粘性流体运动

粘性流体中的应力、N-S方程、流动的两种状态、N-S方程层流解析解、湍流概述

基础知识

张量、牛顿内摩擦定律、牛顿第二定律、理想流体运动欧拉方程、雷诺数

纳维–斯托克斯方程

应力张量、本构方程、定解条件

层流解析解

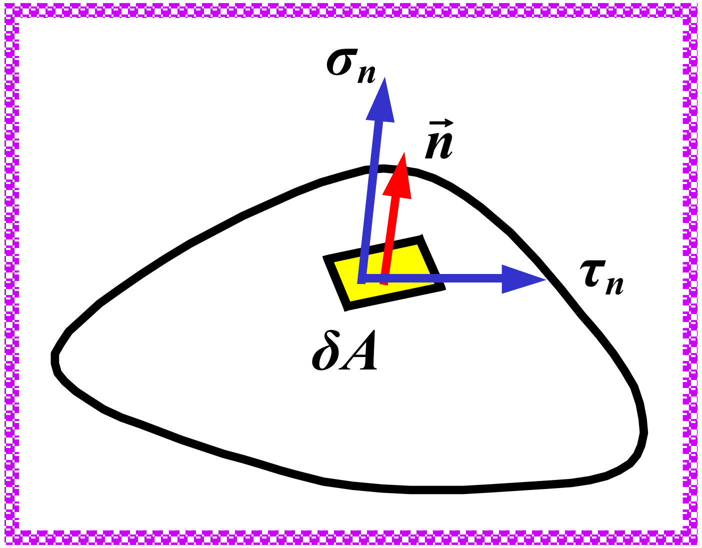
库埃特—泊肃叶流动

湍流概述

雷诺应力、圆管湍流速度分布

理想流体 ⟹ $p = p(x, y, z)$

粘性流体 ⟹
- 法向应力σ_n *normal stress*
- 切向应力τ_n *shear stress*



- 应力大小与作用面方位有关

取 $\vec{n}$ 与 x 正方向一致

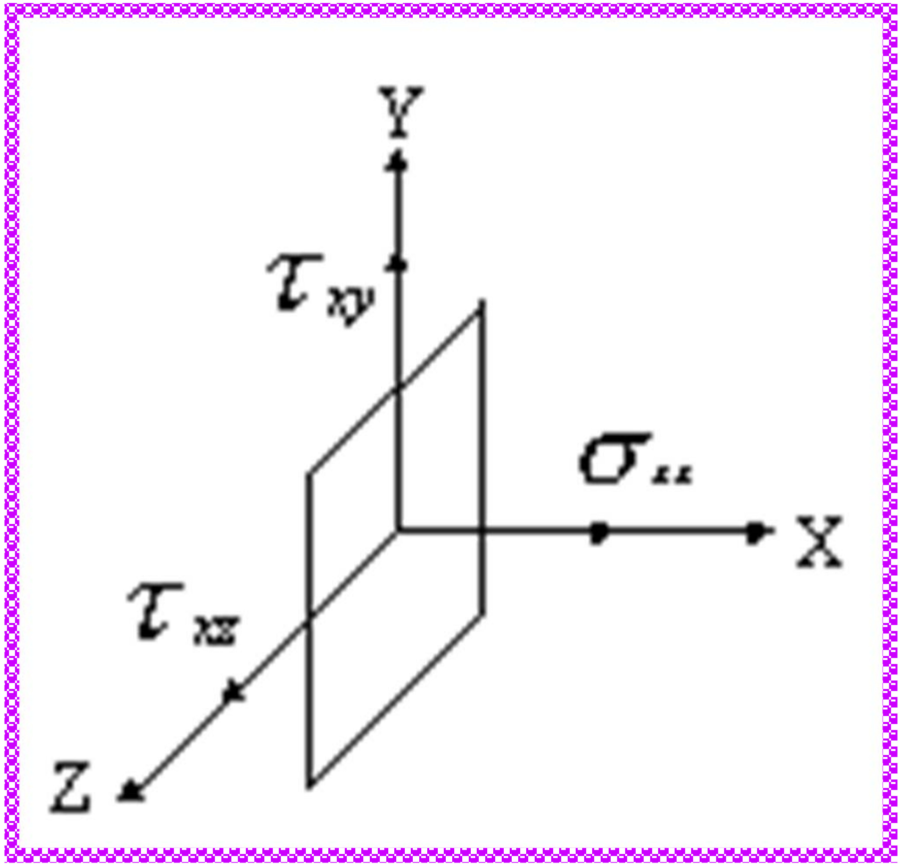
σ_{xx} τ_{xy} τ_{xz}

下标 1

作用面法线方向

下标 2

应力分量的指向



- 过一点作三个相互垂直的平面，则过该点任意方位表面上的应力都可以用这三个平面上的九个应力分量来表示

应力张量 ⇒
$$\begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

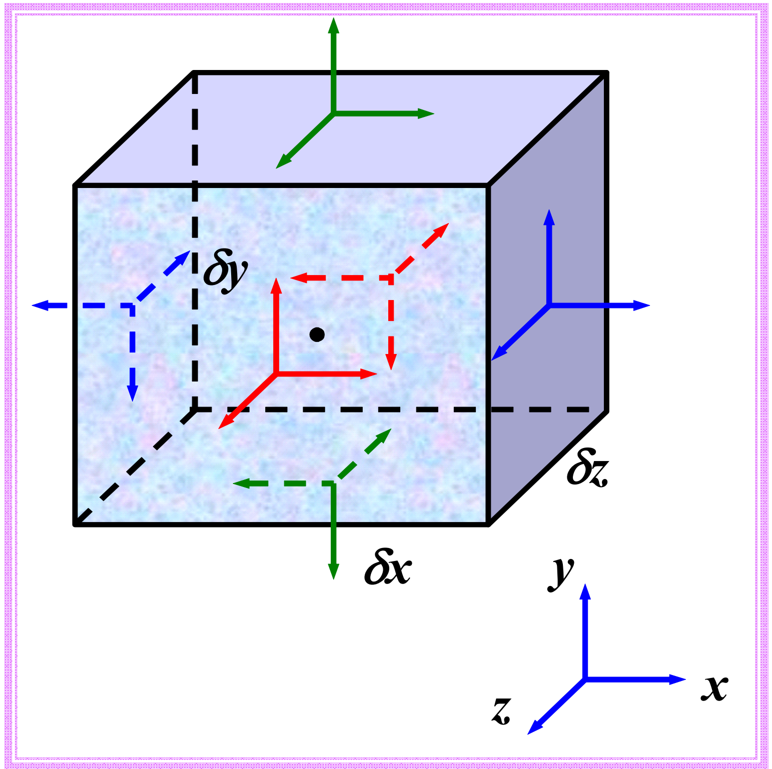
stress tensor

九个应力分量中只有六个是独立的

应力正方向表示规则

- 表面外法线方向和坐标轴正向一致

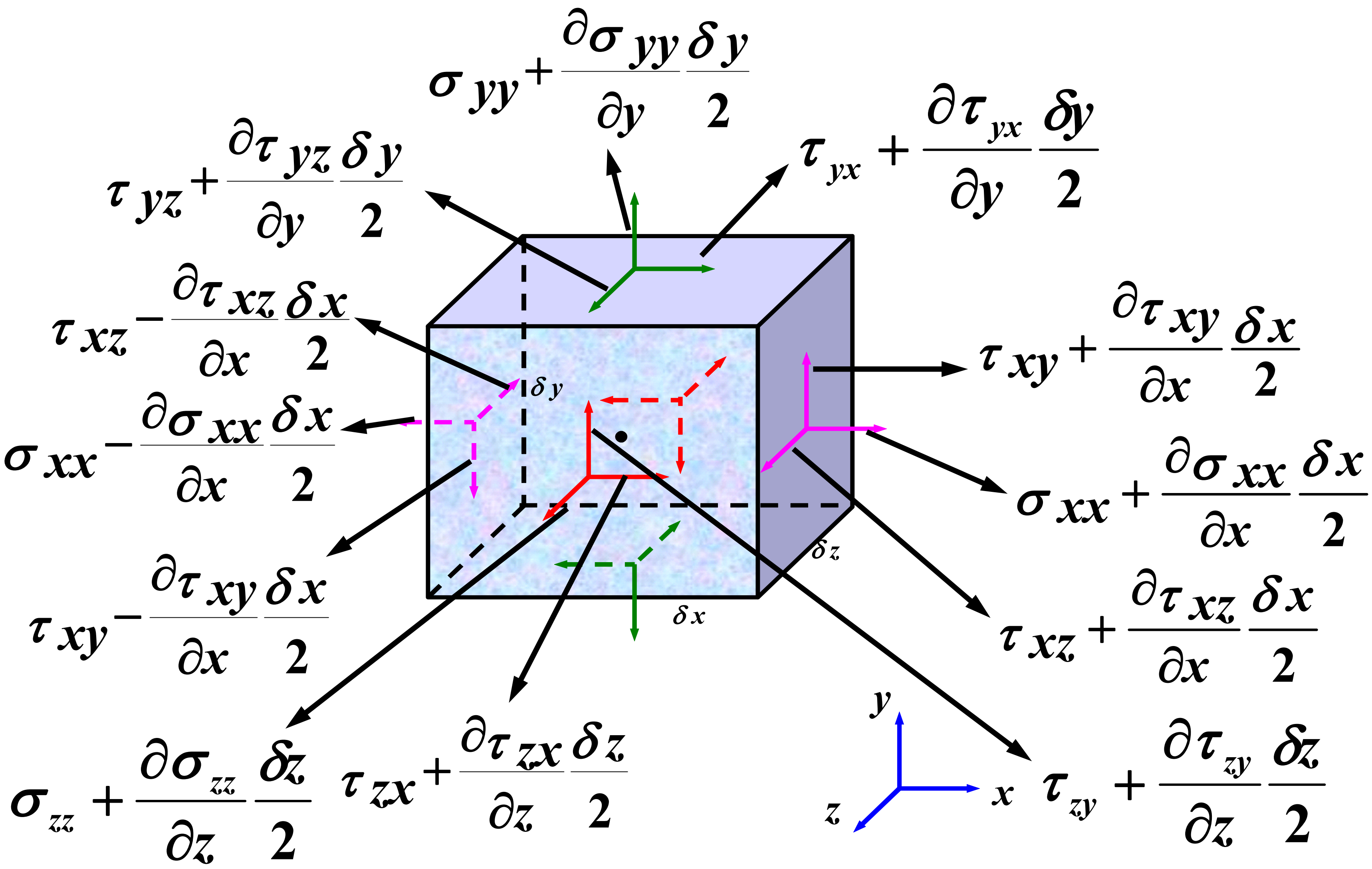
➡ 应力分量正向分别与各坐标轴正向一致



- 表面外法线方向和坐标轴正向相反

➡ 应力分量正向分别与各坐标轴正向相反

六面体流体微团表面力2



表面力合力

$$\delta F_{sx}=\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)\delta x\delta y\delta z$$

$$\delta F_{sy}=\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}\right)\delta x\delta y\delta z$$

$$\delta F_{sz}=\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}\right)\delta x\delta y\delta z$$

微分形式动量方程

differential momentum equation

牛顿第二定律应用于流体微团 $\Sigma\vec{F} = m\vec{a}$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x+\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y+\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho g_z+\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}$$

应力与变形速度的关系 → Stokes假设

- 小变形，应力与变形速度之间成线性关系

- 各向同性，应力与变形速度的关系不随坐标变换而变化

- 当$\mu \to 0$时，应力状态简化为理想流体应力状态

$$\sigma_x = \sigma_y = \sigma_z = -p$$

应力与变形速度的关系

法向应力

$$\sigma_{xx} = -p + 2\mu\frac{\partial u}{\partial x} - \frac{2}{3}\mu\nabla\cdot\vec{V}$$

$$\sigma_{yy} = -p + 2\mu\frac{\partial v}{\partial y} - \frac{2}{3}\mu\nabla\cdot\vec{V}$$

$$\sigma_{zz} = -p + 2\mu\frac{\partial w}{\partial z} - \frac{2}{3}\mu\nabla\cdot\vec{V}$$

其中，p 为压强

$$p = -\frac{1}{3}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right)$$

应力与变形速度的关系

切向应力 ⇒

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$$

$$\tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

- 切向应力与流体的角变形率成正比

微分形式动量方程(运动方程) — N-S方程

Navier-Stokes equations

$$\rho\frac{Du}{Dt}=\rho g_x-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left[2\mu\left(\frac{\partial u}{\partial x}-\frac{1}{3}\nabla\cdot\vec{V}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right]$$

$$\rho\frac{Dv}{Dt}=\rho g_y-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[2\mu\left(\frac{\partial v}{\partial y}-\frac{1}{3}\nabla\cdot\vec{V}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]$$

$$\rho\frac{Dw}{Dt}=\rho g_z-\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]+\frac{\partial}{\partial z}\left[2\mu\left(\frac{\partial w}{\partial z}-\frac{1}{3}\nabla\cdot\vec{V}\right)\right]$$

可压缩流体的控制方程组

- 连续方程、运动方程、能量方程

五个方程，六个未知数 p、u、v、w、ρ、T， 方程组不封闭

- 增加完全气体状态方程

六个方程，六个未知数 p、u、v、w、ρ、T，方程组封闭

不可压缩流体，且 $\mu = \text{const}$

$$\nabla \cdot \vec{V} = 0$$

$$\rho \frac{Du}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$\rho \frac{Dv}{Dt} = \rho g_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\rho \frac{Dw}{Dt} = \rho g_z - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \nabla p + \mu \nabla^2 \vec{V}$$

Navier-Stokes方程4 – 不可压缩

连续方程

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$$

动量方程

$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \nabla p + \mu \nabla^2 \vec{V}$$

- 不可压缩流动：四个未知数 u，v，w，p，四个方程，方程组封闭

理想流体欧拉运动方程，$\mu = 0$

$$\rho \frac{Du}{Dt} = \rho g_x - \frac{\partial p}{\partial x}$$

$$\rho \frac{Dv}{Dt} = \rho g_y - \frac{\partial p}{\partial y}$$

$$\rho \frac{Dw}{Dt} = \rho g_z - \frac{\partial p}{\partial z}$$

Euler's equation

$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \nabla p$$

Navier-Stokes方程6

质量 × 加速度 = 流体微团所受到的合外力

N-S方程 → $$\rho \frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p + \mu\nabla^2\vec{V}$$

欧拉运动微分方程 → $$\rho \frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$$

欧拉平衡方程 → $$0 = \rho\vec{f} - \nabla p$$

粘性流体流动的定解条件

初始条件 *initial conditions* → 非定常流动 $t = 0$ 时刻的物理量场

边界条件 *boundary conditions*

固壁 → 无滑移边界条件 *no-slip conditions* $\vec{V}_{流} = \vec{V}_{固}$

进出口、无穷远处 → 物理量分布

流体相界面 → 速度、压强、粘性应力等连续

求解途径

解析解

第5章：无限大平板及圆管中充分发展层流

圆管充分发展层流，无限大平板间充分发展层流等

近似解

第10章：顺流平板层流边界层流动

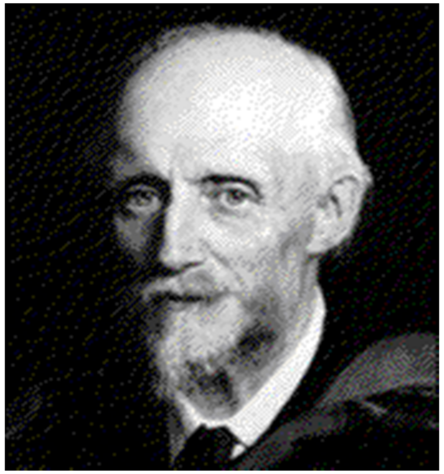
Re数很大（高Re数绕流）和很小（蠕流）两种极端情况下，略去方程中某些次要项

数值解

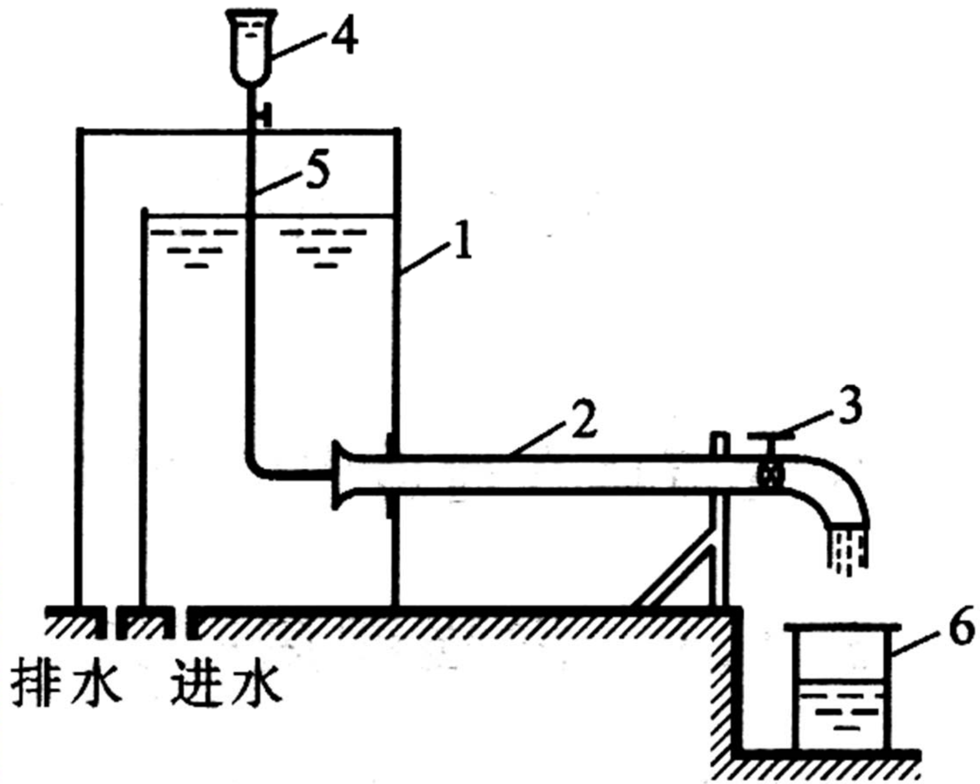
利用数值方法 (CFD)

流动的两种状态

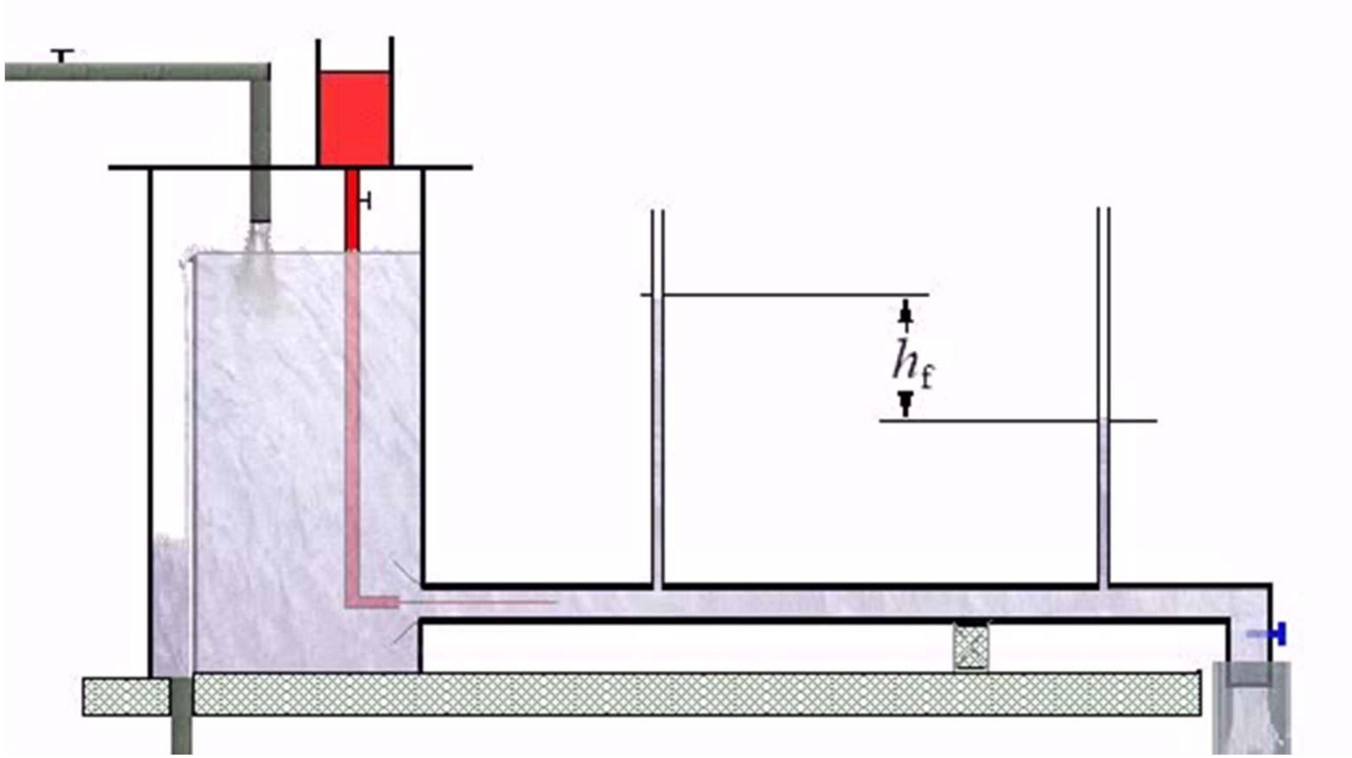
雷诺实验
（1883）

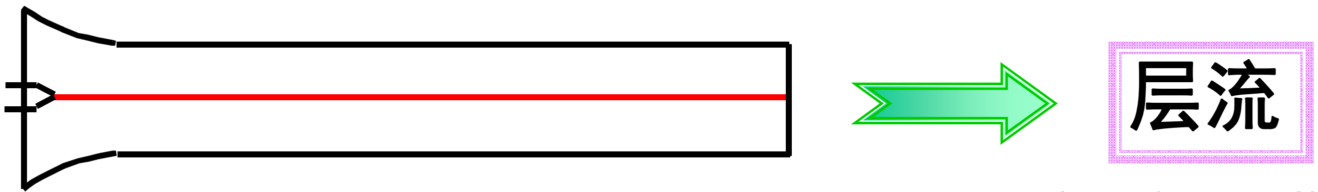


Osborne Reynolds

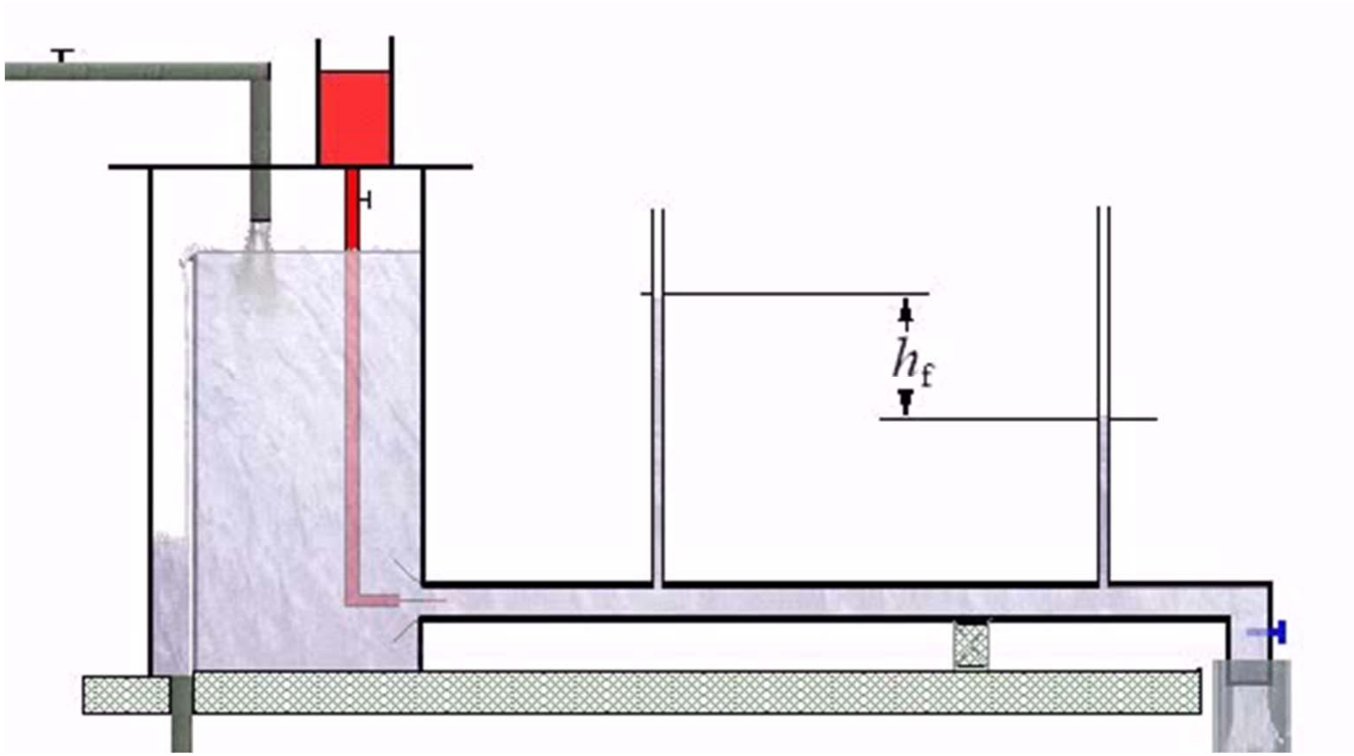


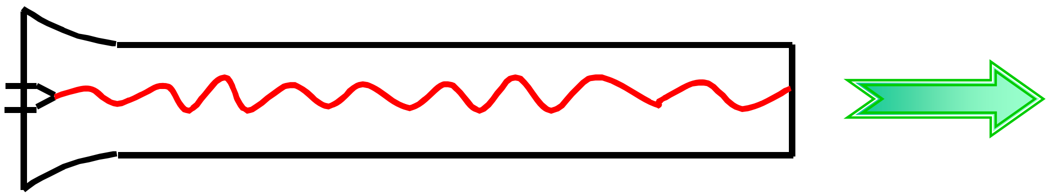
流动的两种状态1





laminar flow

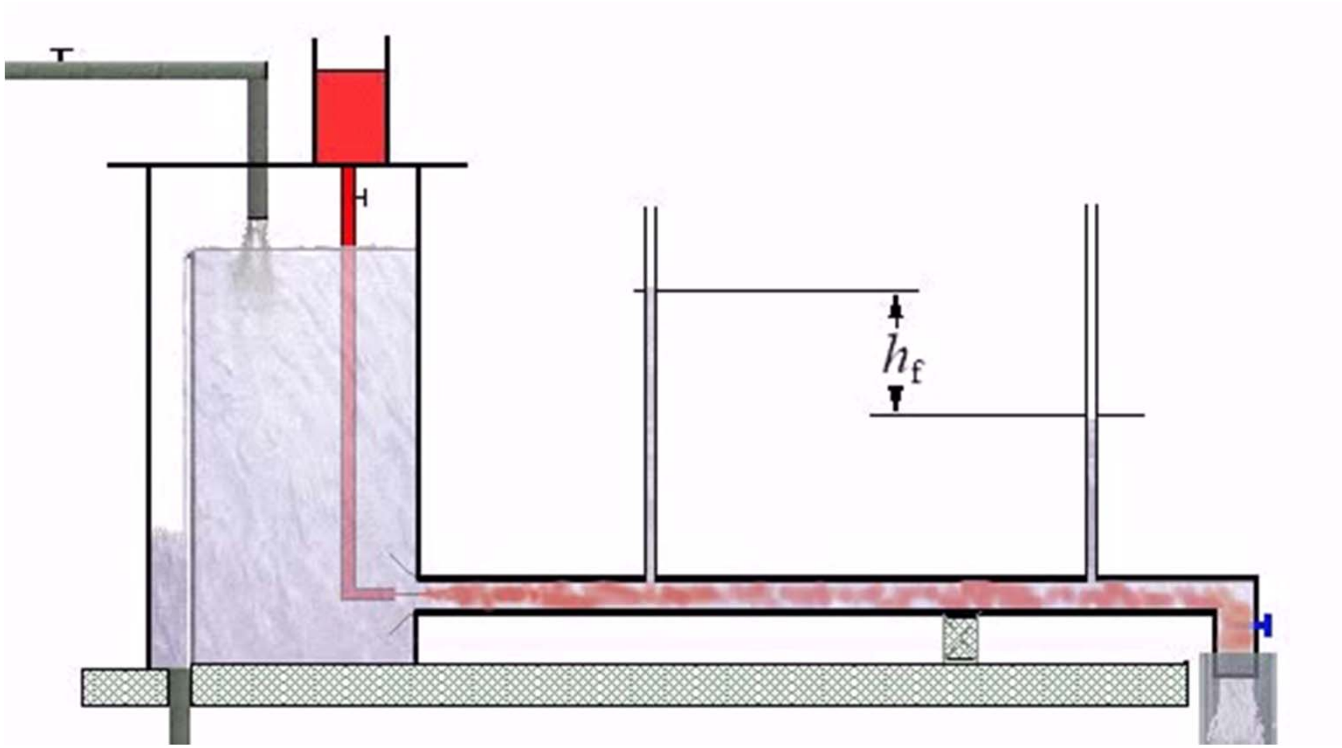




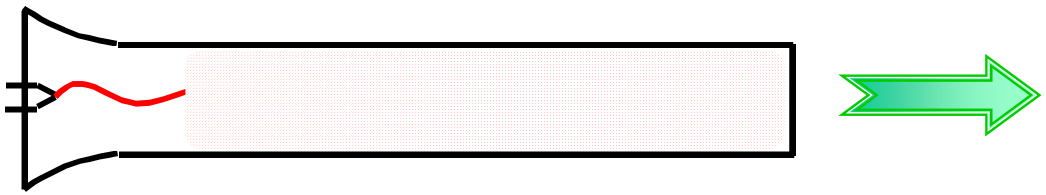
过渡(转捩)

transition

流动的两种状态3



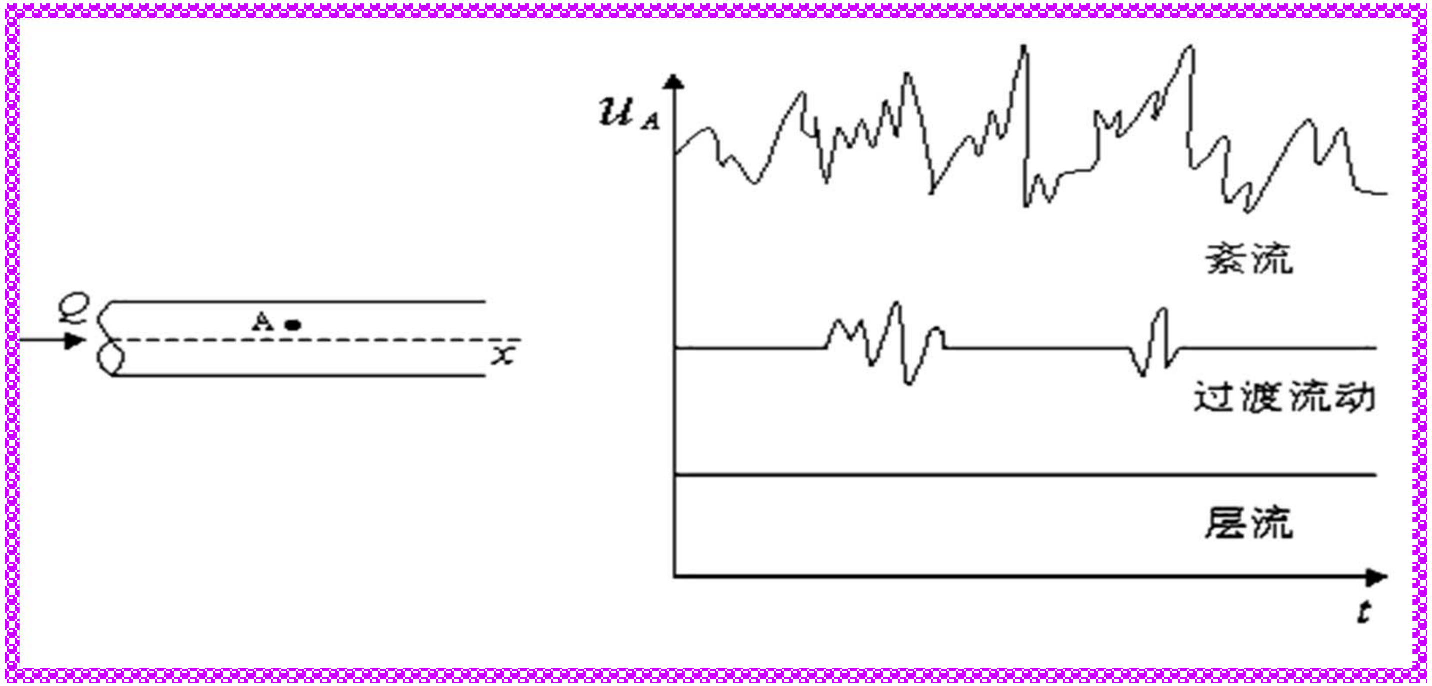
湍流状态下，流体质点存在随机运动



湍流（紊流）

turbulent flow

热线测速仪测量流体速度



- 湍流状态下，物理量随时间存在不规则的随机脉动

流动的两种状态5

层流	湍流
分层流动，各部分互不掺混	各部分激烈掺混
质点轨迹光滑	质点轨迹杂乱无章
流动稳定	流动极不稳定

湍流是随机的三维非定常有旋流动

决定流动状态的判据

$$\mathrm{Re} = \frac{\rho VL}{\mu} = \frac{VL}{\nu}$$

临界雷诺数与实验外部条件有关

平面库埃特流 → $\mathrm{Re}_{cr} = hU/\nu = 1300$

平面泊肃叶流 → $\mathrm{Re}_{cr} = 2h\overline{V}/\nu = 3000$

critical Reynolds number

层流稳定性及其向湍流的过渡

Re较小 → 粘性力影响显著

→ 扰动受粘性阻尼作用衰减

→ 稳定层流

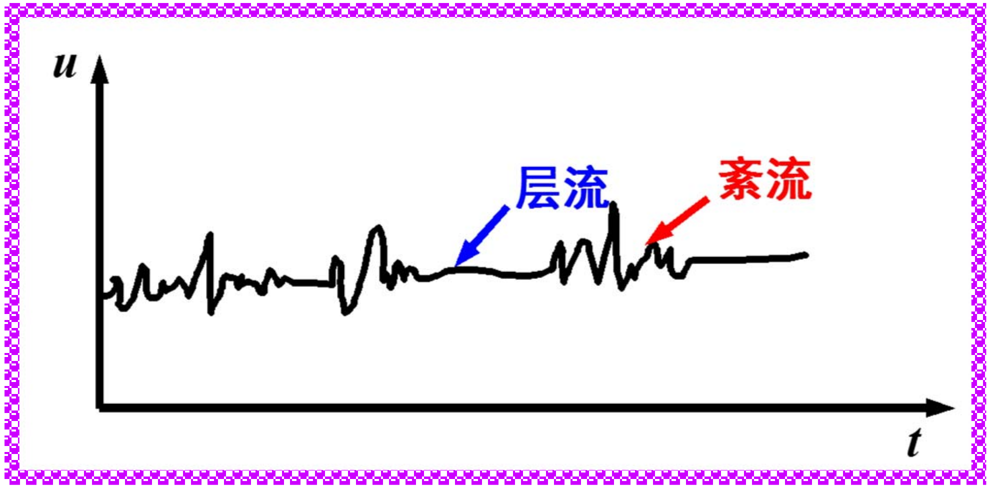
Re较大 → 惯性力远高于粘性力

→ 惯性力对扰动的放大远超过粘性阻尼作用

→ 失稳，层流转化为湍流

过渡区

湍栓、分叉



有$\nu = 1.13\times10^{-6}$ m²/s的水和$\nu = 200\times10^{-6}$ m²/s的油，分别以 $u = 1$ m/s的流速通过直径 $d = 300$ mm的管道，试判别其流动状态.

解：$$\mathrm{Re} = \frac{\rho u d}{\mu} = \frac{ud}{\nu}$$

水 $$\mathrm{Re} = \frac{1\times0.3}{1.13\times10^{-6}} = 265486.7$$ ⟹ 湍流

油 $$\mathrm{Re} = \frac{1\times0.3}{200\times10^{-6}} = 1500$$ ⟹ 层流

层流解析解

粘性、均质不可压、定常

$$\rho = \text{const}\ ,\ \frac{\partial}{\partial t} = 0$$

连续方程

N-S方程

层流

解析解

泊肃叶流（Poiseuille）

库埃特流（Couette）

边界条件

固面无滑移条件

相界面压强、粘性应力连续

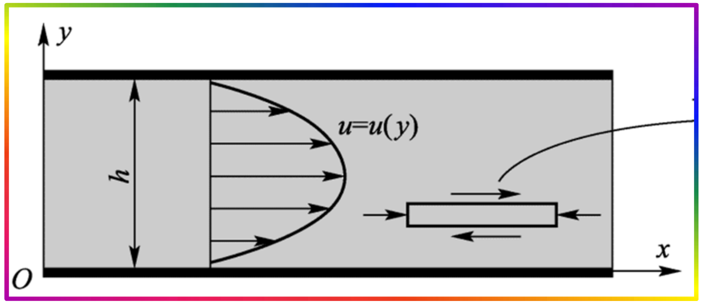
N-S 方程精确解

fully developed laminar flow between parallel plates

假设

- 定常、层流、只有 x 方向流动

$$\frac{\partial}{\partial t}=0 \qquad v=w=0$$

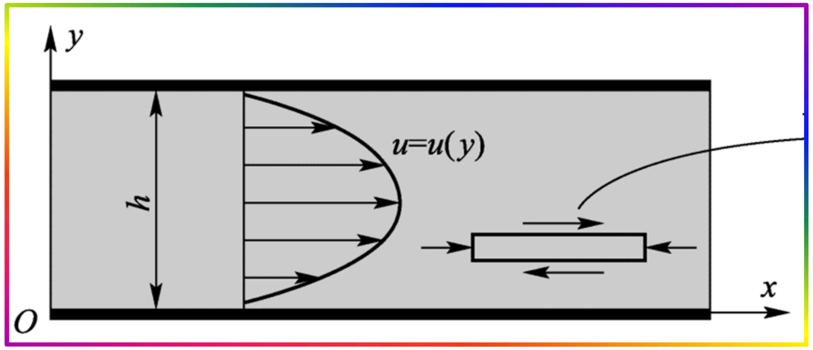


- 均质不可压缩 $\Rightarrow$ $\rho=\text{const}$
- 温度变化不大 $\Rightarrow$ $\mu=\text{const}$
- 无限大平板 $\Rightarrow$ $\dfrac{\partial u}{\partial z}=0$

质量力仅为重力

$$g_x = g_z = 0 \qquad g_y = -g$$

建立方程组



连续方程

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \Rightarrow \frac{\partial u}{\partial x}=0 \Rightarrow u=u(y)$$

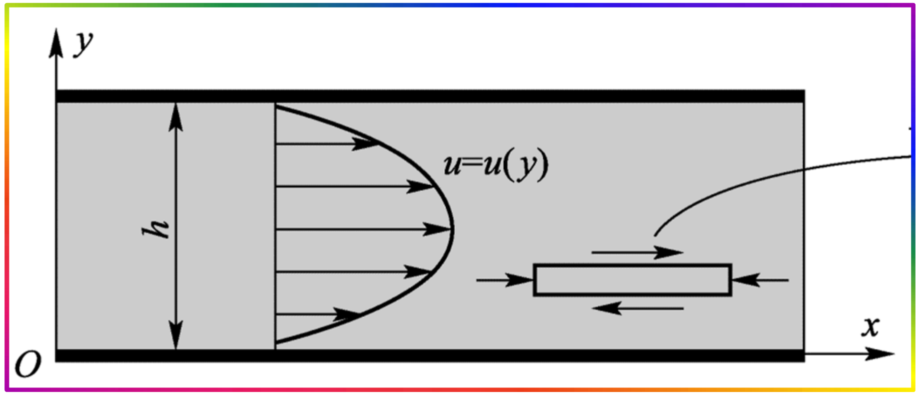
N-S 方程

$$\frac{D\vec{V}}{Dt}=\vec{f}-\frac{1}{\rho}\nabla p+\nu\nabla^2\vec{V}$$

$$0=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial p}{\partial x}$$

$$0=-\rho g-\frac{\partial p}{\partial y}$$

$$0=\frac{\partial p}{\partial z}$$

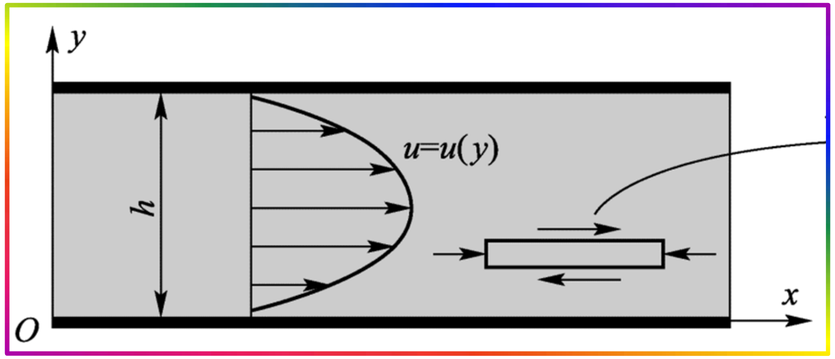


$$u=\frac{1}{2\mu}\frac{\partial p}{\partial x}y^2+c_1 y+c_2$$

恒定压差作用下平板间层流—泊肃叶流 *Poiseuille flow*

- 定解条件 $y=0\ ,\ u=0 \qquad y=h\ ,\ u=0$

$$u=-\frac{h^2}{2\mu}\frac{\partial p}{\partial x}\frac{y}{h}\left(1-\frac{y}{h}\right)$$



设相距为 l 的两点压降为 Δp

$$u=\frac{h^2}{2\mu}\frac{\Delta p}{l}\frac{y}{h}\left(1-\frac{y}{h}\right)$$

抛物线分布

parabolic profile

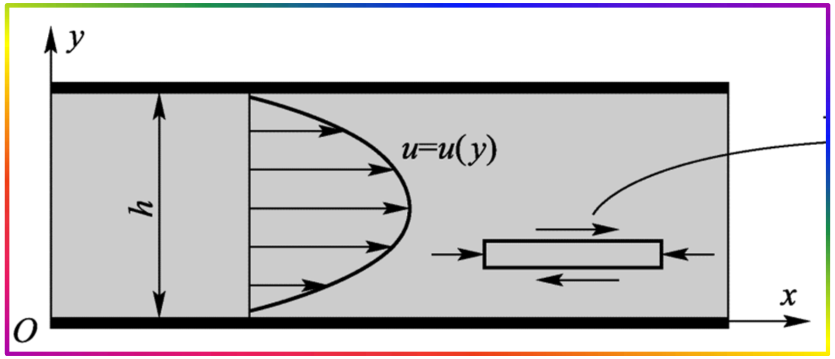
截面最大速度

$$u_{\max}=u\big|_{y=\frac{h}{2}}=\frac{h^2}{8\mu}\frac{\Delta p}{l}$$

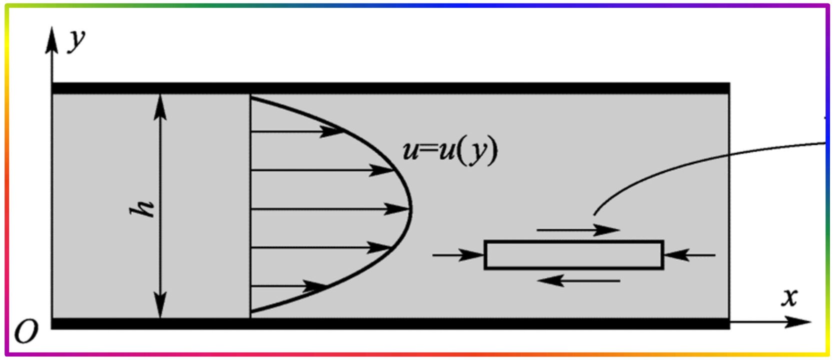
两平板间的体积流量，z 方向为单位长度

$$Q=\int u\,dA=\int_0^h u\,dy$$

$$Q=\frac{h^3}{12\mu}\frac{\Delta p}{l}$$



截面平均速度



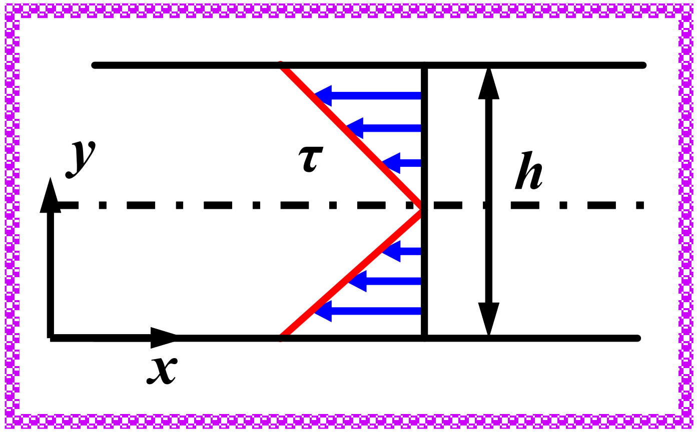
$$\bar{V} = \frac{Q}{A} = \frac{h^2}{12\mu}\frac{\Delta p}{l}$$

$$u_{max} = 1.5\bar{V}$$

切应力分布

$$\tau = \mu \frac{du}{dy}$$

方向与流动方向相反



压强分布 $p = -\rho g y + f(x) \Longrightarrow p = -\rho g y + \frac{\partial p}{\partial x} x + C$

$$\Longrightarrow \boxed{p = p_0 - \rho g y + \frac{\partial p}{\partial x} x}$$

- 流体质点沿流动方向作匀速直线运动

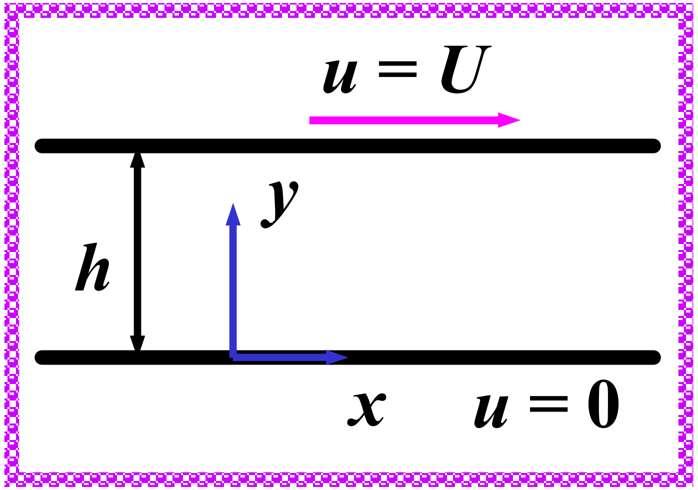
- x 方向粘性力与压力平衡 $\Longrightarrow$ 粘性力导致压强变化

- y 方向重力与压力平衡 $\Longrightarrow$ 重力导致压强变化

恒定压差、剪切作用

$$u=\frac{1}{2\mu}\frac{\partial p}{\partial x}y^2+C_1y+C_2$$

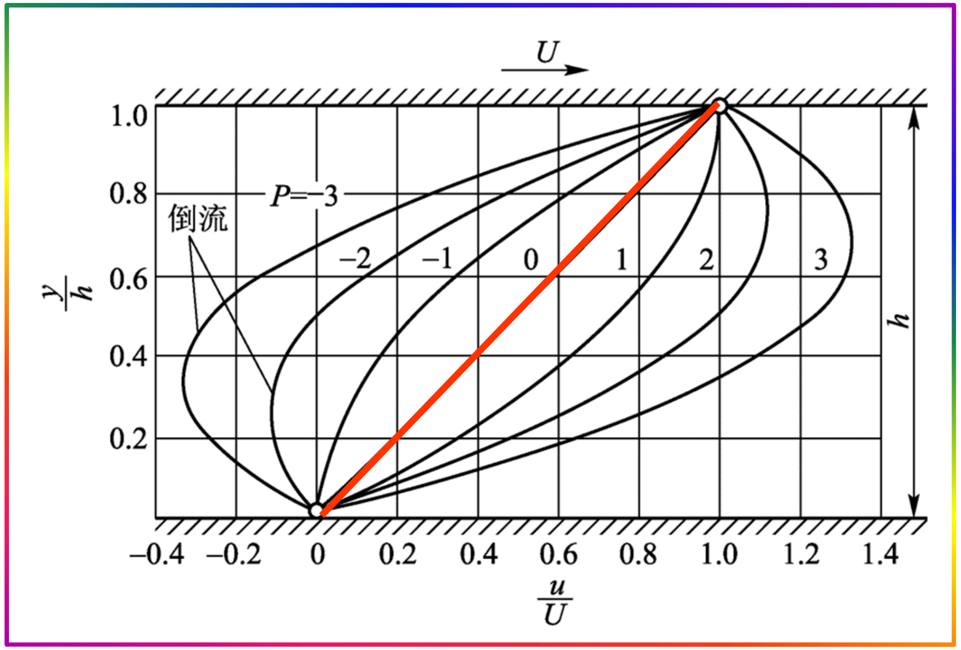
$$p=-\rho gy+f(x)$$



定解条件 $y=0 \Longrightarrow u=0 \qquad y=h \Longrightarrow u=U$

$$\Longrightarrow \frac{u}{U}=\frac{y}{h}-\frac{h^2}{2\mu U}\frac{\partial p}{\partial x}\frac{y}{h}\left(1-\frac{y}{h}\right)=\frac{y}{h}+P\frac{y}{h}\left(1-\frac{y}{h}\right)$$

P—无量纲压强梯度



最早由Couette于1890年分析

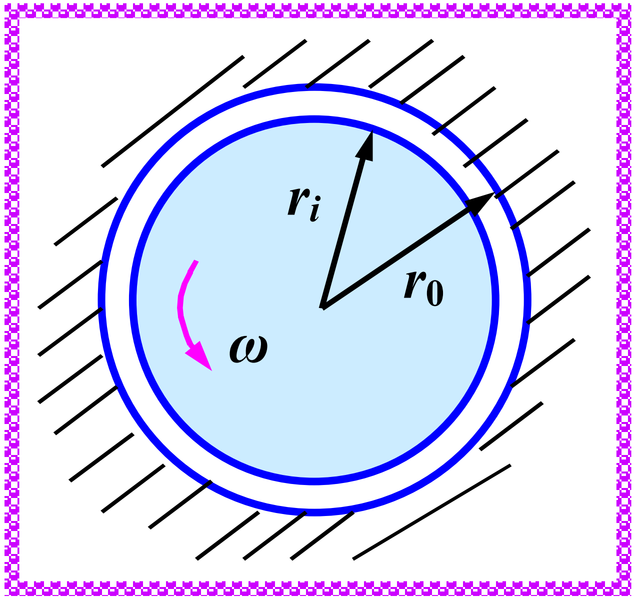
$$\frac{u}{U} = \frac{y}{h} + P\frac{y}{h}\left(1 - \frac{y}{h}\right)$$

$$P = -\frac{h^2}{2\mu U}\frac{\partial p}{\partial x}$$

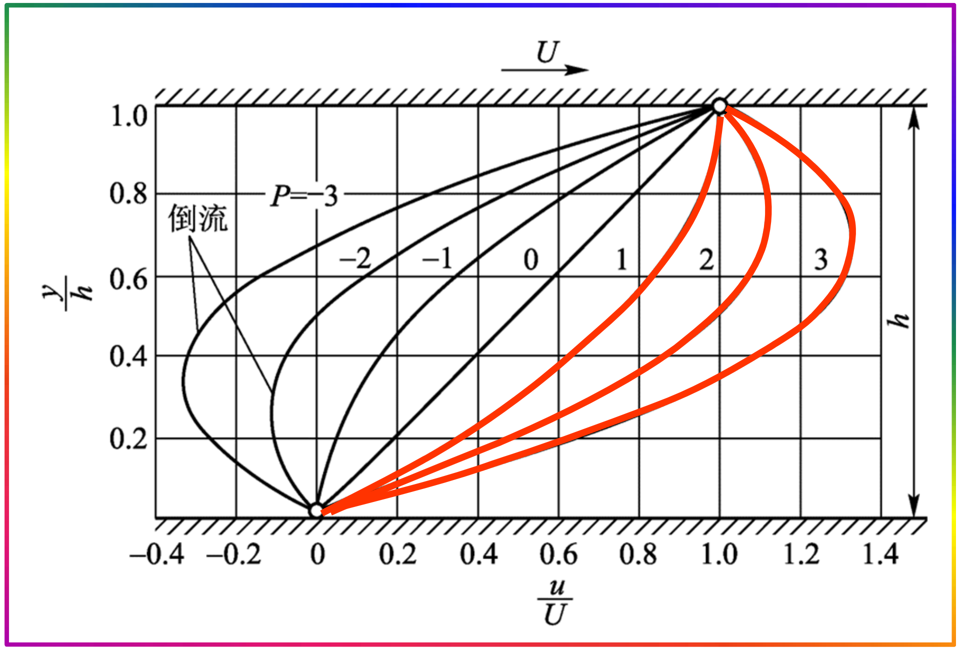
$$\boxed{\frac{\partial p}{\partial x} = 0} \Rightarrow u = \frac{y}{h}U$$

Couette flow

纯剪切流，速度线性分布



平面库埃特—泊肃叶流3



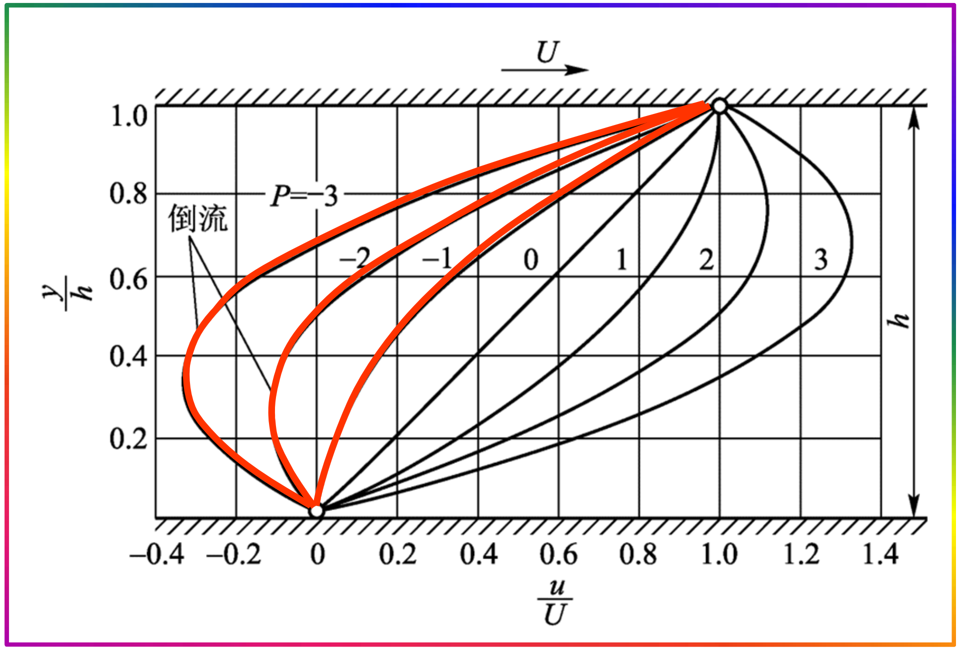
$$\frac{u}{U} = \frac{y}{h} + P\frac{y}{h}\left(1 - \frac{y}{h}\right)$$

$$P = -\frac{h^2}{2\mu U}\frac{\partial p}{\partial x}$$

$$\boxed{\frac{\partial p}{\partial x} < 0}$$ 顺压梯度

negative (favorable) pressure gradient

- 流速廓线是斜直线（纯剪切流）与抛物线（泊肃叶流）的叠加
- 全场速度为正，每点速度大于或等于泊肃叶流



$$\frac{u}{U}=\frac{y}{h}+P\frac{y}{h}\left(1-\frac{y}{h}\right)$$

$$P=-\frac{h^2}{2\mu U}\frac{\partial p}{\partial x}$$

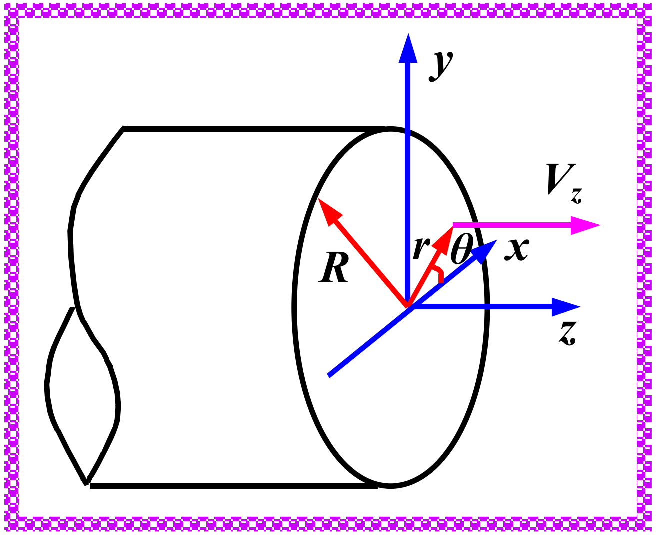
$$\boxed{\frac{\partial p}{\partial x}>0}$$ 逆压梯度

positive (unfavorable) pressure gradient

- 流速廓线是斜直线（纯剪切流）与抛物线（泊肃叶流）相减
- 全场速度有正有负，在固定平板一侧出现倒流

N-S方程精确解

fully developed laminar flow in circular pipe



假设

- 定常、层流

$$\frac{\partial}{\partial t}=0 \qquad V_r=V_\theta=0$$

- 均质不可压缩 ⇒ $\rho=\text{const}$

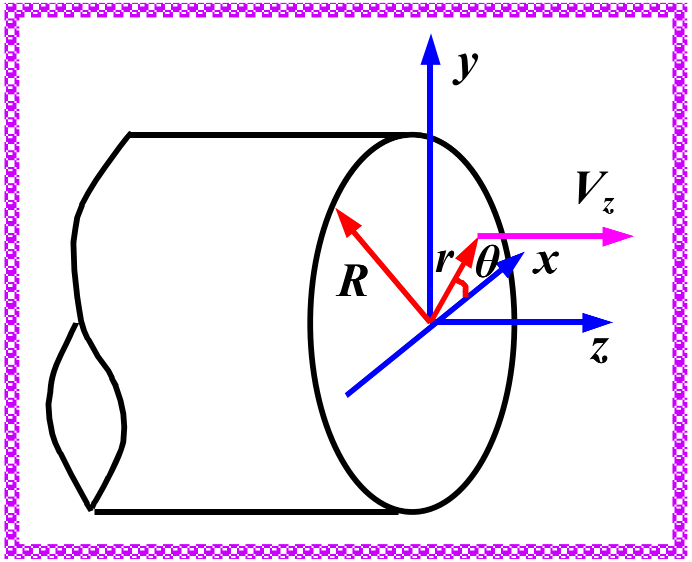
- 圆管内轴对称流动 ⇒ $\dfrac{\partial V_z}{\partial \theta}=0$

- 温度变化不大 ⇒ $\mu=\text{const}$

- 质量力仅为重力

$$g_r = -g\sin\theta \qquad g_\theta = -g\cos\theta \qquad g_z = 0$$

建立方程组



- 连续方程

$$\frac{1}{r}\frac{\partial(rV_r)}{\partial r}+\frac{1}{r}\frac{\partial V_\theta}{\partial\theta}+\frac{\partial V_z}{\partial z}=0$$

$$\Rightarrow \quad \partial V_z/\partial z = 0 \quad \Rightarrow \quad V_z = V_z(r)$$

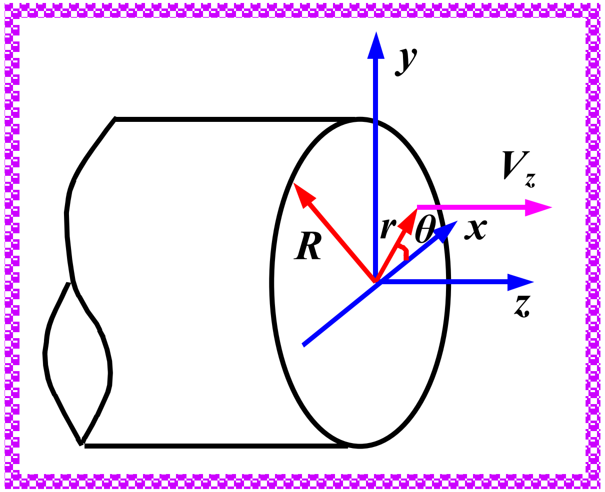
N-S 方程（r 方向）

steady　　$V_r = V_\theta = 0$

$$\rho\left(\cancel{\frac{\partial V_r}{\partial t}} + \cancel{V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial\theta} - \frac{V_\theta^2}{r} + V_z\frac{\partial V_r}{\partial z}}\right) = \rho f_r - \frac{\partial p}{\partial r}$$

$$+\mu\left[\cancel{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_r)\right) + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial\theta^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial\theta} + \frac{\partial^2 V_r}{\partial z^2}}\right] \quad V_r = V_\theta = 0$$

$$-\rho g\sin\theta - \frac{\partial p}{\partial r} = 0$$



圆管内的泊肃叶流动4

θ 方向

steady $\qquad\qquad\qquad V_r = V_\theta = 0$

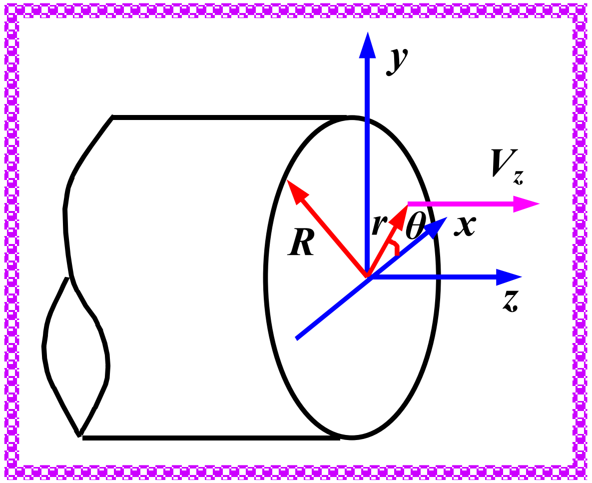
$$\rho\left(\cancel{\frac{\partial V_\theta}{\partial t}} + \cancel{V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta}{r} + V_z\frac{\partial V_\theta}{\partial z}}\right) = \rho f_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta}$$

$V_r = V_\theta = 0$

$$+\mu\left[\cancel{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta} + \frac{\partial^2 V_\theta}{\partial z^2}}\right]$$

$$\Downarrow$$

$$0 = -\rho g\cos\theta - \frac{1}{r}\frac{\partial p}{\partial \theta}$$



z 方向

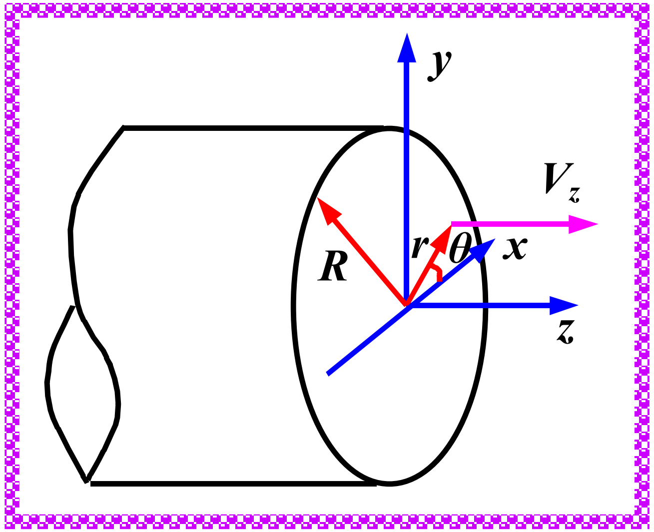
steady　　$V_r = V_\theta = 0$　　fully developed

$$\rho\left(\cancel{\frac{\partial V_z}{\partial t}} + \cancel{V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta}} + V_z\cancel{\frac{\partial V_z}{\partial z}}\right) = \cancel{\rho f_z} - \frac{\partial p}{\partial z}$$

$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 V_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 V_z}{\partial z^2}}\right]$$

fully developed

symmetric flow



$$0 = -\frac{\partial p}{\partial z} + \mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)$$

$$0 = -\rho g \sin\theta - \frac{\partial p}{\partial r}$$

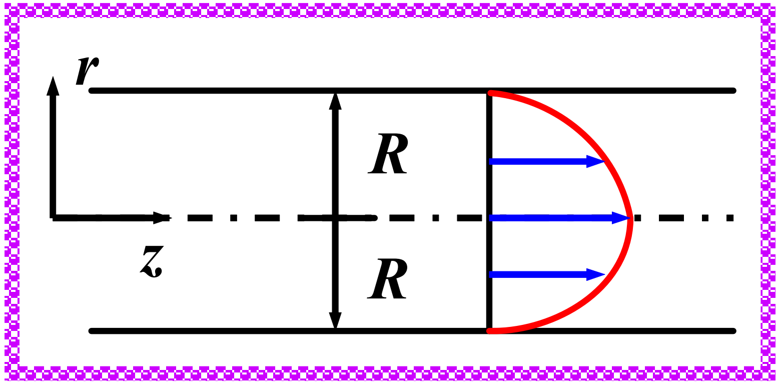
$$0 = -\rho g \cos\theta - \frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$0 = -\frac{\partial p}{\partial z} + \mu \frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial V_z}{\partial r} \right]$$

$$V_z = \frac{1}{4\mu}\frac{\partial p}{\partial z} r^2 + C_1 \ln r + C_2$$

$$V_z = \frac{1}{4\mu}\frac{\partial p}{\partial z}r^2 + C_1 \ln r + C_2$$

Poiseuille flow



边界条件

$r = R$ ⇨ $V_z = 0$

$r = 0$ ⇨ V_z 为有限值 ⇨ $C_1 = 0$

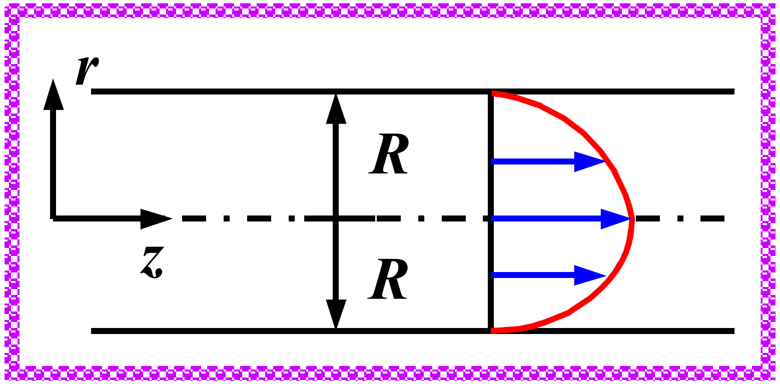
⇨ $$\boxed{V_z = \frac{1}{4\mu}\frac{\partial p}{\partial z}(r^2 - R^2)}$$

抛物线分布

parabolic profile

设相距为 l 的两点压降为Δp

$$V_z = \frac{1}{4\mu}\frac{\Delta p}{l}\left(R^2 - r^2\right)$$



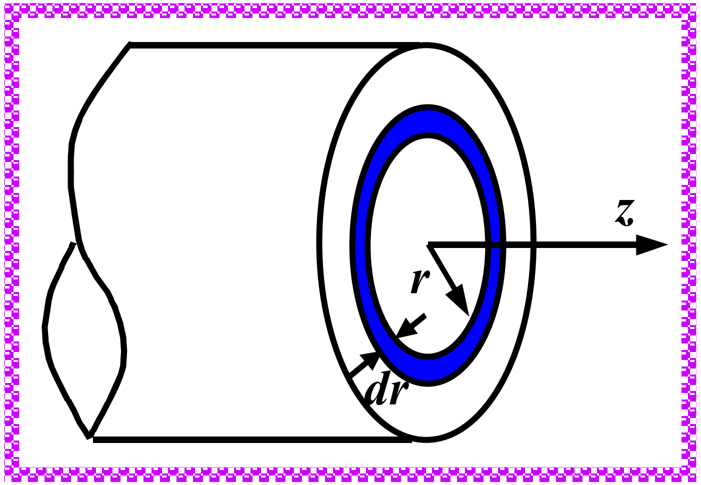
截面最大速度

$$V_{z\max} = \frac{R^2}{4\mu}\frac{\Delta p}{l}$$

哈根—泊肃叶（Hagen-Poiseuille）定律

$$dQ = V_z 2\pi r dr$$

$$Q = \frac{\pi R^4}{8\mu}\frac{\Delta p}{l} = \frac{\pi D^4}{128\mu}\frac{\Delta p}{l}$$



- 与精密实验的测定结果完全一致，验证了层流理论和实践的完美一致性

截面平均速度 $\Rightarrow$ $\bar{V}=\dfrac{Q}{A}=\dfrac{R^2\Delta p}{8\mu l}$

$\Rightarrow$ $\boxed{V_{z\max}=2\bar{V}_z}$

- 层流状态下圆管过流断面上的速度分布很不均匀
- $\alpha=2.0$（动能修正系数）

压强分布

$$p = -\rho g r \sin\theta + f(z)$$

$$\Rightarrow \quad p = -\rho g r \sin\theta + \frac{\partial p}{\partial z} z + C$$

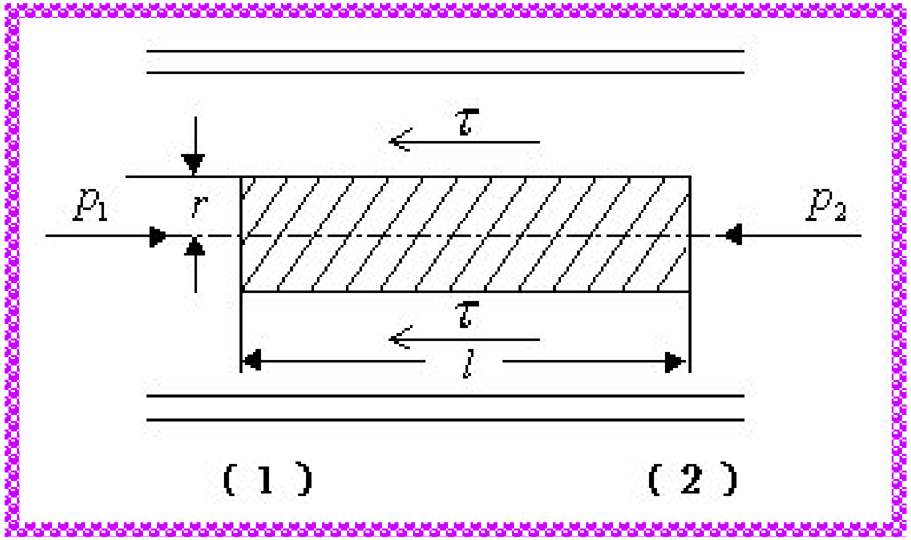
设 $z = 0$，$r = 0$ 时，$p = p_0$

$$\Rightarrow \quad \boxed{p = p_0 - \rho g r \sin\theta + \frac{\partial p}{\partial z} z}$$

粘性力和重力导致压强变化

解二： *elemental approach*

圆柱体在圆管轴线方向受力平衡



$$\tau = \frac{\Delta p}{2l} r$$

当 $r = R$ $\Rightarrow$ $\tau_w = \frac{\Delta p}{2l} R$ $\Rightarrow$ $\tau = \frac{r}{R} \tau_w$

园管过流断面上切应力沿径向成线性分布

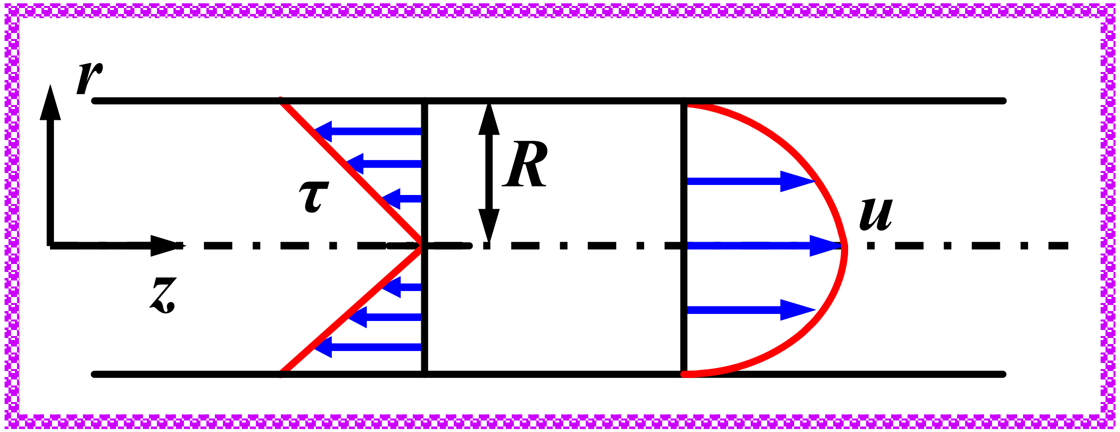
速度分布

$$\boxed{\tau = -\mu \frac{dV_z}{dr}}$$

$$\boxed{\frac{\Delta p}{l} = \frac{2\tau}{r}}$$

$$\Rightarrow \quad \frac{dV_z}{dr} = -\frac{r\Delta p}{2\mu l} \quad \Rightarrow \quad V_z = -\frac{1}{4\mu}\frac{\Delta p}{l} r^2 + C$$

$$\Rightarrow \quad V_z = \frac{1}{4\mu}\frac{\Delta p}{l}(R^2 - r^2)$$

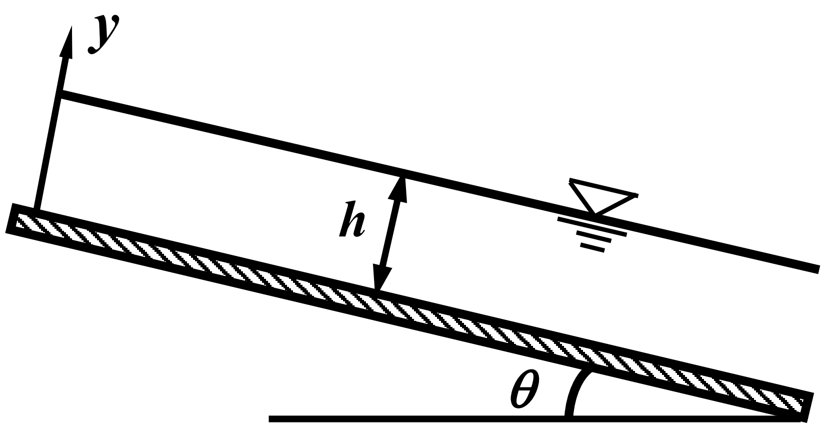


层流－降膜流动1

降膜流动：不可压缩流体在倾斜平板上呈液膜状向下流动，液膜厚度 h 不变，表面与大气接触。流动是定常层流流动

- 降膜流动在湿壁塔、冷凝器、蒸发器及产品涂层方面有广泛的应用

靠重力产生，特点是液膜的一侧与大气接触，沿流动方向没有压力差



层流－降膜流动2

解： 定常、层流 ⟹ $v = w = 0 \quad \dfrac{\partial u}{\partial t} = \dfrac{\partial u}{\partial z} = 0$

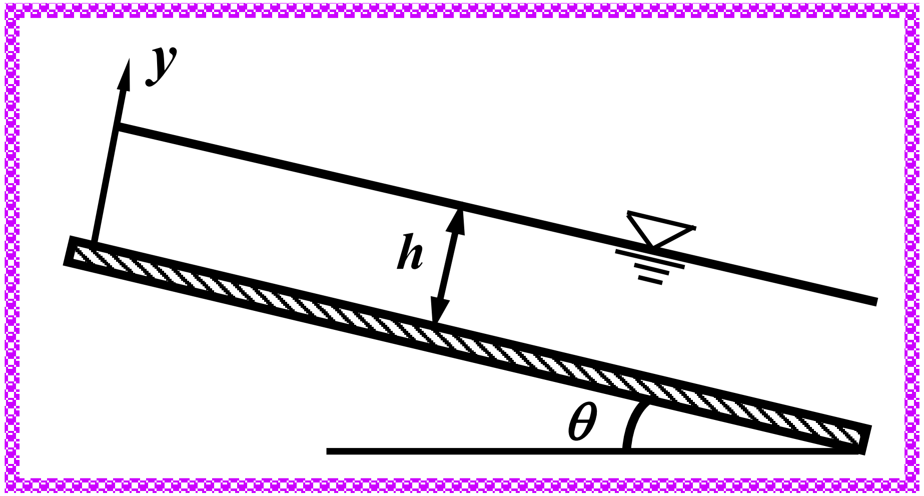
连续方程 ⟹ $\dfrac{\partial u}{\partial x} = 0 \Rightarrow u = u(y)$

N-S方程

$$0 = \rho g \sin\theta - \frac{\partial p}{\partial x} + \mu \frac{d^2 u}{dy^2}$$

$$0 = -\rho g \cos\theta - \frac{\partial p}{\partial y}$$

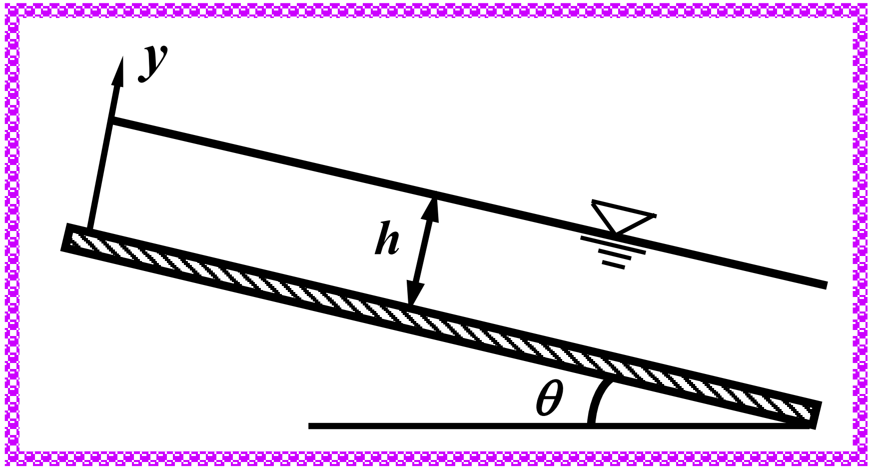
$$0 = \frac{\partial p}{\partial z}$$



$$\frac{\partial p}{\partial y}=-\rho g\cos\theta \Longrightarrow p=-\rho g y\cos\theta+f(x)$$

$$y=h \Longrightarrow p=p_0$$

$$p=-\rho g(y-h)\cos\theta+p_0$$



$$\boxed{\frac{\partial p}{\partial x}=0} \Longrightarrow \frac{d^2u}{dy^2}=-\frac{\rho g\sin\theta}{\mu}$$

$$u = -\frac{\rho g \sin\theta}{\mu}\frac{y^2}{2} + C_1 y + C_2$$

定解条件

$y = 0 \Rightarrow u = 0$ $\qquad$ $y = h \Rightarrow \dfrac{du}{dy} = 0$

$$\Rightarrow \quad u = \frac{\rho g \sin\theta}{\mu} y\left(h - \frac{y}{2}\right)$$

体积流量 $\Rightarrow dQ = udy \Rightarrow \boxed{Q = \frac{\rho g \sin\theta}{3\mu} h^3}$

时均速度 → $$\bar{u}=\frac{1}{T}\int_{t_0}^{t_0+T}u(t)dt$$ *time-averaged velocity*

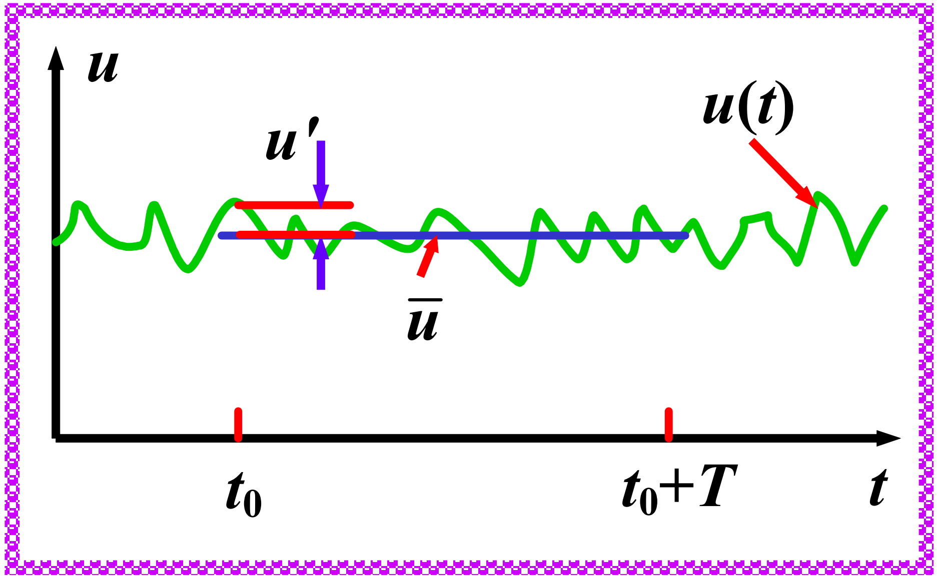
速度脉动周期 << T << 时均速度的不稳定变化周期

瞬时速度 → $u(t)$

instantaneous velocity

脉动速度 *fluctuating velocity*

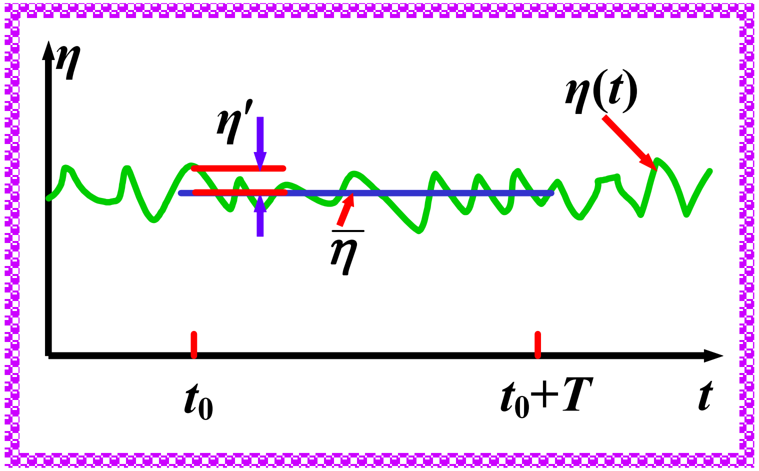
→ $u'=u-\bar{u}$



时均物理量 → $$\overline{\eta} = \frac{1}{T}\int_{t_0}^{t_0+T} \eta(t)dt$$ *time-averaged variables*

脉动物理量 → $$\eta' = \eta - \overline{\eta}$$

$$p = \overline{p} + p' \quad , \quad T = \overline{T} + T'$$
$$\rho = \overline{\rho} + \rho'$$



脉动物理量的时均值

→ $\overline{\eta'} = 0$　脉动在时均物理量两侧的分布机会均等

湍动能 $\Rightarrow$

$$K=\frac{\overline{\left|\vec{V}'\right|^2}}{2}=\frac{\overline{(u')^2}+\overline{(v')^2}+\overline{(w')^2}}{2}$$

湍流度 $\Rightarrow$

$$I=\sqrt{\frac{1}{3}\left[\overline{(u')^2}+\overline{(v')^2}+\overline{(w')^2}\right]}\Big/\overline{U}_\infty$$

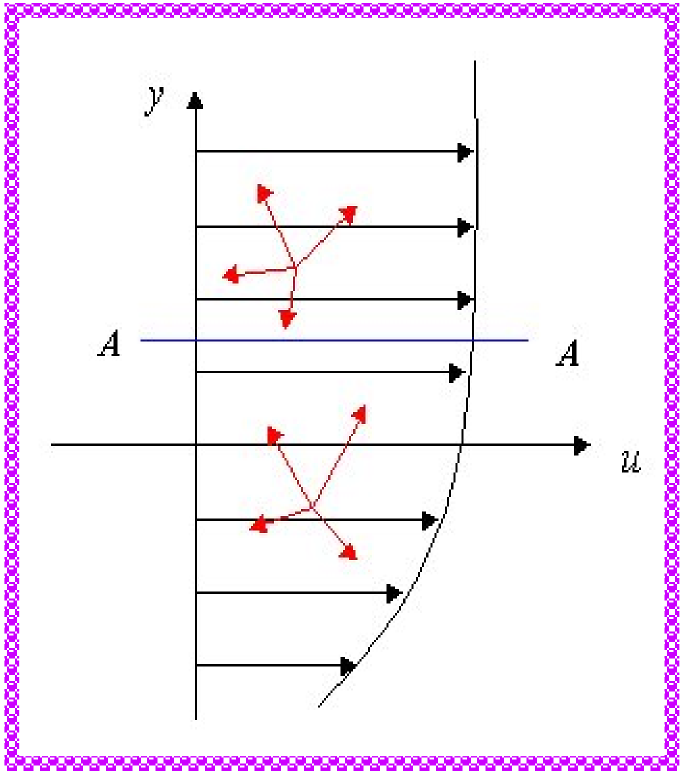
其中 $$\overline{(u')^2}=\frac{1}{T}\int_{t_0}^{t_0+T}(u')^2\,dt\quad>0$$

- 湍流度越大，速度的脉动幅度越大，相应的其它参数的脉动幅度也越大

层流切应力 ⇐ 分子粘性应力

$$\tau_{\text{lam}} = \mu \frac{du}{dy}$$

laminar (viscous) shear stress



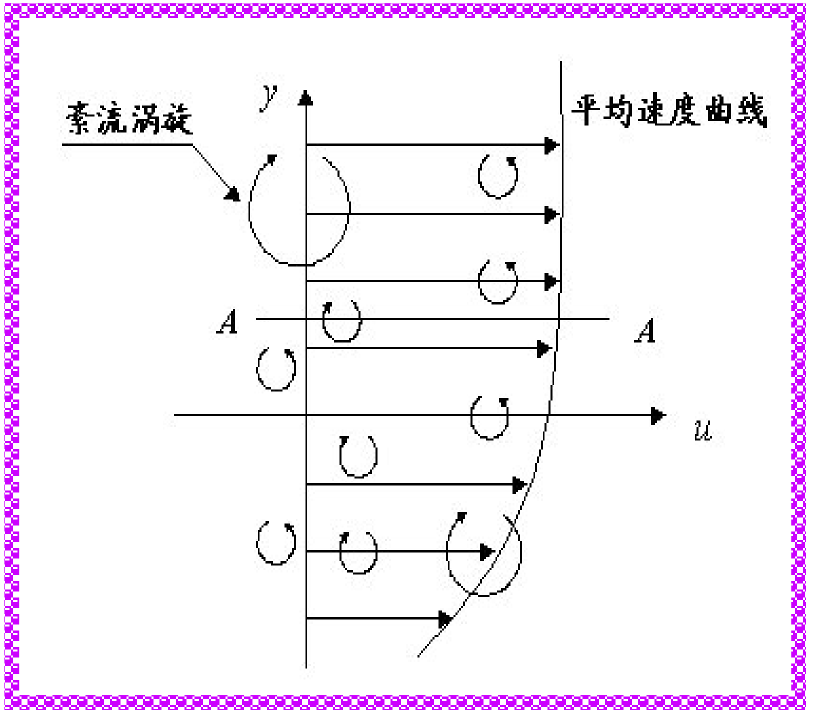
液体 ⇒ 分子间内聚力

气体 ⇒ 分子热运动

湍流切应力

分子间内聚力，分子热运动

$$\tau_{\text{lam}} = \mu \frac{d\bar{u}}{dy}$$



流体质点的随机运动

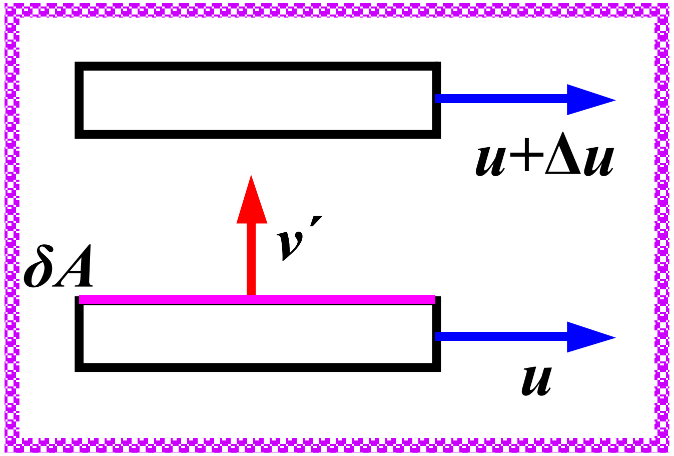
湍流附加应力(雷诺应力)

turbulent shear stress

Reynolds stress

雷诺应力

单位时间通过 δA 的流体微团 x 方向动量的时均值



$$\Rightarrow \quad \overline{\rho v'\delta A(\bar{u}+u')} = \rho\overline{u'v'}\delta A$$

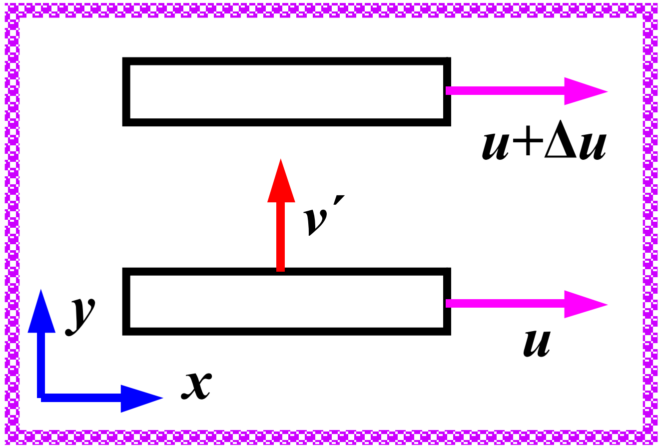
由动量对时间的变化率＝外力之和

$$\Rightarrow \quad \rho\overline{u'v'} \qquad \text{应力}$$

流体质点由下层→上层

$v' > 0 \Rightarrow u' < 0$

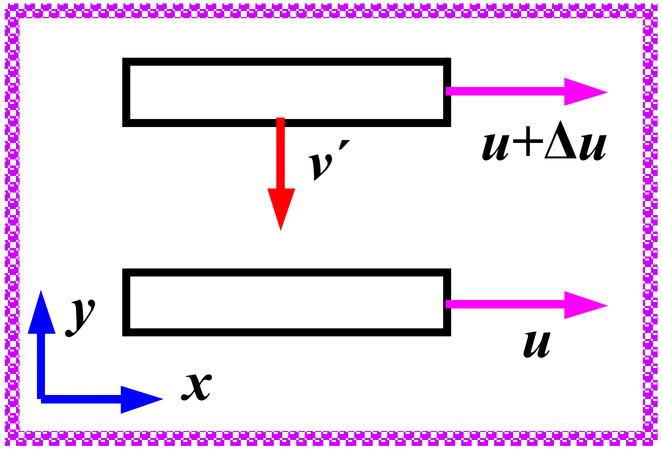
$\Rightarrow \overline{u'v'} < 0$



流体质点由上层→下层

$v' < 0 \Rightarrow u' > 0$

$\Rightarrow \overline{u'v'} < 0$



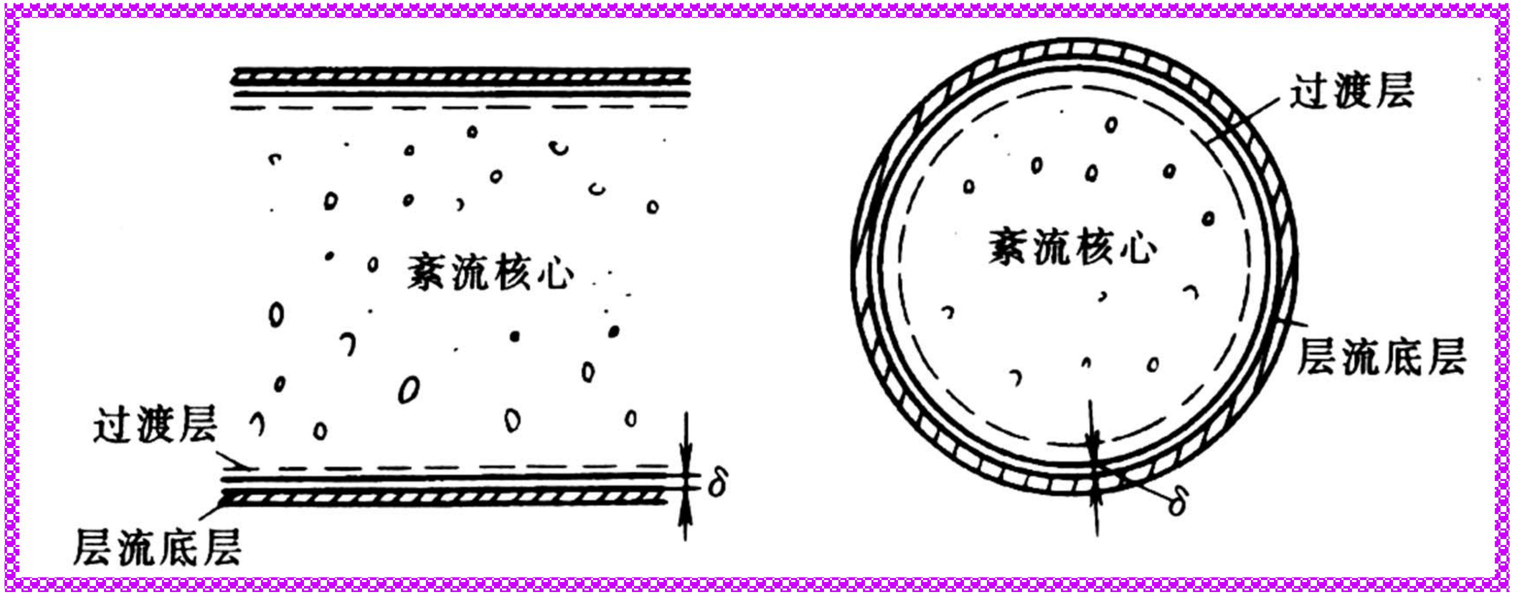
雷诺（湍流附加）应力 $\Rightarrow$ $\tau = -\rho\overline{u'v'}$

$$\tau = \tau_{\text{lam}} + \tau_{\text{turb}} \boxed{= \mu \frac{d\bar{u}}{dy} - \rho \overline{u'v'}}$$

分子粘性应力（层流切应力、粘性切应力）是由流体层间**分子内聚力及分子热运动**引起的

雷诺应力（湍流附加应力）是由**流体微团的脉动**进而产生动量横向传递引起的

湍流切应力2



层流底层的厚度

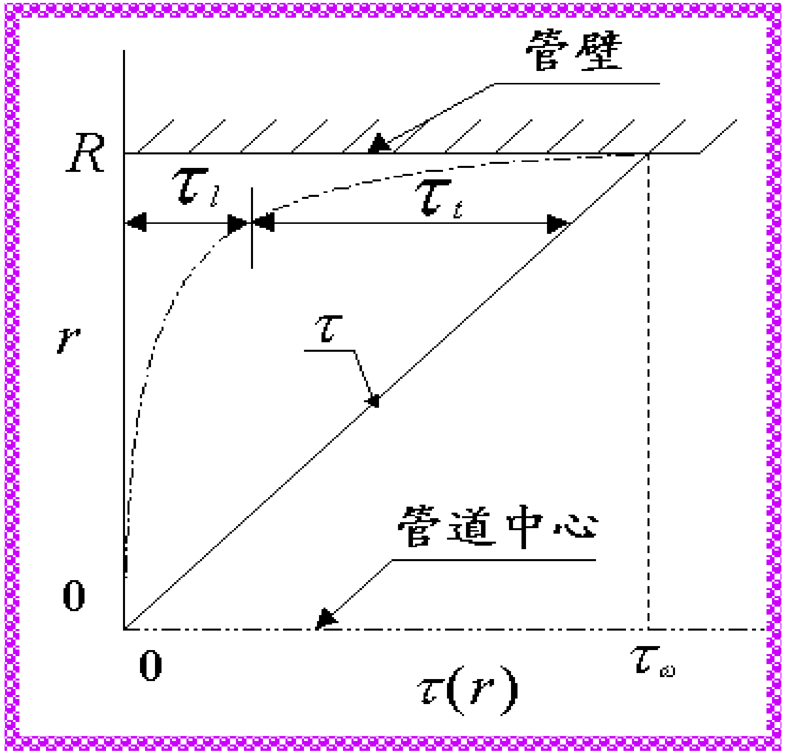
$$\delta = \frac{14.1d}{\mathrm{Re}\sqrt{f}}$$

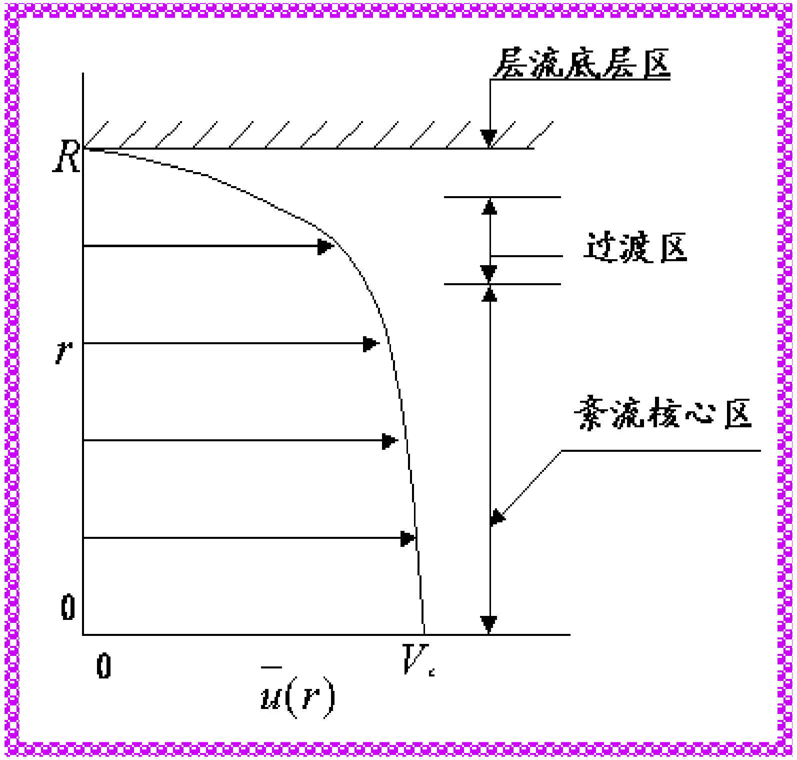
Re 数越大，层流底层越薄

viscous sublayer

viscous wall layer

$$\tau=\tau_{\text{lam}}+\tau_{\text{turb}}=\mu\frac{d\bar{u}}{dy}-\rho\overline{u'v'}$$





应力张量

$$\begin{bmatrix} \bar{\sigma}_{xx} & \bar{\tau}_{xy} & \bar{\tau}_{xz} \\ \bar{\tau}_{yx} & \bar{\sigma}_{yy} & \bar{\tau}_{yz} \\ \bar{\tau}_{zx} & \bar{\tau}_{zy} & \bar{\sigma}_{zz} \end{bmatrix} + \begin{bmatrix} -\rho\overline{u'^2} & -\rho\overline{u'v'} & -\rho\overline{u'w'} \\ -\rho\overline{u'v'} & -\rho\overline{v'^2} & -\rho\overline{v'w'} \\ -\rho\overline{u'w'} & -\rho\overline{v'w'} & -\rho\overline{w'^2} \end{bmatrix}$$

分子粘性应力　　　　雷诺应力

- 六个独立的雷诺应力分量，需补充六个本构方程

雷诺平均数值模拟（RANS）

Reynolds Averaged Navier-Stokes simulation

- 以时均流动控制方程组为基础
- 湍流模式理论

Boussinesq 涡粘性假设 ⇒ $$-\rho\overline{u'v'}=\eta\frac{d\bar{u}}{dy}$$

- η 取决于流体的种类、流场的结构、流动条件等 *eddy viscosity*

零方程模型（代数模型） → 只需补充代数方程

普朗特（Prandtl）混合长度模型

$$\eta = \rho l^2 \left| \frac{d\bar{u}}{dy} \right|$$

其中 $l = Ky$ → 混合长度 *mixing length*

两方程模型 → 需补充两个微分方程

K - ε 模型 ⇒ $$\eta = C_\mu \rho \frac{K^2}{\varepsilon}$$

其中

$$K = \frac{1}{2}\overline{u'^2 + v'^2 + w'^2}$$

湍动能

turbulent kinetic energy

$$\varepsilon = \nu\overline{\left[\left(\frac{\partial u'}{\partial x}\right)^2 + \left(\frac{\partial u'}{\partial y}\right)^2 + \left(\frac{\partial u'}{\partial z}\right)^2 + \left(\frac{\partial v'}{\partial x}\right)^2 + \left(\frac{\partial v'}{\partial y}\right)^2 + \left(\frac{\partial v'}{\partial z}\right)^2 + \left(\frac{\partial w'}{\partial x}\right)^2 + \left(\frac{\partial w'}{\partial y}\right)^2 + \left(\frac{\partial w'}{\partial z}\right)^2\right]}$$

湍动能耗散率

dissipation rate of turbulent kinetic energy

直接数值模拟 (DNS)

Direct numerical simulation

- 直接求解三维非定常的N-S方程，可以获得流场的全部信息

大涡模拟 (LES)

Large eddy simulation

大尺度 – 直接求解

小尺度 – 建立模型

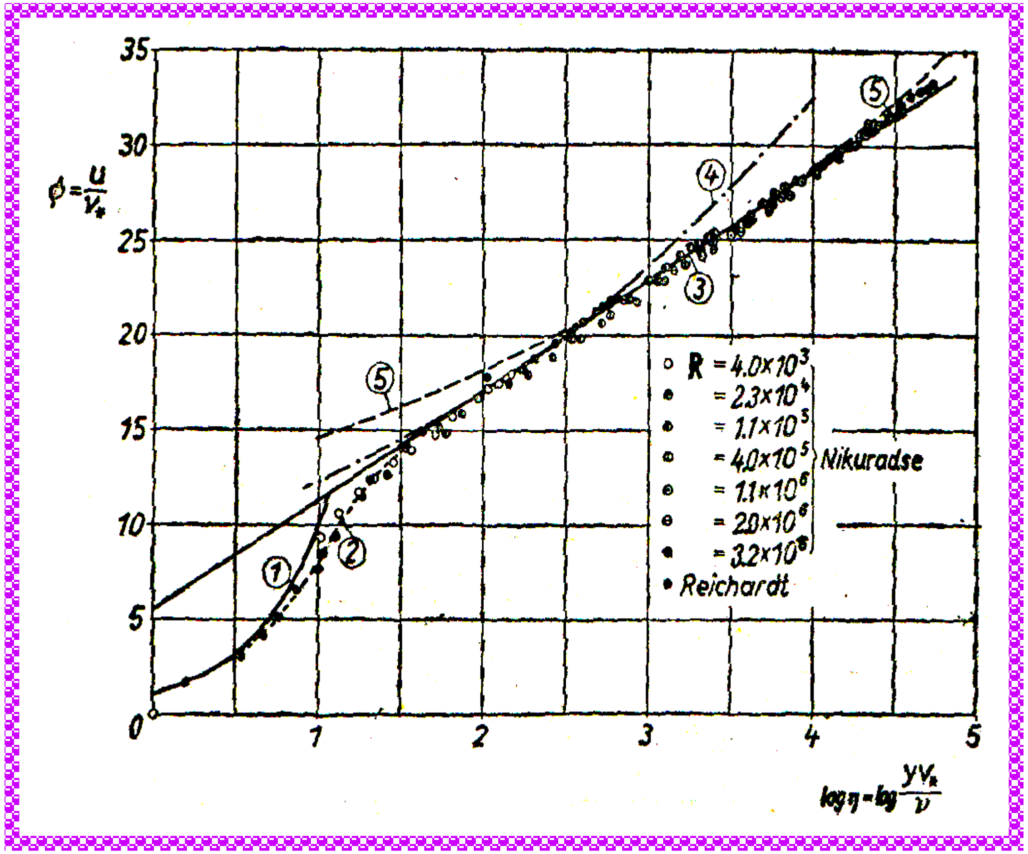
光滑圆管—速度分布主要依据实验测量确定

近壁区层流底层

$$\frac{\overline{u}}{u_*} = \frac{y u_*}{\nu}$$

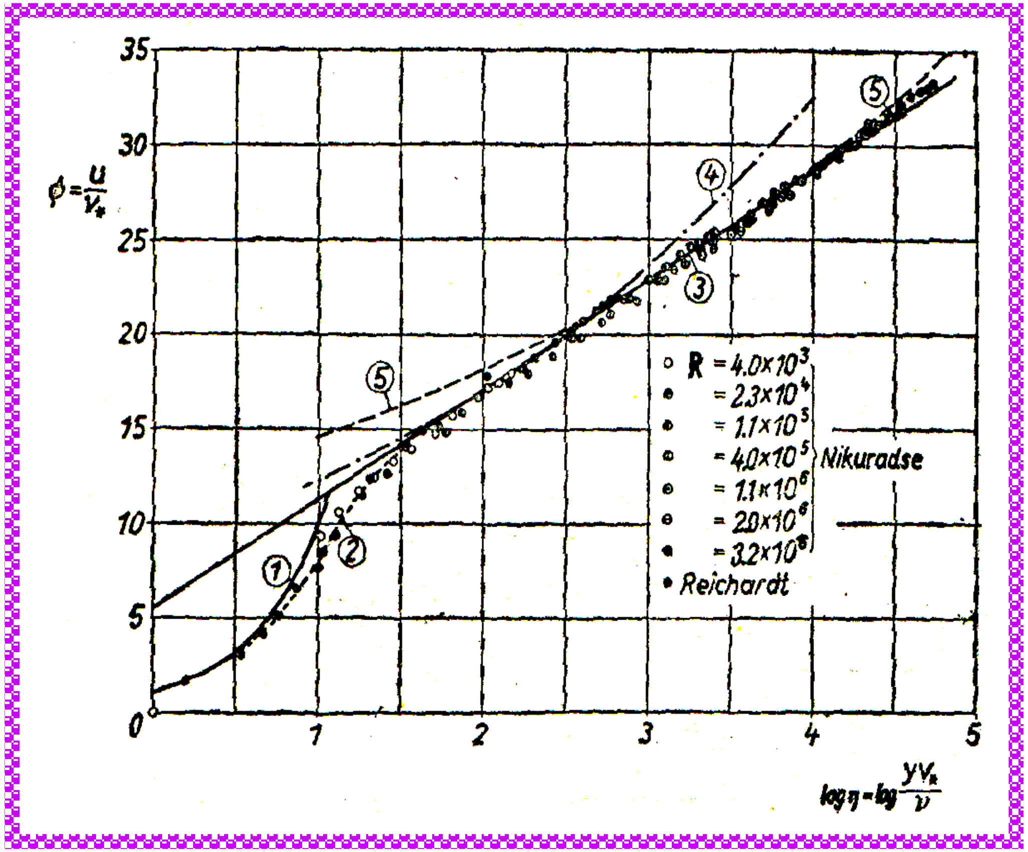
$$y^+ = \frac{y u_*}{\nu} \le 5$$

y：到壁面的距离



u_*： 摩擦速度　*friction velocity*

$$u_* = \sqrt{\frac{\tau_w}{\rho}}$$



层流底层厚度 δ_s

$$\boxed{\delta_s = \frac{5\nu}{u_*}}$$

过渡区　$5 < y^+ < 30$

光滑圆管内的湍流速度分布3

湍流核心区

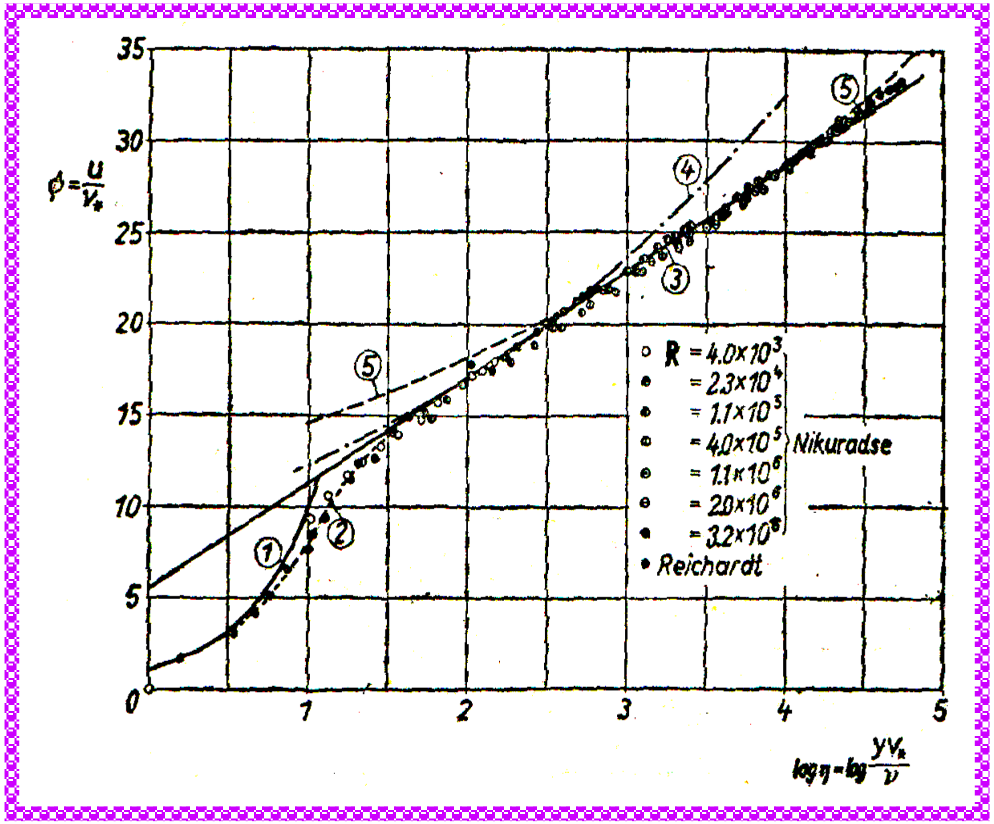
$y^+ \geq 30$

对数分布律

logarithm law profile

$$\frac{\bar{u}}{u_*} = 5.75\lg\frac{yu_*}{\nu} + 5.5$$

$$= 2.5\ln\frac{yu_*}{\nu} + 5.5$$



管中心区域

$$\frac{U - \bar{u}}{u_*} = 2.5\ln\frac{R}{y}$$

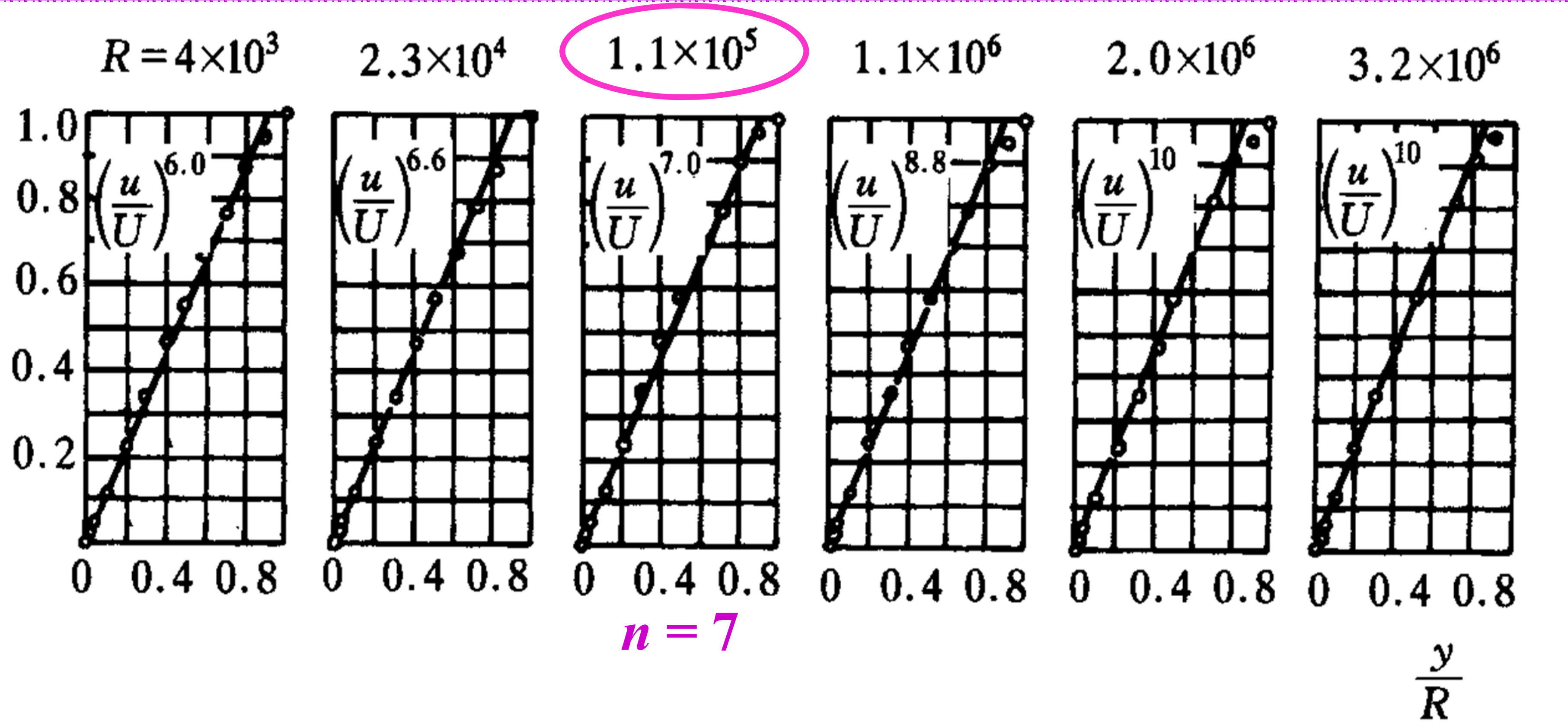
velocity defect law

光滑圆管内的湍流速度分布4

幂次分布律

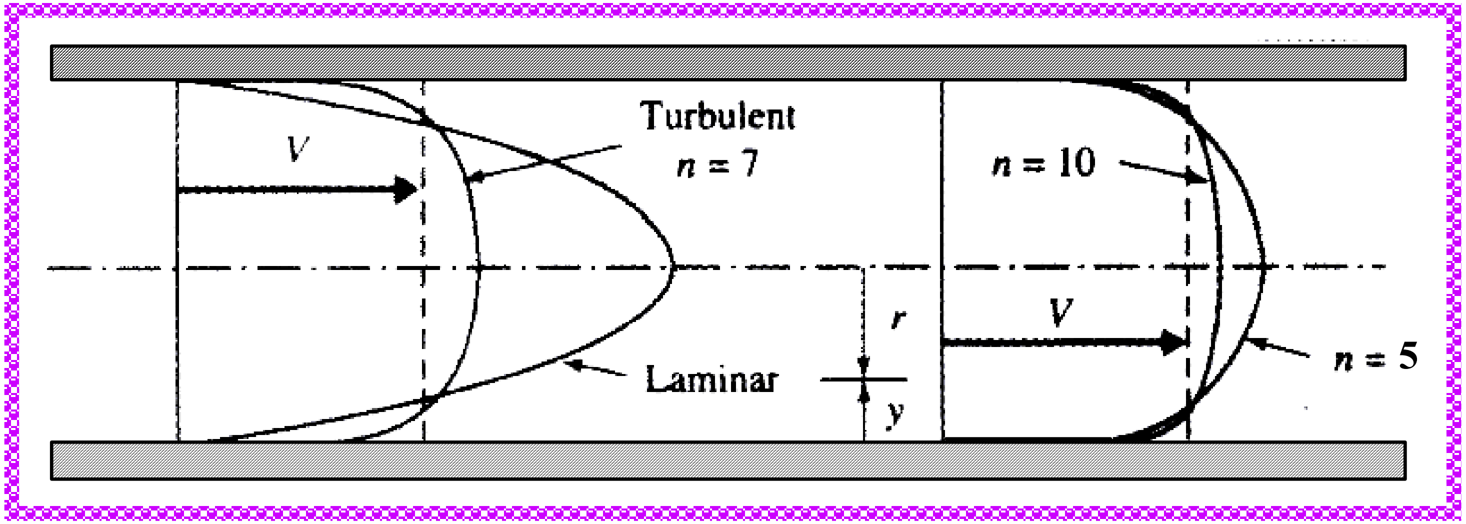
power law profile

$$\frac{\bar{u}}{U}=\left(\frac{y}{R}\right)^{1/n}=\left(1-\frac{r}{R}\right)^{1/n}$$



平均速度与管中心速度的比

$$\frac{\bar{V}}{U}=0.8\sim0.85$$



$$\bar{V}=\frac{Q}{A}=\frac{1}{\pi R^2}\int_0^R 2\pi r\bar{u}dr$$

$$\Rightarrow \frac{\bar{V}}{U}=\frac{2n^2}{(n+1)(2n+1)}$$

$n=6 \Rightarrow \bar{V}/U=0.79$

$n=10 \Rightarrow \bar{V}/U=0.87$

作业：P.182～184

- 5 . 2
- 5 . 9
- 5 . 14

小结1

粘性流体中一点的应力状态

由九个应力分量确定，其中六个是独立的

本构方程

应力与变形速度的关系

小结2

流动的两种状态

层流、湍流、流动状态的判据 Re

N-S方程精确解

- 简化模型：定常、均质不可压缩、质量力只有重力、二维流动
- 边界条件：无滑移、园管中心、自由面等

小结3

切应力

- 层流：分子间内聚力、分子热运动

$$\tau = \mu \frac{du}{dy}$$

- 湍流：分子间内聚力、分子热运动、流体质点随机运动导致的紊流附加应力

$$\tau = \mu \frac{d\bar{u}}{dy} - \rho \overline{u'v'}$$

小结4

两无限大平板间的充分发展层流

- 速度分布规律
- 平均速度与最大速度之间的关系
- 压强分布规律
- 库埃特流速度分布与压强梯度的关系

圆管内充分发展层流

- 速度分布规律
- 平均速度与最大速度之间的关系
- 切应力沿径向的分布规律
- 湍流时，两种不同原因引起的切应力在层流底层及湍流核心区中的分布

小结6

有关湍流的几个概念

- 时均物理量及其与平均物理量的区别
- 脉动物理量及其时均值、湍动能、湍流度

小结7

其它公式

圆管内层流速度分布 ⇒

$$V_z = \frac{1}{4\mu}\frac{\Delta p}{l}\left(R^2 - r^2\right)$$

圆管内切应力分布

⇒

$$\tau = \frac{\Delta p}{2l} r$$

$$\tau = \frac{r}{R}\tau_w$$