HVAC Energy Cost Optimization for a Multizone Building via a Decentralized Approach

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***Abstract*— The control of heating, ventilation, and air-conditioning (HVAC) systems has raised extensive attention due to their high energy consumption cost and operation patterns far from being energy-efficient. However, most of the existing methods suffer limitations in scalability and computational efficiency for large buildings due to the centralized structures. To compensate for such defects, this article studies the scalable control of multizone HVAC systems with the target to reduce energy cost while maintaining thermal comfort. In particular, the thermal couplings due to heat transfer among the adjacent zones are incorporated, which has been ignored or not well studied due to complexity in the literature. To overcome the computational challenges of the nonlinear and nonconvex problem caused by the complex system dynamics, this article proposes a decentralized approach composed of three main steps: 1) relaxing the bilinear system dynamics; 2) solving the relaxed problem in a decentralized manner using the accelerated distributed augmented Lagrangian (ADAL) method; and 3) recovering the recursive feasibility of the solution. Through a comparison with the centralized method, the suboptimality of this approach is demonstrated. In addition, the superior performance of this approach is illustrated through a comparison with the distributed token-based scheduling strategy (DTBSS). The numerical results imply that for buildings with a relatively small number of zones (less than 20), the two methods are competitive. However, for larger cases, the proposed approach performs better with a considerable reduction both in energy cost and computation time.**

***Note to Practitioners*—This article develops a scalable computing framework for multizone HVAC controller, which can help reduce the electricity cost while maintaining the comfortable temperature bands set by the occupants. The proposed computing paradigm is scalable to large buildings as it mainly requires local zone-level computation. Specifically, for multizone buildings with a central shared HVAC system, the zone controllers determine the operations of their local variable air volume (VAV) boxes mainly through local computation and some necessary communications with the other entities. Consequently, this control technique can be implemented via local controllers deployed on simple hardware while applied to large buildings. The performance (electricity cost and computation time) of the controller is validated through a comparison with the centralized method and the DTBSS by using simulations.**

***Index Terms*—Decentralized methods, energy cost, heating, ventilation, and air-conditioning (HVAC) system, multizone buildings, thermal couplings.**

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Nomenclature

c_p	The specific heat of air $[\mathrm{kJ/(kg \cdot K)}]$.
T_t^i	Temperature of zone i at time t (°C).
$\underline{T}^i/\overline{T}^i$	The lower/upper comfortable temperature bound for zone i (°C).
$\underline{m}^{zi}/\overline{m}^{zi}$	The lower/upper zone air flow rate bound for zone i $(\mathrm{kg\,s^{-1}})$.
$\overline{m}$	The upper bound of supply air flow rate by the AHU $(\mathrm{kg\,s^{-1}})$.
c_t	The electricity price at time t (s\$/kW).
Q_t^i	The internal heat generation of zone i at time t (kW).
R_{oi}	The thermal resistance between zone i and the outside (K/kW).
R_{ij}	The thermal resistance between zone i and zone j (K/kW).
$\mathcal{N}_i$	The collection of adjacent zones of zone i.
d_r	The fraction of return air delivered to AHU.
T_c	The set-point temperature of the AHU (°C).
P_t^f	The fan power of AHU at time t (kW).
P_t^c	The cooling power of AHU at time t (kW).
η	The reciprocal of coefficient of performance (COP) of the chiller.
κ_f	The efficiency of fan within the AHU.

I. Introduction

As WE may know, buildings are responsible for a large proportion of the world's energy consumption [3], wherein about 40%–50% is attributed to heating, ventilation, and air-conditioning (HVAC) [3]. The number is more threatening for tropical countries like Singapore due to the perennial demand [2]. Such a high proportion of the HVAC system's energy consumption considerably ascribes to the operation patterns far from being energy-efficient [2].

Therefore, the control of HVAC systems has raised extensive investigations in the literature. The available methods include model predictive control (MPC) [4]–[7], mixed-integer linear programming (MILP) [8], [9], sequential quadratic programming (SQP) [10]–[12], intelligent control based on fuzzy logic or genetic algorithm [13]–[15], and rule-based methods [16], [17]. As with centralized structure, most of these methods tend to be computationally prohibitive or not scalable to large buildings. Decentralized methods have been recognized as viable solutions to such an issue and raised considerable concerns. However, most of the existing decentralized methods focus on temperature regulation with the control of HVAC systems circumvented and not discussed due to the system complexity [18]–[20]. The available results of scalable control of the multizone HVAC system are fairly limited and in deficit except for [2], [21]–[23]. Moreover, the thermal couplings among the neighboring zones are usually ignored [21], [22] or not well-tackled [2], [23] due to the computational complexity, which tend to the result in the performance deviations in application.

To compensate for such defect, this article aims to develop a scalable (decentralized) control method for multizone HVAC systems to reduce the energy cost while respecting thermal comfort. In particular, the thermal couplings among the adjacent zones are explicitly studied. It is challenging and nontrivial to achieve such a goal concerning the following complexity. *First*, there exist various couplings in the multizone network. The couplings both arise from the heat transfer among adjacent zones and the operation limits of the air handling unit (AHU), which leads to various temporally and spatially coupled constraints. *Second*, the problem is nonlinear and nonconvex due to complex system behaviors. Consequently, most of the existing decentralized methods established for convex problems with linear dynamics (see [24]–[26]) cannot be directly applied to solve the problem.

Main Contributions: To overcome the challenges, this article develops a decentralized approach, which mainly contains three steps: 1) the first step relaxes the bilinear system dynamics by introducing some auxiliary decision variables; 2) the second step solves a relaxed nonconvex problem in a decentralized manner based on an accelerated distributed augmented Lagrangian (ADAL) method proposed in [1]; 3) the last step recovers the recursive feasibility of solutions by the proposed heuristic method. As it allows the zones to compute their mass flow rates for local variable air volume (VAV) boxes by solving small-scale QP problems in parallel, this method is scalable to large buildings. To evaluate its performance, the approach is compared with a centralized method, which can obtain the optimal solutions for small-scale cases. The results imply that the decentralized approach can achieve a *suboptimal* solution. In addition, the superior performance of the proposed approach is illustrated through a comparison with the distributed token-based scheduling strategy (DTBSS) [2]. The numerical results report that for buildings with relatively a small number of zones (less than 20), the decentralized approach slightly outperforms DTBSS with about $2\% - 4\%$ reduction in energy cost. However, for larger cases, the proposed method performs better with a considerable reduction both in energy cost and computation time.

The remainder of this article is outlined. In Section II, the related works are reviewed. In Section III, the problem formulation is presented. In Section IV, the decentralized approach is introduced. In Section V, the performance and scalability of the decentralized approach are validated through simulations. In Section VI, we briefly conclude this article.

II. Related Works

This section reviews the available decentralized methods on thermal comfort control. Generally, these methods can be broadly divided into two categories based on the decision variables. The first category is mainly focused on the zone temperature regulation [18]–[20]. Specifically, the zone temperature is regarded as the decision variables to be computed. However, the control of HVAC systems (e.g., AHU and VAV boxes) to achieve such zone temperature has been circumvented or not discussed mainly due to the complex system dynamics. Sophisticatedly, the other category usually directly focuses on the control of HVAC systems. For such cases, a challenging and nontrivial problem that involves the complex system dynamics and various zone couplings usually needs to be handled. To simplify the discussions and alleviate the computational challenges, most of the existing works ignored the thermal couplings among the neighboring zones [21], [22] or regarded them as external disturbances that assumed to be measured through sensors or learned from data [2], [23]. As a scarce attempt, reference [27] explicitly discussed the thermal couplings while developing a distributed MPC strategy for the multizone HVAC system. To cope with the difficulties due to the nonlinearity and nonconvexity, a distributed alternating direction method of multipliers (ADMM) method [26] was applied based on some convexity approximations.

In general, thermal couplings among neighboring zones have not been well-studied due to the complexity while developing scalable control methods for multizone HVAC systems, which may lead to performance deviations both in energy cost and thermal conditions in practice. Complementary to the existing works, this article studies the decentralized control of multizone HVAC system, which incorporates the thermal couplings among the neighboring zones in the optimization.

III. Problem Formulation

A. HVAC Systems for Multizone Buildings

A typical schematic of a multizone HVAC system is shown in Fig. 1. The main parts include the AHU, the VAV boxes, and the chiller. The central AHU is shared by multizones, which is equipped with a damper, a cooling/heating coil, and a supply fan. The damper is responsible for mixing the return air from inside and the fresh air from outside. The heating/cooling coil can cool down/heat up the mixed air to a set-point temperature. Without loss of generality, this article investigates the cooling mode. In such case, the set-point temperature of AHU is 12 °–16 °C. There usually exists a local VAV box composed

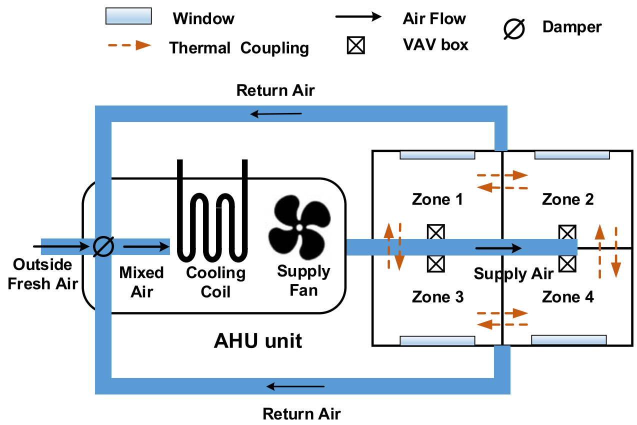

Fig. 1. Schematic of multizone HVAC systems.

of a damper and a heating coil attached to each zone. This damper can regulate zone air flow rates, and the heating coil can reheat the supply air if necessary (not discussed in this article). We refer the readers to [10], [28] for more details about the multizone HVAC systems.

This article aims to reduce the energy cost of multizone HVAC systems while still maintaining zone thermal comfort over the optimization horizon. The problem is discussed in a discrete-time framework with the optimization horizon (one day) equally divided into $T = 48$ stages, corresponding to a decision interval $\Delta_t = 30$ mins.

B. Zone Thermal Dynamics

This article studies the control of a multizone building with I thermal zones indexed by $\mathcal{I} = \{1, 2, \ldots, I\}$ over the optimization horizon $\mathcal{T} = \{0, 1, \ldots, T-1\}$. The thermal dynamics of zone i ($\forall i \in \mathcal{I}$) can be described by the resistance–capacitance (RC) network [29], [30], that is

$$\begin{aligned} C_i\left(T_{t+1}^i - T_t^i\right) &= \sum_{j \in \mathcal{N}_i} \frac{T_t^j - T_t^i}{R_{ij}} \Delta_t \\ &\quad + \frac{T_t^o - T_t^i}{R_{oi}} \Delta_t + c_p m_t^{zi}\left(T_c - T_t^i\right)\Delta_t + Q_t^i \Delta_t \end{aligned} \tag{1}$$

where C_i is the zone air heat capacity. The internal zone heat generation Q_t^i is affected by multiple factors, such as occupancy, devices, and solar radiation.

If we define $A^{ii} = 1 - (\sum_{j \in \mathcal{N}_i}(\Delta_t / R_{ij} C_i) + (\Delta_t / C_i R_{oi}))$, $A^{ij} = (\Delta_t / C_i R_{ij})$, $C^{ii} = -(\Delta_t \cdot c_p / C_i)$, and $D_t^{ii} = (\Delta_t T_t^o / C_i R_{oi}) + (\Delta_t \cdot Q_t^i / C_i)$, the zone thermal dynamics in (2) can be compactly described as follows:

$$T_{t+1}^i = A^{ii} T_t^i + \sum_{j \in \mathcal{N}_i} A^{ij} T_t^j + C^{ii} m_t^{zi}\left(T_t^i - T_c\right) + D_t^{ii}. \tag{2}$$

C. AHU

As introduced in Section II-A, the AHU is responsible for cooling down the mixed air to the set-point temperature. The main parameters related to the AHU include the following: 1) d_r ($0 \le d_r \le 1$): the fraction of the return air from inside; 2) T_c: the set-point temperature of the supply air; 3) T_t^r: the average temperature of the return air from inside; 4) T_t^m: the average temperature of the mixed air. The settings of d_r and T_c are usually fixed and based on experiences, whereas T_t^r and T_t^m are dynamically changing with the system state (i.e., zone temperature) and the control of the HVAC system (i.e., zone air flow rates). Specifically, the average temperature of the return air from inside at time t can be estimated as follows:

$$T_t^r = \frac{\sum_{i=1}^I m_t^{zi} T_t^i}{\sum_{i=1}^I m_t^{zi}}. \tag{3}$$

Accordingly, the average temperature of the mixed air at time t can be calculated as follows:

$$\begin{aligned} T_t^m &= (1 - d_r) T_t^o + d_r T_t^r \\ &= (1 - d_r) T_t^o + d_r \frac{\sum_{i=1}^I m_t^{zi} T_t^i}{\sum_{i=1}^I m_t^{zi}}. \end{aligned} \tag{4}$$

As the AHU has mainly composed of the cooling coil and the supply fan, its total energy consumption is mainly produced by the two parts. Specifically, the cooling power of the cooling coil can be calculated as follows:

$$\begin{aligned} P_t^c &= c_p \eta \left(\sum_{i=1}^I m_t^{zi}\right)\left(T_t^m - T_c\right) \\ &= c_p \eta (1 - d_r) \sum_{i=1}^I m_t^{zi}\left(T_t^o - T_c\right) + c_p \eta d_r \sum_{i=1}^I m_t^{zi}\left(T_t^i - T_c\right). \end{aligned} \tag{5}$$

As one may note, the cooling power in (5) may be interpreted as two parts, i.e.: 1) the first part results from the proportion of fresh air from outside and 2) the second part caused by the proportion of return air from inside.

According to [22] and [27], the energy consumption of the supply fan within the AHU can be estimated as follows:

$$P_t^f = \kappa_f \left(\sum_{i=1}^I m_t^{zi}\right)^3. \tag{6}$$

Thus, we collect the total power consumption of the HVAC system at time t as follows:

$$\begin{aligned} P_t^c + P_t^f &= c_p (1 - d_r) \sum_{i=1}^I m_t^{zi}\left(T_t^o - T_c\right) \\ &\quad + c_p d_r \sum_{i=1}^I m_t^{zi}\left(T_t^i - T_c\right) + \kappa_f \left(\sum_{i=1}^I m_t^{zi}\right)^3. \end{aligned} \tag{7}$$

From above, one may note that the power consumption of the HVAC system is a nonlinear function with respect to the control and state variables m_t^{zi} and T_t^i ($i \in \mathcal{I}$).

D. System Constraints

The operation of the HVAC system is subject to: 1) the zone thermal comfort requirements in (8a); 2) the operation limits of the local VAV boxes in (8b); and 3) the operation limits of the AHU in (8c)

$$\underline{T}^i \leq T_t^i \leq \overline{T}^i \quad \forall i \in \mathcal{I},\ t \in \mathcal{T}. \tag{8a}$$

$$\underline{m}^{zi} \leq m_t^{zi} \leq \overline{m}^{zi} \quad \forall i \in \mathcal{I},\ t \in \mathcal{T}. \tag{8b}$$

$$\textstyle\sum_{i=1}^{I} m_t^{zi} \leq \overline{m} \quad \forall t \in \mathcal{T}. \tag{8c}$$

As shown in (8a), this article uses zone temperature ranges to describe the zone thermal comfort requirements. To accommodate personalized thermal comfort, $\underline{T}^i$ and $\overline{T}^i$ may be customized. The lower/upper bounds of zone air flow rates ($\underline{m}^{zi}/\overline{m}^{zi}$) depend on the pressure supplied by the fan in the duct system [2].

E. Optimization Problem

Overall, the optimization problem on minimizing the HVAC system's energy cost while maintaining the zone thermal comfort is presented as follows:

$$\min_{m_t^{zi}, T_t^i} J = \sum_{t=0}^{T-1} c_t \cdot \left\{ P_t^c + P_t^f \right\} \cdot \Delta_t$$

$$\textbf{Constraints: } (2), (8a) - (8c). \tag{$\mathcal{P}_1$}$$

One may note that (9) is nonlinear and nonconvex. The nonlinearity and nonconvexity both arise from the objective function and the constraints. Moreover, the objective function is nonseparable and nondecomposable with respect to the zones. Such characteristics on the problem structures make it challenging and nontrivial to find a scalable solution method.

IV. Decentralized Approach

To cope with the computational challenges, this section proposes a decentralized approach for solving (9), which mainly contains three steps: 1) the first step relaxes problem (9) by introducing some auxiliary decision variables; 2) the second step solves a relaxed nonconvex problem in a decentralized manner using the ADAL method [1]; and 3) the third step recovers the recursive feasibility of the solution based on a heuristic method. The details of the three steps are specified in the following.

Step 1: To deal with the nonlinear (bilinear) constraints in (9), we first introduce the following auxiliary decision variables, i.e., $X_t^i = m_t^{zi}(T_t^i - T_c) \geq 0$ ($\forall i \in \mathcal{I}, t \in \mathcal{T}$) and $Y_t = \sum_{i=1}^{I} m_t^{zi}$ ($\forall t \in \mathcal{T}$). We note that X_t^i can be interpreted as the "cooling power" of zone i at time t, whereas Y_t can be regarded as the total zone air flow rate supplied by the AHU at time t.

According to [31] and [27], these auxiliary decision variables X_t^i ($\forall i \in \mathcal{I}$, $t \in \mathcal{T}$) are bounded by their convex and concave envelopes, that is

$$\begin{aligned}
X_t^i &= m_t^{zi}\left(T_t^i - T_c\right) \\
&\geq \max\Big\{\underline{m}^{zi}(T_t^i - T_c) + m_t^{zi}(\underline{T}^i - T_c) - \underline{m}^{zi}(\underline{T}^i - T_c), \\
&\qquad \overline{m}^{zi}(T_t^i - T_c) + m_t^{zi}(\overline{T}^i - T_c) - \overline{m}^{zi}(\overline{T}^i - T_c)\Big\} \\
X_t^i &= m_t^{zi}\left(T_t^i - T_c\right) \\
&\leq \min\Big\{m_t^{zi}(\overline{T}^i - T_c) + \underline{m}^{zi}(T_t^i - T_c) - \underline{m}^{zi}(\overline{T}^i - T_c), \\
&\qquad \overline{m}^{zi}(T_t^i - T_c) + m^{zi}(\underline{T}^i - T_c) - \overline{m}^{zi}(\underline{T}^i - T_c)\Big\} \\
&\forall i \in \mathcal{I}, \quad t \in \mathcal{T}.
\end{aligned} \tag{9}$$

Consequently, by introducing those auxiliary decision variables, we can obtain the relaxed problem (10a), that is

$$\begin{aligned}
\min_{m_t^{zi}, T_t^i, X_t^i, Y_t} J = \sum_{t=0}^{T-1} c_t
&\cdot \Big\{ c_p \eta (1 - d_r)\left(T_t^o - T_c\right) Y_t \\
&+ \kappa_f (Y_t)^3 + c_p \eta d_r \sum_{i=1}^{I} X_t^i \Big\} \cdot \Delta_t
\end{aligned}$$

$$\text{s. t. } (8a) - (8c) \tag{$\mathcal{P}_2$}$$

$$T_{t+1}^i = A^{ii} T_t^i + \sum_{j \in \mathcal{N}_i} A^{ij} T_t^j + C_{ii} X_t^i + D_t^{ii} \tag{10a}$$

$$X_t^i \geq \underline{m}^{zi}(T_t^i - T_c) + m_t^{zi}(\underline{T}^i - T_c) - \underline{m}^{zi}(\underline{T}^i - T_c) \tag{10b}$$

$$X_t^i \geq \overline{m}^{zi}\left(T_t^i - T_c\right) + m_t^{zi}(\overline{T}^i - T_c) - \overline{m}^{zi}(\overline{T}^i - T_c) \tag{10c}$$

$$X_t^i \leq m_t^{zi}(\overline{T}^i - T_c) + \underline{m}^{zi}\left(T_t^i - T_c\right) - \underline{m}^{zi}(\overline{T}^i - T_c) \tag{10d}$$

$$X_t^i \leq \overline{m}^{zi}\left(T_t^i - T_c\right) + m^{zi}(\underline{T}^i - T_c) - \overline{m}^{zi}(\underline{T}^i - T_c) \tag{10e}$$

$$\forall i \in \mathcal{I},\ t \in \mathcal{T}.$$

$$\sum_{i=1}^{I} m_t^{zi} = Y_t \quad \forall t \in \mathcal{T} \tag{10f}$$

where constraints (10a) inherit constraints (2) of (9). The constraints (10b)–(10e) and (10f) related to the auxiliary decision variables X_t^i and Y_t directly duplicate constraints (9) and the definitions, respectively.

Step 2: We note that (10a) is still nonconvex due to the global objective function. However, there only exist linear (coupled) constraints, and this problem can be tackled by the ADAL method proposed in [1] in a decentralized manner. More specifically, we assume that there are $I + 1$ agents indexed by $\mathcal{I} \cup \{0\}$, wherein the collection of agents in $\mathcal{I}$ corresponds to the I zones, and the virtual Agent 0 is defined to manage the total zone air flow rates supplied by the AHU.

For Agent i associated with zone i ($\forall i \in \mathcal{I}$), the local decision variables at time t can be gathered as follows:

$$\boldsymbol{x}_t^i = \left(T_t^i, m_t^{zi}, X_t^i\right)^T \quad \forall i \in \mathcal{I},\ t \in \mathcal{T}. \tag{11}$$

For Agent 0, the local decision variable can be defined as follows:

$$\boldsymbol{x}_t^0 = Y_t \quad \forall t \in \mathcal{T}. \tag{12}$$

For notation, we use the vector $\boldsymbol{x}^i = [(\boldsymbol{x}_0^i)^T, (\boldsymbol{x}_1^i)^T, \ldots, (\boldsymbol{x}_{T-1}^i)^T]^T$ to represent the local decision variables for Agent i ($\forall i \in \mathcal{I} \cup \{0\}$) over the optimization horizon $\mathcal{T}$.

In this case, the global objective function in (10a) is decomposable with respect to the agents $\mathcal{I} \cup \{0\}$, that is

$$J_0(\boldsymbol{x}^0) = \sum_{t=0}^{T-1} c_t \cdot \left\{ c_p \eta (1-d_r)\left(T_t^o - T_c\right) Y_t + \kappa_f (Y_t)^3 \right\} \cdot \Delta_t.$$
$$J_i(\boldsymbol{x}^i) = \sum_{t=0}^{T-1} c_t \cdot \left\{ c_p \eta d_r X_t^i \right\} \cdot \Delta_t \quad \forall i \in \mathcal{I} \tag{13}$$

where $J_i(\boldsymbol{x}^i)$ ($\forall i \in \mathcal{I} \cup \{0\}$) can be regarded as the local objective function of Agent i.

Problem (10a) can be written in a compact form as (14). The details of the transformation are given in the Appendix

$$\begin{aligned} &\min_{\boldsymbol{x}^i, i=0,1,2,\dots,I} \sum_{i=0}^{I} J_i(\boldsymbol{x}_i) \\ &\text{s.t. } \boldsymbol{A}_d^{ii} \boldsymbol{x}^i + \sum_{j \in \mathcal{N}_i} \boldsymbol{A}_d^{ij} \boldsymbol{x}^j = \boldsymbol{b}_d^i \quad \forall i \in \mathcal{I}. \\ &\quad \sum_{i=0}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i = \boldsymbol{0}. \\ &\quad \sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i \leq \boldsymbol{c}_d. \\ &\quad \boldsymbol{x}^i \in \mathcal{X}^i \quad \forall i \in \mathcal{I} \end{aligned} \tag{$\mathcal{P}_3$}$$

where we have

$$\boldsymbol{A}_d^{ii} = \begin{pmatrix} \overline{A}^{ii} & -\mathbf{I}_1 & \mathbf{0} & \cdots & \cdots\cdots \\ \mathbf{0} & \overline{A}^{ii} & -\mathbf{I}_1 & \mathbf{0} & \cdots\cdots \\ \mathbf{0} & \mathbf{0} & \overline{A}^{ii} & -\mathbf{I}_1 & \mathbf{0} & \cdots \end{pmatrix} \in \mathbb{R}^{(T-1)\times 3T}$$
$$\boldsymbol{A}_d^{ij} = \begin{pmatrix} \overline{A}^{ij} & \mathbf{0} & \cdots & \cdots\cdots \\ \mathbf{0} & \overline{A}^{ij} & \mathbf{0} & \cdots\cdots \\ \mathbf{0} & \mathbf{0} & \overline{A}^{ij} & \mathbf{0} & \cdots \end{pmatrix} \in \mathbb{R}^{(T-1)\times 3T}$$

and $\boldsymbol{b}_d^i = (\overline{D}_0^{ii}, \overline{D}_1^{ii}, \dots, \overline{D}_{T-1}^{ii})^T \in \mathbb{R}^{T-1}$, with $\overline{A}^{ii} = (A^{ii} \;\; 0 \;\; C^{ii})$, $\overline{A}^{ij} = (A^{ij} \;\; 0 \;\; 0)$ $(i, j \in \mathcal{I})$. $\mathbf{I}_1 = (1 \; 0 \; 0)$. $\overline{D}_t^{ii} = -D_t^{ii}$.

$$\boldsymbol{B}_d^i = \begin{pmatrix} \overline{B}^i & \mathbf{0} & \cdots\cdots\cdots \\ \mathbf{0} & \overline{B}^i & \mathbf{0} & \cdots\cdots \\ \mathbf{0} & \mathbf{0} & \overline{B}^i & \mathbf{0} & \cdots \end{pmatrix} \in \mathbb{R}^{T\times 3T}$$

with $\overline{B}^i = (0 \; 1 \; 0)$ $(\forall i \in \mathcal{I})$

$$\boldsymbol{B}_d^0 = \begin{pmatrix} \overline{B}^0 & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \overline{B}^0 & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \overline{B}^0 & \cdots \end{pmatrix} \in \mathbb{R}^{T\times T}$$

with $\overline{B}^0 = (-1)\boldsymbol{c}_d = (\overline{m} \;\; \overline{m} \; \cdots \; \overline{m})^T \in \mathbb{R}^T$. The sets $\mathcal{X}^i$ $(i \in \mathcal{I})$ represent the collection of admissible control trajectories for Agent i, which is constructed by the local constraints (8a)–(8c) and (10b)–(10e) related to zone i.

As the ADAL method [1] is effective in tackling equality constraints, we transform the (coupled) inequality constraints in (14) into (coupled) equality constraints by resorting to some nonnegative slack decision variables $\boldsymbol{s}$ as discussed in [32]. In this regard, (14) is equivalent to (14), that is

$$\begin{aligned} &\min_{\boldsymbol{x}^i, i=0,1,2,\dots,I, \boldsymbol{s}^1, \boldsymbol{s}^2} \sum_{i=0}^{I} J_i(\boldsymbol{x}_i) \\ &\text{s.t. } \boldsymbol{A}_d^{ii} \boldsymbol{x}^i + \sum_{j \in \mathcal{N}_i} \boldsymbol{A}_d^{ij} \boldsymbol{x}^j = \boldsymbol{b}_d^i, \quad \forall i \in \mathcal{I}. \\ &\quad \sum_{i=0}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i = \boldsymbol{0}. \\ &\quad \sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i - \boldsymbol{c}_d + \boldsymbol{s} = \boldsymbol{0}. \\ &\quad \boldsymbol{x}^i \in \mathcal{X}^i \quad \forall i \in \mathcal{I}. \\ &\quad \boldsymbol{s} \geq \boldsymbol{0}. \end{aligned} \tag{$\mathcal{P}_4$}$$

When the ADAL method is applied to tackle (14), we can define the following augmented Lagrangian function to eliminate the coupled equality constraints, that is

$$\begin{aligned} &\mathbb{L}_\rho(\boldsymbol{x}^0, \boldsymbol{x}^1, \dots, \boldsymbol{x}^I, \boldsymbol{s}, \boldsymbol{\lambda}, \boldsymbol{\gamma}, \boldsymbol{\eta}) \\ &= \sum_{i=0}^{I} J_i + \sum_{i=1}^{I} (\boldsymbol{\lambda}^i)^T \Big(\boldsymbol{A}_d^{ii} \boldsymbol{x}^i + \sum_{j \in \mathcal{N}_i} \boldsymbol{A}_d^{ij} \boldsymbol{x}^j - \boldsymbol{b}_d^i \Big) \\ &\quad + \sum_{i=1}^{I} \frac{\rho}{2} \Big\| \boldsymbol{A}_d^{ii} \boldsymbol{x}^i + \sum_{j \in \mathcal{N}_i} \boldsymbol{A}_d^{ij} \boldsymbol{x}^j - \boldsymbol{b}_d^i \Big\|^2 \\ &\quad + \boldsymbol{\gamma}^T \Big(\sum_{i=0}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i + \boldsymbol{s}^1 \Big) + \frac{\rho}{2} \Big\| \sum_{i=0}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i + \boldsymbol{s}^1 \Big\|^2 \\ &\quad + \boldsymbol{\eta}^T \Big(\sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i - \boldsymbol{c}_d + \boldsymbol{s}^2 \Big) + \frac{\rho}{2} \Big\| \sum_{i=1}^{I} \boldsymbol{B}_d^i \boldsymbol{x}^i - \boldsymbol{c}_d + \boldsymbol{s}^2 \Big\|^2 \end{aligned} \tag{14}$$

where $\boldsymbol{\lambda} = [(\boldsymbol{\lambda}^1)^T, (\boldsymbol{\lambda}^2)^T, \dots, (\boldsymbol{\lambda}^I)^T]^T$ with $\boldsymbol{\lambda}^i \in \mathbb{R}^{(T-1)\times 3T}$, and $\boldsymbol{\gamma}, \boldsymbol{\eta} \in \mathbb{R}^T$ are the Lagrangian multipliers. $\rho > 0$ is the penalty parameter.

Thus, the primal problem of (14) can be described as follows:

$$\begin{aligned} &\min_{\boldsymbol{x}^0, \boldsymbol{x}^1, \dots, \boldsymbol{x}^I, \boldsymbol{s}} \mathbb{L}_\rho(\boldsymbol{x}^0, \boldsymbol{x}^1, \dots, \boldsymbol{x}^I, \boldsymbol{s}^1, \boldsymbol{s}^2, \boldsymbol{\lambda}, \boldsymbol{\gamma}, \boldsymbol{\eta}) \\ &\text{s.t. } \boldsymbol{x}^i \in \mathcal{X}^i \quad \forall i \in \mathcal{I} \\ &\quad \boldsymbol{s} \geq \boldsymbol{0}. \end{aligned} \tag{15}$$

Similar to the standard method of multipliers (MMs), such as ADMM [26], the ADAL mainly includes two steps while applied to solve (14): 1) solving the primal problem (15) in a decentralized manner and 2) updating the Lagrangian multipliers $\boldsymbol{\gamma}, \boldsymbol{\eta}$. The details of the algorithm are displayed in Algorithm 1. The superscript k represents the iteration. We use $\boldsymbol{x}^k = ((\boldsymbol{x}^{0,k})^T, (\boldsymbol{x}^{1,k})^T, \dots, (\boldsymbol{x}^{I,k})^T)^T$ to represent the collection of control trajectories for all zones at iteration k, and $\boldsymbol{x}^{-i,k} = ((\boldsymbol{x}^{0,k})^T, \dots, (\boldsymbol{x}^{i-1,k})^T, (\boldsymbol{x}^{i+1,k})^T, \dots, (\boldsymbol{x}^{I,k})^T)^T$ denotes the collection of control trajectories for all zones except zone i. The local objective functions for Agent i $(i \in \mathcal{I} \cup \{0\})$ at iteration k are defined as

Algorithm 1 ADAL

1: **Initialization:** $k \leftarrow 0$, set $\boldsymbol{\lambda}^0$, $\boldsymbol{\gamma}^0$, $\boldsymbol{\eta}^0$, $\boldsymbol{x}^{i,0}$ $(\forall i \in \{0\} \cup \mathcal{I})$ and $s^{1,0}$, $\boldsymbol{s}^{2,0}$.

2: **Iteration:**

3: **Solve the primal problem** (15)**:**

4: Solve Subproblem 0, that is,

$$\begin{aligned}(\boldsymbol{x}^{0,k+1}, \boldsymbol{s}^{k+1}) = \ & \arg\min_{\boldsymbol{x}^0, \boldsymbol{s}} \mathbb{L}_\rho^0(\boldsymbol{x}^0, \boldsymbol{s}, \boldsymbol{x}^{-0,k}, \boldsymbol{\lambda}^k, \boldsymbol{\gamma}^k, \boldsymbol{\eta}^k)\\ & \text{s. t. } \ \boldsymbol{x}^0 \in \mathbb{R}.\\ & \qquad \boldsymbol{s} \geq \boldsymbol{0}. \end{aligned} \tag{16}$$

5: Solve the subproblems in $\mathcal{I}$ in parallel, that is,

$$\begin{aligned}\boldsymbol{x}^{i,k+1} = \ & \arg\min_{\boldsymbol{x}^i} \mathbb{L}_\rho^i(\boldsymbol{x}^i, \boldsymbol{x}^{-i,k}, \boldsymbol{s}^k, \boldsymbol{\lambda}^k, \boldsymbol{\gamma}^k, \boldsymbol{\eta}^k)\\ & \text{s. t. } \ \boldsymbol{x}^i \in \mathcal{X}^i. \end{aligned} \tag{17}$$

6: If the stopping criterion (20) is satisfied, then stop with the solution $\boldsymbol{x}^{k+1,*}$, otherwise continue.

7: **Update the Lagrangian multipliers:**

$$\begin{aligned}\boldsymbol{\lambda}^{i,k+1} &= \boldsymbol{\lambda}^{i,k} + \rho\left(\boldsymbol{A}_d^{ii}\boldsymbol{x}^{i,k+1} + \sum_{j\in\mathcal{N}_i}\boldsymbol{A}_d^{ij}\boldsymbol{x}^{j,k+1} - \boldsymbol{b}_d^i\right)\\ &\quad \forall i \in \mathcal{I}\\ \boldsymbol{\gamma}^{k+1} &= \boldsymbol{\gamma}^k + \rho\left(\sum_{i=0}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^{i,k+1}\right)\\ \boldsymbol{\eta}^{k+1} &= \boldsymbol{\eta}^k + \rho\left(\sum_{i=1}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^{i,k+1} - \boldsymbol{c}_d + \boldsymbol{s}^{k+1}\right)\end{aligned}$$

8: Set $k \to k+1$ and go to **Step** 3.

follows:

$$\begin{aligned}&\mathbb{L}_\rho^0(\boldsymbol{x}^0, \boldsymbol{s}, \boldsymbol{x}^{-0,k}, \boldsymbol{\lambda}^k, \boldsymbol{\gamma}^k, \boldsymbol{\eta}^k)\\ &= J_0(\boldsymbol{x}^0) + \boldsymbol{\gamma}^T(\boldsymbol{B}_d^0\boldsymbol{x}^0) + \frac{\rho}{2}\left\|\boldsymbol{B}_d^0\boldsymbol{x}^0 + \sum_{i=1}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^{i,k}\right\|^2\\ &\quad + \boldsymbol{\eta}^T\boldsymbol{s} + \frac{\rho}{2}\left\|\sum_{i=1}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^{i,k} - \boldsymbol{c}_d + \boldsymbol{s}\right\|^2 \end{aligned} \tag{18}$$

$$\begin{aligned}&\mathbb{L}_\rho^i\left(\boldsymbol{x}^i, \boldsymbol{x}^{-i,k}, \boldsymbol{s}^k, \boldsymbol{\lambda}^k, \boldsymbol{\gamma}^k, \boldsymbol{\eta}^k\right)\\ &= J_i(\boldsymbol{x}^i) + \sum_{j\in\mathcal{N}_i\cup\{i\}}\left(\boldsymbol{\lambda}_j^k\right)^T\boldsymbol{A}_d^{ji}\boldsymbol{x}^i\\ &\quad + \frac{\rho}{2}\sum_{j\in\mathcal{N}_i\cup\{i\}}\left\|\boldsymbol{A}_d^{ji}\boldsymbol{x}^i + \sum_{l\in\mathcal{N}_j}\boldsymbol{A}^{jl}\boldsymbol{x}^{l,k} - \boldsymbol{b}_d^j\right\|^2\\ &\quad + (\boldsymbol{\gamma}^k)^T\boldsymbol{B}_d^i\boldsymbol{x}^i + \frac{\rho}{2}\left\|\boldsymbol{B}_d^i\boldsymbol{x}^i + \sum_{l=0,l\neq i}^{I}\boldsymbol{B}_d^l\boldsymbol{x}^{l,k}\right\|^2\\ &\quad + (\boldsymbol{\eta}^k)^T\boldsymbol{B}_d^i\boldsymbol{x}^i + \frac{\rho}{2}\left\|\boldsymbol{B}_d^i\boldsymbol{x}^i + \sum_{l=1,l\neq i}^{I}\boldsymbol{B}_d^l\boldsymbol{x}^l - \boldsymbol{c}_d + \boldsymbol{s}^k\right\|^2\\ &\forall i \in \mathcal{I}. \end{aligned} \tag{19}$$

The residual error of all the coupled constraints is selected as the stopping criterion with ϵ a constant threshold, that is

$$\begin{aligned}&r(\boldsymbol{x}^k, \boldsymbol{s}^k)\\ &= \sum_{i=1}^{I}\left\|\boldsymbol{A}_d^{ii}\boldsymbol{x}^{i,k} + \sum_{j\in\mathcal{N}_i}\boldsymbol{A}_d^{ij}\boldsymbol{x}^{j,k} - \boldsymbol{b}_d^i\right\|_2\\ &\quad + \left\|\sum_{i=0}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^{i,k}\right\|_2 + \left\|\sum_{i=1}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^{i,k} - \boldsymbol{c}_d + \boldsymbol{s}^k\right\|_2 \leq \epsilon. \end{aligned} \tag{20}$$

We note that the subproblems in $\mathcal{I}$ are quadratic programming (QP) problems, which can be tackled efficiently by many existing toolboxes, such as CPLEX, and Subproblem 0 is a small-scale nonlinear optimization problem, which can also be solved efficiently.

Step 3: As introduced, (14) is a relaxed version of the original optimization problem (9). It is not difficult to note that both the zone thermal comfort and the operation limits of the HVAC system can be guaranteed by solving (14). However, the recursive feasibility of the solution cannot be assured due to the introduction of the auxiliary decision variables X_t^i $(i \in \mathcal{I})$. To address such an issue, this section proposes a heuristic method to recover the recursive feasibility of the solution while still guaranteeing a satisfactory performance (the HVAC system's energy cost and zone thermal comfort) by exploring the special problem structures. Specifically, we note that the decision variables X_t^i $(i \in \mathcal{I})$ not only "dominate" the HVAC system's cost (compared with the decision variable m_t^{zi}, κ_f is relatively small) but also determine the zone temperature. Therefore, if high priority is distributed to X_t^i when recovering the recursive feasibility of the solution, the performance of the recovered solution can be retained. By adopting such idea, a heuristic method is proposed in Algorithm 2. We use $\hat{m}_t^{zi}$, $\hat{T}_t^i$, and $\hat{X}_t^i$ to represent the recovered solution for (9), which is obtained stage by stage. Specifically, at each time t, the zone air flow rate $\hat{m}_t^{zi}$ is first determined by approaching $X_t^{i,*}$ while complying with the upper and lower bounds of zone air flow rates (Step 5). After that, the zone temperature is updated accordingly (Step 7). The control sequence ($\hat{T}_t^i$, m_t^{zi}, and X_t^i) $(i \in \mathcal{I})$ can be obtained by repeating this process until the end of the optimization horizon $\mathcal{T}$.

V. Numerical Results

This section evaluates the performance of the proposed decentralized approach through simulations. First of all, we compare the proposed method with a centralized method to capture its suboptimality in Section V-A. Subsequently, the scalability and computational advantages of the proposed method are reported through comparisons with the DTBSS method [2] in Section V-B.

A. Performance Evaluation

The outside air temperature fluctuates over the time, as shown in Fig. 2(a). The time-of-use (TOU) electricity price is shown in Fig. 2(b), which refers to [8]. We first consider some small-scale case studies (i.e., 2 and 5 zones). Similar to

Algorithm 2 Heuristic Method to Recover the Recursive Feasibility of the Solution

Obtain the optimal solution $\boldsymbol{x}^* = ((\boldsymbol{x}^{0,*})^T, (\boldsymbol{x}^{1,*})^T, \cdots, (\boldsymbol{x}^{I,*})^T)^T$, $\boldsymbol{X}^* = ((\boldsymbol{X}^{0,*})^T, (\boldsymbol{X}^{1,*})^T, \cdots, (\boldsymbol{X}^{I,*})^T)^T$ of the relaxed problem (14) according to **Algorithm** 1.
Obtain the initial zone temperature $\hat{T}_0^i = T_0^i$ $(\forall i \in \mathcal{I})$.
for $t \in \mathcal{T}$ **do**
  for $i \in \mathcal{I}$ **do**
    Determine the air flow rate of zone i by

$$\begin{aligned} \overline{m}_t^{zi} &= \min\left(\overline{m}^{zi}, X_t^{i,*}/(\hat{T}_t^i - T_c)\right) \\ \hat{m}_t^{zi} &= \max\left(\overline{m}_t^{zi}, \underline{m}^{zi}\right). \end{aligned} \tag{21}$$

    Determine the auxiliary variable $\hat{X}_t^i$ of zone i by

$$\hat{X}_t^i = \hat{m}_t^{zi}(\hat{T}_t^i - T_c) \tag{22}$$

    Update the temperature of zone i by

$$\hat{T}_{t+1}^i = A^{ii}\hat{T}_t^i + \sum_{j \in \mathcal{N}_i} A^{ij}\hat{T}_t^j + C^i i\hat{X}_t^i + D_t^{ii} \tag{23}$$

  end for
end for
Output the recovered solution $\hat{m}_t^{zi}$ and $\hat{T}_t^i$ $(\forall i \in \mathcal{I}, t \in \mathcal{T})$ for problem (9).

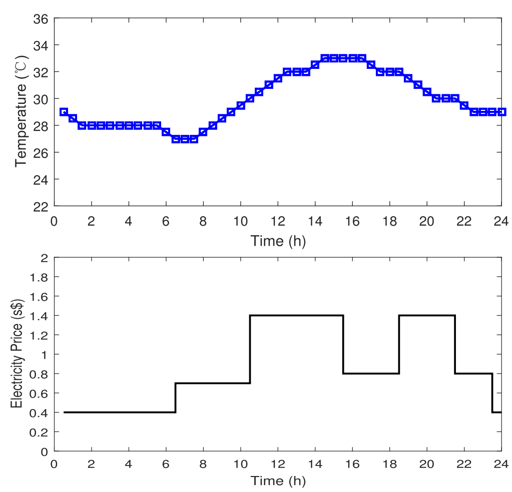

Fig. 2. (a) Outside air temperature. (b) TOU electricity price.

[33] and [34], the comfortable zone temperature bounds are set as 24°–26°C. The set-point temperature of the AHU is $T_c = 15$ °C. Consider that the internal zone heat generation is affected by various factors, we randomly generate thermal load curves for all zones according to a uniform distribution, i.e., $Q_t^i \sim U[0, 1]$ kW $(i \in \mathcal{I}, t \in \mathcal{I})$. As an instance, the thermal load scenarios for the 5-zone case study are exhibited in Fig. 3. We suppose the initial zone temperature as [26, 28] °C (2-zone case) and [26, 28, 28, 27, 24] °C (5-zone case). In addition, we assume that there exists heat transfer among each pair of zones in both the two case studies. The zone air flow rate bounds are set as $[0, 0.5]$ $\text{kg}\,\text{s}^{-1}$ (2-zone case and 5-zone case). In particular, to verify the versatility of the proposed method, we consider an extreme case, where

TABLE I

SYSTEM PARAMETERS

Param.	Value	Units
$C_i(i \in \mathcal{I})$	1.375×10^3	kJ/K
c_p	1.012	kJ/(kg · K)
R^{oi}	50	K/kW
$R^{ij}(i, j \in \mathcal{I})$	14	K/kW
k_f	0.08	-
η	1	-

TABLE II

DECENTRALIZED APPROACH VERSUS CENTRALIZED METHOD

#Zones	Centralized Method		Decentralized Approach	
	Cost (s\$)	Computation (s)	Cost (s\$)	Computation (s)
2	24.01	52.71	24.20	3.09
5	51.75	-	53.44	6.95

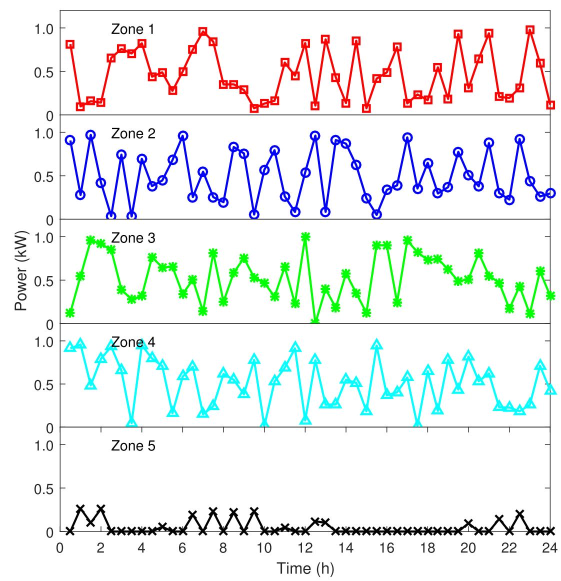

Fig. 3. Zone thermal load.

the local VAV box for zone 5 is forced to be closed ($m_t^{zi} = 0$) over the optimization horizon in the 5-zone case study. The other parameters are gathered in Table I.

We compare the decentralized approach with a centralized method, which possibly obtains the optimal solutions of such two small cases by solving small-scale nonlinear problems (9) using the IPOPT solver. Both the HVAC system's total energy cost and the average computation time for each stage incurred by the two methods are shown in Table II. One may note that the total energy cost under the two methods is comparable, and there only exists a slight performance degradation (less than 3.3%) for the decentralized approach. However, we observe a substantial improvement in computational efficiency as the average computation time is reduced.

In addition, the zone temperature (Fig. 4) and zone air flow rate (Fig. 5) are inspected under the decentralized approach for the 5-zone case study. We see that both the zone temperature and zone air flow rates are maintained in the desired ranges 24 °C–26 °C and $[0, 0.5]$ $\text{kg}\,\text{s}^{-1}$, respectively. In particular, the air flow rates of zone 5 are kept at *zero* as prescribed. This

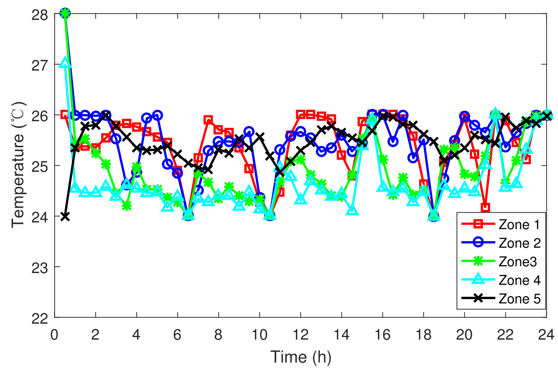

Fig. 4. Zone temperature.

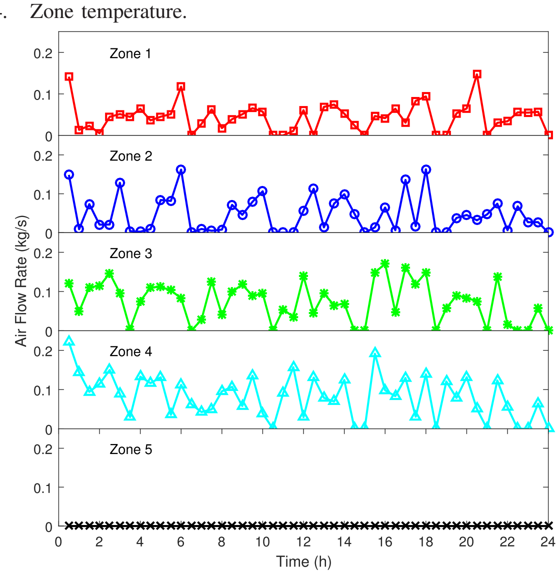

Fig. 5. Zone air mass flow rates.

implies that the proposed method can be applied to the general cases with customized zone settings in practice. Exceptionally, one may note some valley points in zone temperature curves (Fig. 4) corresponding well to the time instances that the electricity price is going to rise. This phenomenon is reasonable as our HVAC systems are designed to save electricity cost while still maintaining thermal comfort. Therefore, they are expected to pre-cool the zones to a relatively lower temperature (close to the lower temperature bounds) before the coming higher electricity price so as to save bills.

Furthermore, to improve the computational efficiency, we explore the convergence rate of the decentralized approach under a different penalty parameter ρ. In the 5-zone case study, the convergence rates of the primal cost and the residual error of the coupled constraints under different penalty parameters, i.e., $\rho = 1$, $\rho = 3$, $\rho = 5$, $\rho = 10$, $\rho = 15$, and $\rho = 20$ are studied, as shown in Fig. 6(a) and (b). The results imply that a larger penalty parameter ρ tends to lead to a faster convergence rate, whereas we observe that the convergence rate seems to be saturated with $\rho = 15$. Therefore, in the following case studies, the penalty parameter is set as $\rho = 15$.

B. Scalability

The scalability of the proposed method is demonstrated through a comparison with the DTBSS [2] developed for

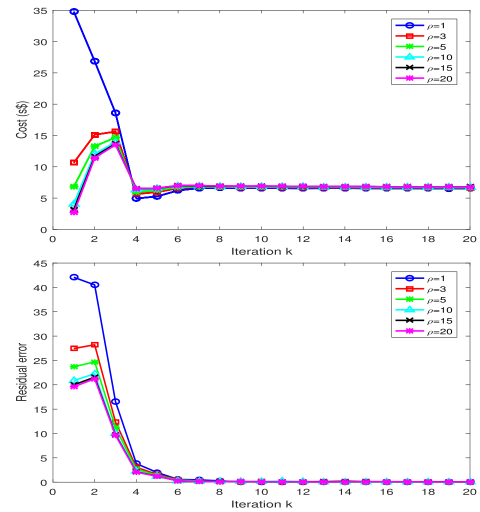

Fig. 6. (a) Convergence rate of primal cost under different penalty parameters ρ. (b) Convergence rate of residual error of the coupled constraints with different penalty parameters ρ.

multizone HVAC control. In general, DTBSS is a heuristic hierarchical-distributed method, in which (9) is divided into three-level subproblems, which manage each part of the overall cost function and constraints. We refer readers to [2] for more details about DTBSS. To guarantee a fair comparison, the constraints caused by duct pressure and chiller efficiency mentioned in [2] are circumvented in DTBSS. In addition, the two methods are both carried out under model predictive control (MPC) framework with the convenience to estimate the disturbances caused by thermal couplings among neighboring zones as required by DTBSS. Specifically, over each planning horizon, the disturbances are estimated based on the control trajectories computed over the previous one for DTBSS. For the DTBSS discussed in [2], the first and third steps are retained for (9). However, the second step can be skipped as the constraints due to duct pressure and chiller efficiency are not discussed in this article. In these case studies, we select the planning horizon for the decentralized approach and the first step of DTBSS as $H = 10$ [replace T by H in (9)], whereas the planning horizon for the third step of DTBSS is shortened to $H' = 5$ to reduce computation as suggested [2].

We consider a number of case studies with a different number of zones (i.e., 5, 20, 50, 100, 200, 300, and 500 zones) in this section. We randomly generate some networks to represent the thermal coupling among different zones, and the maximum number of adjacent zones for each zone is set as 4. The zone air flow rare bounds are set as $[0, 0.5]$ $\mathrm{kg\,s^{-1}}$. The other parameters are shown in Table I.

Both the HVAC system's energy cost and average computation time for each zone at each stage incurred by the two methods are shown in Table III. We see that for the cases with relatively small numbers of zones (less than 20), the two methods are comparable and our proposed decentralized approach performs slightly better with about $2\% - 4\%$ reduction in energy cost. However, for a large number of zones, the decentralized approach outperforms DTBSS both in reducing energy cost and improving computational efficiency.

TABLE III
PERFORMANCE OF THE DECENTRALIZED APPROACH AND THE DTBSS

#Zones	DTBSS		Decentralized Approach		Cost
	Cost(s\$)	Time(s)	Cost(s\$)	Time(s)	Reduced (%)
5	63.57	2.11	62.05	1.39	2.39
20	266.09	2.71	258.79	1.38	2.74
50	713.07	4.47	687.23	2.35	3.62
100	1.69×10^3	6.90	1.61×10^3	2.56	4.73
200	5.73×10^3	10.32	5.10×10^3	5.04	11.00
300	1.51×10^4	17.89	1.27×10^4	8.80	15.89
500	5.57×10^4	28.01	4.49×10^4	13.46	19.39

In particular, the total energy cost is cut down by around 19.39% for the 500-zone case study. The superior performance of the decentralized approach in reducing energy cost is attributed to the fact that the cooling power and fan power of AHU are coordinated in one problem in contrast to DTBSS, where they are successively optimized in different levels of subproblems. This also illuminates the slight difference in the energy cost inducted by the two methods for the small case studies. Specifically, for a small number of zones, the fan power of the AHU (depends on the cube of the total zone air flow rate) is negligible compared with the cooling power. Therefore, we only observe a small amount of energy cost reduction in our proposed method compared with the DTBSS. However, for increasing numbers of zones, the part of energy cost caused by fan power increases rapidly, which results in an apparent difference in energy cost for the two methods as observed in our numerical results.

Moreover, from Table III, we observe that the proposed decentralized approach is computationally advantageous (less average computation time) compared with the DTBSS. The preferable performance can generally be attributed to: 1) the superior convergence rate of the ADAL that used in our decentralized approach (see Fig. 6); 2) the subproblems to be solved are generally QPs for our proposed decentralized approach *versus* the nonlinear and nonconvex subproblems for the DTBSS; and 3) the *fully* decentralized structure of our proposed method *versus* the *partially* decentralized computing paradigm for the DTBSS (a centralized problem that coupled all zones has to be solved in the third step of the DTBSS).

VI. CONCLUSION

This article studies the scalable control of multizone HVAC systems with the objective to reduce energy cost while maintaining zone thermal comfort. As centralized methods are usually computationally intensive or prohibitive for large buildings, a decentralized approach based on the ADAL method [1] was developed. Through a comparison with the centralized method, we demonstrated the suboptimality of the proposed approach. In addition, the scalability and computational efficiency of the proposed method were illustrated through comparisons with the DTBSS method [2] developed for multizone HVAC control. The results implied that when a number of zones are relatively small (less than 20), our proposed decentralized approach performs slightly better with a minor reduction in the energy cost ($2\% - 4\%$). However, with an increasing number of zones, the proposed method outperforms the DTBSS by reducing the HVAC system's energy cost and improving computational efficiency.

APPENDIX
TRANSFORMATION OF (P2) TO (P3)

The temperature dynamics of zone i can be described as follows:

$$T_{t+1}^i = \begin{pmatrix} A^{ii} & 0 & C^{ii} \end{pmatrix} \begin{pmatrix} T_t^i \\ m_t^{zi} \\ X_t^i \end{pmatrix} + \sum_{j\in\mathcal{N}_i} \begin{pmatrix} A^{ij} & 0 & 0 \end{pmatrix} \begin{pmatrix} T_t^j \\ m_t^{zj} \\ X_t^j \end{pmatrix} + D_t^{ii}. \tag{24}$$

If we define $\overline{A}^{ii} = \begin{pmatrix} A^{ii} & 0 & C^{ii} \end{pmatrix}$, $\overline{A}^{ij} = \begin{pmatrix} A^{ij} \; 0 \; 0 \end{pmatrix}$ and $\overline{D}_t^{ii} = -D_t^{ii}$, and the decision variable $\boldsymbol{x}^i = ((\boldsymbol{x}_0^i)^T, (\boldsymbol{x}_1^i)^T, \ldots, (\boldsymbol{x}_{T-1}^i)^T)^T$ $(\forall i \in \mathcal{I})$ with $\boldsymbol{x}_t^i = (T_t^i, m_t^{zi}, X_t^i)$, the temperature dynamics for all the thermal zones can be combined by

$$\begin{pmatrix} \overline{A}_{ii} & -\boldsymbol{I}_1 & \cdots & \cdots \\ \boldsymbol{0} & \overline{A}_{ii} & -\boldsymbol{I}_1 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \boldsymbol{x}^i + \sum_{j\in\mathcal{N}_i} \begin{pmatrix} \overline{A}_{ij} & \boldsymbol{0} & \cdots & \cdots \\ \boldsymbol{0} & \overline{A}_{ij} & \boldsymbol{0} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \boldsymbol{x}^j + \begin{pmatrix} \overline{D}_0^{ii} \\ \vdots \\ \overline{D}_{T-1}^{ii} \end{pmatrix} = \boldsymbol{0}. \tag{25}$$

Furthermore, we define

$$\boldsymbol{A}_d^{ii} = \begin{pmatrix} \overline{A}^{ii} & -\boldsymbol{I}_1 & \boldsymbol{0} & \cdots & \cdots \\ \boldsymbol{0} & \overline{A}^{ii} & -\boldsymbol{I}_1 & \boldsymbol{0} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \in \mathbb{R}^{(T-1)\times 3T}$$

$$\boldsymbol{A}_d^{ij} = \begin{pmatrix} \overline{A}^{ij} & \boldsymbol{0} & \cdots & \cdots \\ \boldsymbol{0} & \overline{A}^{ij} & \boldsymbol{0} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \in \mathbb{R}^{(T-1)\times 3T}$$

and $\boldsymbol{b}_d^i = (\overline{D}_0^{ii}, \; \overline{D}_1^{ii}, \ldots, \overline{D}_{ii}^{T-1})^T \in \mathbb{R}^{(T-1)}$.

Thus, the zone thermal dynamics of (25) can be described in a compact form as follows:

$$\boldsymbol{A}_d^i \boldsymbol{x}^i + \sum_{j\in\mathcal{N}_i} \boldsymbol{A}_d^{ij} \boldsymbol{x}^j = \boldsymbol{b}_d^i. \tag{26}$$

Similarly, the coupled constraints in (10f) can be written as follows:

$$\sum_{i=0}^{I} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} T_t^i \\ m_t^{zi} \\ X_t^i \end{pmatrix} - Y_t = 0. \tag{27}$$

If we define $\overline{B}^i = (0 \quad 1 \quad 0)$ $(\forall i \in \mathcal{I})$ and $\overline{B}^0 = (-1)$, the coupled constraints in (27) over the optimization horizon

$\mathcal{T}$ can be collected as follows:

$$\sum_{i=0}^{I}\begin{pmatrix}\overline{B}^i & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \overline{B}^i & \mathbf{0} & \cdots \\ \cdots & \cdots & \cdots & \cdots\end{pmatrix}\boldsymbol{x}^i = \mathbf{0}. \tag{28}$$

If we define

$$\boldsymbol{B}_d^i = \begin{pmatrix}\overline{B}^i & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \overline{B}^i & \mathbf{0} & \cdots \\ \cdots & \cdots & \cdots & \cdots\end{pmatrix} \in \mathbb{R}^{T\times 3T} \quad (\forall i \in \mathcal{I})$$

(28) is equivalent to

$$\sum_{i=0}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^i = 0.$$

Accordingly, the constraints (8c) can be described as follows:

$$\sum_{i=1}^{I}\boldsymbol{B}_d^i\boldsymbol{x}^i \le \boldsymbol{c}_d \tag{29}$$

where we have $\boldsymbol{c}_d = (\overline{m}, \overline{m}\cdots\overline{m})^T \in \mathbb{R}^T$.

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