ll}$	0.4048	0.4269	0.1841
$\phi_{l,lt}$	0.5532	-0.4754	0.1481
$\phi_{l,tl}$	0.5665	-0.4586	0.1413
$\phi_{l,tt}$	0.6162	-0.5063	0.124

Hemeida-Sumait model

- *Hemeida-Sumait model* enabled the development of a computer program for the analysis of data using the Lockhart-Martinelli (1949) correlation. Using this program, they analyzed field data from Saudi flow lines. The results showed that the improved Lockhart-Martinelli correlation predicted accurately the downstream pressure in flow lines with an average percent difference of 5.1 and standard deviation of 9.6%.
- It should be noted that the Hemeida-Sumait (1988) model is not famous in the literature like other models such as the Chisholm (1967) model although it gave an accurate prediction of two-phase frictional pressure gradient.

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Other models

- *Modified Turner model* (Awad and Muzychka (2004b))

$$\left(\frac{dp}{dz}\right)_f = \left[\left(\frac{dp}{dz}\right)_{f,l}^p + \left(\frac{dp}{dz}\right)_{f,g}^p\right]^{1/p} \qquad \phi_l^2 = \left[1 + \left(\frac{1}{X^2}\right)^p\right]^{1/p} \qquad \phi_g^2 = [1 + (X^2)^p]^{1/p}$$

- *Modified Chisholm models* (Saisorn and Wongwises (2008, 2009, and 2010))

$$\phi_l^2 = 1 + \frac{A}{X^m} + \frac{1}{X^2}$$

$$\phi_g^2 = 1 + AX^m + X^2$$

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Outline

- Homogeneous flow model
- Separated flow model
- Drift-flux model
- Two-fluid model
- CFD

The drift flux model

- The drift flux model is a type of separated flow model. In the drift flux model, attention is focused on the **relative motion** rather than on the motion of the individual phases.
- This model is cast in a form that corrects homogeneous model. A closure is found for α rather than s.
- The drift flux model has widespread application to **bubble flow** and **plug flow**. The drift flux model is not particularly suitable to a flow such as **annular flow that has two characteristic velocities in one phase**: the liquid film velocity and the liquid drop velocity.

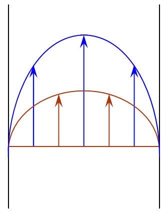
The drift flux model

- Drift velocity

$$u_d = u_{gj} = u_g - j$$

- Drift flux: Volume flux of gas relative a surface moving with a velocity j

$$j_{gl} = \alpha u_{gj} = \alpha(u_g - j) = \alpha u_g - \alpha j = j_g - \alpha j$$



二维二速度模型

$$j_g = \alpha j + j_{gl}$$

- The volume flux of gas = Conc. × average volume flux + relative velocity correction

The drift flux model

$$j_g = \alpha j + j_{gl}$$

$$\frac{j_g}{\alpha j} = 1 + \frac{j_{gl}}{\alpha j}$$

$$\Rightarrow \frac{\beta}{\alpha} = 1 + \frac{u_{gj}}{j} \xrightarrow{j\uparrow} 1$$

- Homogeneous model over-predicts the void fraction due to the neglect of void fraction distribution
- To define the local phase velocities, there is a need to appreciate the time averaged local parameters

The drift flux model

- Zuber-Findlay (1965) introduced the void weighted averages:

$$Q_g = \int_A \tilde{u}_g dA_g = \int_A \tilde{u}_g \tilde{\alpha} dA$$

- The phase averaged velocity is the weighted average denoted by:

$$\hat{u}_g = \frac{Q_g}{A_g} = \frac{\int_A \tilde{u}_g \tilde{\alpha} dA}{\int_A \tilde{\alpha} dA}$$

- For a general variable

$$\hat{F} = \frac{\frac{1}{A}\int_A \tilde{F}\tilde{\alpha} dA}{\frac{1}{A}\int_A \tilde{\alpha} dA} = \frac{\langle \tilde{F}\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle}$$

The drift flux model

$$\frac{j_g}{\alpha} = \hat{u}_g = \frac{\langle \tilde{u}_g \tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle} = \frac{\langle (\tilde{j} + \tilde{u}_{gj})\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle} = \frac{\langle \tilde{j}\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle} + \frac{\langle \tilde{u}_{gj}\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle}$$

$$\langle \tilde{j}\tilde{\alpha}\rangle \neq \langle \tilde{j}\rangle\langle\tilde{\alpha}\rangle \qquad C_0 = \frac{\langle \tilde{j}\tilde{\alpha}\rangle}{\langle \tilde{j}\rangle\langle\tilde{\alpha}\rangle}$$

$$\frac{j_g}{\alpha} = \frac{C_0\langle \tilde{j}\rangle\langle\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle} + \frac{\langle \tilde{u}_{gj}\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle} = C_0\langle \tilde{j}\rangle + \hat{u}_{gj}$$

$$\frac{\beta}{\alpha} = \frac{j_g}{\langle \tilde{j}\rangle\alpha} = C_0 + \frac{\hat{u}_{gj}}{\langle \tilde{j}\rangle}$$

流型		C_0
泡状	圆管 Di>50mm	1-0.5pr
	圆管 De<50mm	1.2(pr<0.5)
	圆管 De<50mm	1.4-0.4pr (pr>0.5)
	矩形管	1.4-0.4pr
弹状		1.2
环状		1.0

Distribution coefficient C_0

Vertical channel

Condition	C_0	u_{gj}	author
Elevated pressures	1.13	$1.41\left[\frac{\sigma g(\rho_l-\rho_g)}{\rho_l^2}\right]^{1/4}$	Zuber et al. 1967
Bubble flow (vertical, no coalescence)	1.0	$1.53\left[\frac{\sigma g(\rho_l-\rho_g)}{\rho_l^2}\right]^{1/4}$	Wallis 1969
Slug flow	1.2	$0.35\left[\frac{g(\rho_l-\rho_g)d_i}{\rho_l^2}\right]^{1/4}$	
Annular flow	1.0	$23\left(\frac{\mu_l U_l}{\rho_g d_i}\right)\frac{(\rho_l-\rho_g)}{\rho_l}$	Ishii et al. 1976
	1.1 (j>200kg/m^2s) 1.54 (j<200kg/m^2s)	$1.18\left[\frac{g(\rho_l-\rho_g)}{\rho_l^2}\right]^{1/4}$	Rouhani & Axelsson 1970

Distribution coefficient C_0

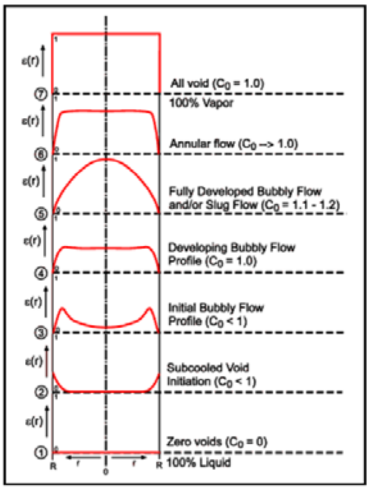
$$\frac{u}{u_c}=1-\left(\frac{r}{R}\right)^m$$

$$\frac{\alpha-\alpha_w}{\alpha_c-\alpha_w}=1-\left(\frac{r}{R}\right)^n$$

$$C_0=1+\frac{2}{m+n+2}\left[1-\frac{\alpha_w}{\langle\alpha\rangle}\right]$$

$$=\frac{m+2}{m+n+2}\left[1+\frac{\alpha_c}{\langle\alpha\rangle}\left(\frac{n}{m+2}\right)\right]$$

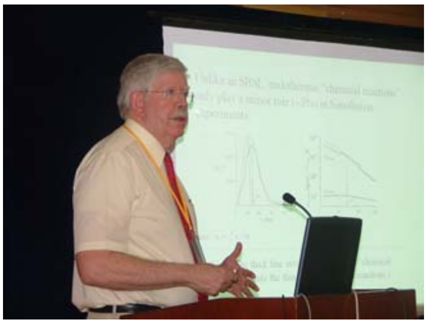
$$C_0\begin{cases}=1 & \alpha_c=\alpha_w=\langle\alpha\rangle\\ >1 & \alpha_c>\alpha_w\\ <1 & \alpha_c<\alpha_w\\ \frac{n+2}{n+1} & m=n,\alpha_w=0\end{cases}$$



Lahey 1974

Richard T. Lahey

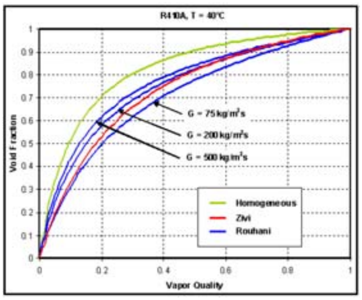
- **<u>Dr. Richard T. Lahey, Jr.</u>**
- *The Edward E. Hood*
  Professor Emeritus of Engineering
- Rensselaer Polytechnic Institute
- member of the National Academy of Engineering (NAE), the Russian Academy of Science-Bashkortostan (RAS), a Fellow of the ANS and ASME

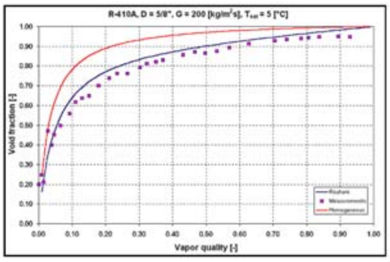


Distribution coefficient C_0

Horizontal channel

C_0	u_{gj}	author
$1+c_0(1-x)$	$1.18(1-x)\left[\frac{\sigma g(\rho_l-\rho_g)}{\rho_l^2}\right]^{1/4}$	Steiner 1993





R410A, Wojtan et al. 2003

The drift flux model

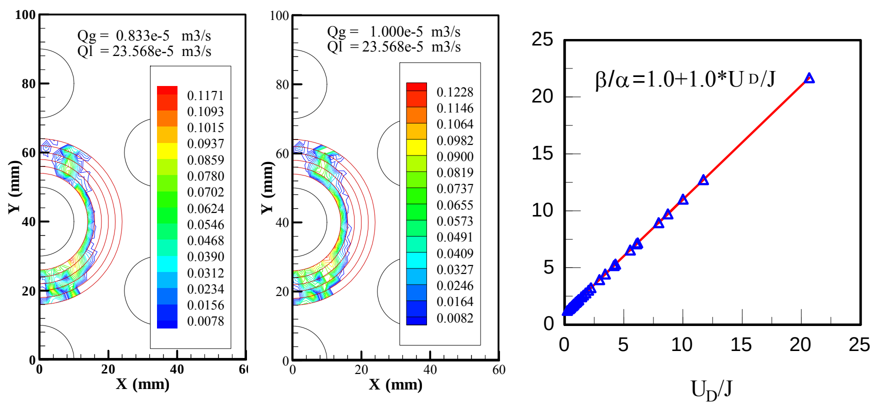
- The drift flux model reduces to the homogeneous void fraction when C_0=1 and either u_{gj} = 0 or the mass velocity becomes very large
- The drift flux model has widespread application to **bubble flow** and **plug flow**. The drift flux model is not particularly suitable to a flow such as **annular flow that has two characteristic velocities in one phase**: the liquid film velocity and the liquid drop velocity.
- The terms 'drift flux' and 'drift velocity' had been previously introduced by Graham Wallis (1969), who developed the continuity relationships and introduced the idea of continuity waves into two-phase flow.

Dr. Novak Zuber

- Dr. Zuber was a Montenegrin (黑山), a member of the Balkan Air Force. The unit was active from June 1944 until July 1945 under the command of Royal Air Vice Marshalls William Elliot and George Mills.
- He found his way to the University of California at Los Angeles and was able to enroll in their mechanical engineering program. In order to complete his education, he performed odd jobs such as washing dishes, washing cars or gardening.
- He was the first recipient of ASME Heat Transfer Division's Memorial Award in 1961, the Technical Achievement Award from the Thermal Hydraulics Division of the American Nuclear Society in 1990, fellow of the ASME.

Novak Zuber
1922-2013
General Electric
New York University
Georgia Institute of Technology
U.S. Nuclear Regulatory Commission

The drift flux model



The drift flux model

- Ishii 1977

$$V_{dj} = j - \hat{v}_d = v_\infty (1 - \alpha_d)^n - \frac{D_d^\alpha}{\alpha_d} \nabla \alpha_d$$

- where D_d^α is the drift coefficient based on the void fraction α_d. The first term on the right-hand side takes into account for the *effect of gravity and forces which is usually the dominant part of the drift velocity*.

- v_∞ is the terminal velocity of a single particle in an infinite medium

The drift flux model

- For a viscous regime

$$V_{dj} \simeq 10.8\left(\frac{\mu_c g \Delta\rho}{\rho_c^2}\right)^{1/3} \frac{(1-\alpha_d)^{1.5} f(\alpha_d)}{r_d^*} \times \frac{\psi(r_d^*)^{4/3}\{1+\psi(r_d^*)\}}{1+\psi(r_d^*)\{f(\alpha_d)\}^{6/7}}$$

$$f(\alpha_d) = (1-\alpha_d)^{1/2}\frac{\mu_c}{\mu_m}$$

$$r_d^* \equiv r_d\left(\frac{\rho_c g \Delta\rho}{\mu_c^2}\right)^{1/3}$$

$$\psi(r_d^*) = 0.55\left\{\left(1+0.08 r_d^{*3}\right)^{4/7} - 1\right\}^{0.75}$$

The drift flux model

- For a Newton's regime ($r_d^* \geq 34.65$)

$$V_{dj} = 2.43\left(\frac{r_d g \Delta\rho}{\rho_c}\right)^{1/2}(1-\alpha_d)^{1.5} f(\alpha_d) \times \frac{18.67}{1+17.67\{f(\alpha_d)\}^{6/7}}$$

- For a distorted-fluid-particle regime

$$V_{dj} \simeq \sqrt{2}\left(\frac{\sigma g \Delta\rho}{\rho_c^2}\right)^{1/4} \times \begin{cases} (1-\alpha_d)^{1.75} & \mu_c >> \mu_d \\ (1-\alpha_d)^2 & \mu_c \simeq \mu_d \\ (1-\alpha_d)^{2.25} & \mu_d >> \mu_c \end{cases}$$

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The drift flux model

- For a churn-turbulent flow regime

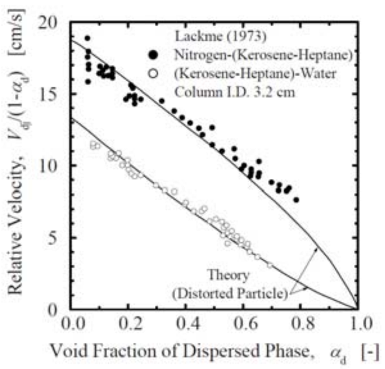
$$V_{dj} = \begin{Bmatrix} \sqrt{2} \\ \text{or } 1.57 \end{Bmatrix} \left(\frac{\sigma g \Delta\rho}{\rho_c^2}\right)^{1/4} \frac{\rho_c - \rho_d}{\Delta\rho} (1-\alpha_d)^{1/4}$$

$$\simeq \sqrt{2} \left(\frac{\sigma g \Delta\rho}{\rho_c^2}\right)^{1/4} \frac{\rho_c - \rho_d}{\Delta\rho}$$

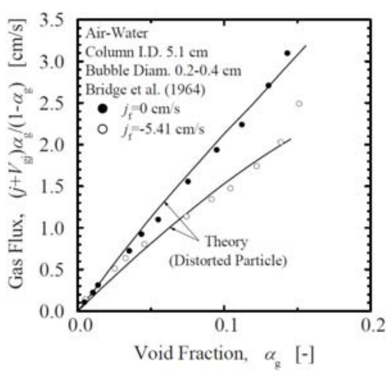
- the proportionality constant 2 is applicable for bubbly flows and 1.57 for droplet flows
- For a slug flow regime

$$V_{dj} = 0.35 \left(\frac{g \Delta\rho D}{\rho_c}\right)^{1/2}$$

The drift flux model

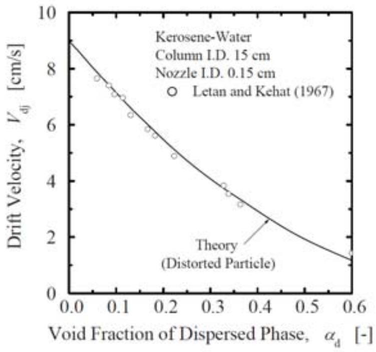


Difference between bubble system and droplet-dispersion system in distorted particle regime (Ishii and Chawla, 1979)

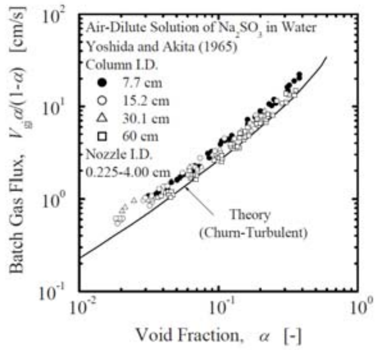


Comparison of gas volumetric flux with distorted-bubble-regime data in a flowing system (Ishii and Chawla, 1979)

The drift flux model



Comparison of predicted distorted-particle-regime drift velocity to data in a Kerosene-water system (Ishii and Chawla, 1979)



Comparison of predicted gas volumetric flux based on churn-flow-regime drift velocity to data (Ishii and Chawla, 1979)

Outline

- Homogeneous flow model
- Separated flow model
- Drift-flux model
- Two-fluid model
- CFD

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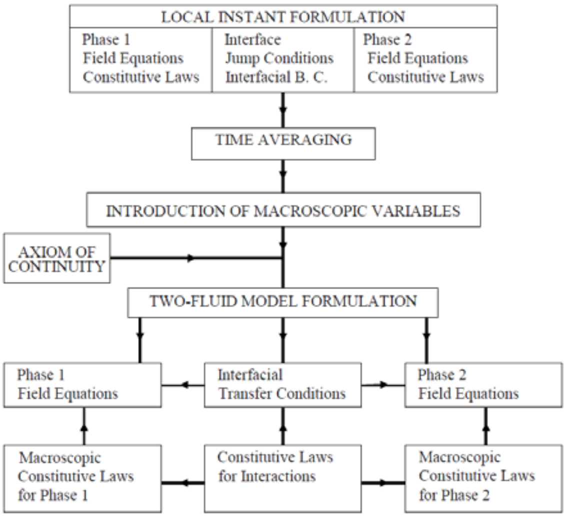
Two-fluid model

- It is based on the mass, momentum and energy balance equations for every phase (Ishii, 1987).
- In this model, every phase or component is treated as a **separate fluid** with its own set of governing balance equations. In general, every phase has its own velocity, temperature and pressure.
- This approach enables the prediction of **important non-equilibrium phenomena** of two-phase flow like the velocity difference between liquid and gas phase.
- This prediction is important for two-phase flows in large shell sides of steam generators and kettle reboilers, where **even different gas and liquid velocity directions exist**.

Two-fluid model

	Two-fluid model	Mixture model
对象	considering each phase separately	considering the mixture as a whole
控制方程	two sets of conservation equations of mass, momentum and energy (6)	three-mixture conservation equations of mass, momentum, and energy with one additional diffusion (continuity) equation which takes account of the concentration changes (4)
界面耦合	an interaction term coupling the two phases through jump conditions	A mixture conservation equation can be obtained by adding two corresponding conservation equations for each phase with an appropriate jump condition

Two-fluid model



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Ishii M. & Hibiki T., Thermo-Fluid Dynamics of Two-Phase Flow. Springer, 2011

Two-fluid Model

- Continuity equations

$$\frac{\partial \alpha_k \overline{\overline{\rho_k}}}{\partial t} + \nabla \cdot \left(\alpha_k \overline{\overline{\rho_k}} \widehat{\boldsymbol{v}_k}\right) = \Gamma_k \qquad \sum_{k=1}^{2} \Gamma_k = 0$$

- Γ represents the rate of production of kth-phase mass from the phase changes at the interfaces per unit volume.
- If each phase is originally incompressible

$$\frac{\partial \alpha_k}{\partial t} + \nabla \cdot \left(\alpha_k \widehat{\boldsymbol{v}_k}\right) = \frac{\Gamma_k}{\overline{\overline{\rho_k}}}$$

- For low speed two-phase flow without phase change

$$\frac{\partial \alpha_k}{\partial t} + \nabla \cdot \left(\alpha_k \widehat{\boldsymbol{v}_k}\right) = 0$$

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Two-fluid Model

- Momentum equations

$$\frac{\partial \alpha_k \overline{\overline{\rho_k}} \widehat{\boldsymbol{v}_k}}{\partial t} + \nabla \cdot \left(\alpha_k \overline{\overline{\rho_k}} \widehat{\boldsymbol{v}_k} \widehat{\boldsymbol{v}_k} \right)$$
$$= -\nabla \left(\alpha_k \overline{\overline{p_k}} \right) + \nabla \cdot \left[\alpha_k \left(\overline{\overline{\boldsymbol{\mathcal{T}}}}_k + \boldsymbol{\mathcal{T}}_k^T \right) \right] + \alpha_k \overline{\overline{\rho_k}} \widehat{\boldsymbol{g}_k} + \boldsymbol{M}_k$$

$$\sum_{k=1}^{2} \boldsymbol{M}_k = \boldsymbol{M}_m = 2\overline{\overline{H_{21}}}\, \overline{\overline{\sigma}} \nabla \alpha_2 + \boldsymbol{M}_m^H$$

- the momentum equation for each phase has an interfacial source term ***Mk*** that couples the motions of two phases. H_{21} and σ are the average mean curvature of interfaces and the surface tension, whereas the term given by ***Mm*** takes account for the effect of the changes in the mean curvature.

Two-fluid Model

- Energy equations

$$\frac{\partial}{\partial t}\left[\alpha_k \overline{\overline{\rho_k}} \left(\widehat{e_k} + \frac{\widehat{v_k}^2}{2} \right) \right] + \nabla \cdot \left[\alpha_k \overline{\overline{\rho_k}} \left(\widehat{e_k} + \frac{\widehat{v_k}^2}{2} \right) \widehat{\boldsymbol{v}_k} \right]$$
$$= -\nabla \cdot \left[\alpha_k \left(\overline{\overline{\boldsymbol{q}_k}} + \boldsymbol{q}_k^T \right) \right] + \nabla \cdot \left(\alpha_k \overline{\overline{T_k}} \cdot \widehat{\boldsymbol{v}_k} \right)$$

$$\sum_{k=1}^{2} E_k = E_m = \overline{\overline{T_i}} \left(\frac{d\sigma}{dT} \right) \frac{D_i a_i}{Dt} + 2\overline{\overline{H_{21}}}\, \overline{\overline{\sigma}} \frac{\partial \alpha_1}{\partial t} + E_m^H$$

- the sum of the interfacial energy transfer terms E_k for each phase balances with the time rate of change of surface energy and the work done by the surface tension.

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- Fellow, American Nuclear Society (ANS)
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- Atomic Energy Society of Japan
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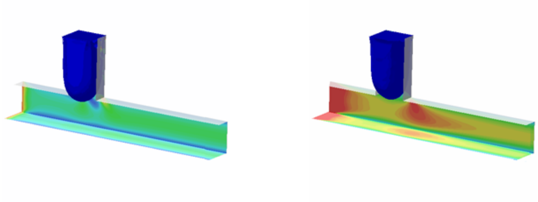
Outline

- Homogeneous flow model
- Separated flow model
- Drift-flux model
- Two-fluid model
- CFD

CFD



Ca=8.75×10^{-4} Ca=1.4×10^{-2}

To be continued……

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