Optimal Network Charge for Peer-to-Peer Energy Trading: A Grid Perspective

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Abstract—Peer-to-peer (P2P) energy trading is a promising market scheme to accommodate the increasing distributed energy resources (DERs). However, how P2P to be integrated into the existing power systems remains to be investigated. In this paper, we apply network charge as a means for the grid operator to attribute transmission loss and ensure network constraints for empowering P2P transaction. The interaction between the grid operator and the prosumers is modeled as a Stackelberg game which yields a bi-level optimization problem. We prove that the Stackelberg game admits an *equilibrium* network charge price. Besides, we propose a method to obtain the network charge price by converting the bi-level optimization into a single-level mixed-integer quadratic programming (MIQP) that can handle a reasonable scale of prosumers efficiently. Simulations on the IEEE bus systems show that the network charge mechanism is favorable as it can benefit both the grid operator while securing the prosumers' profit, and achieves *near-optimal* social welfare. Moreover, the results show that the presence of energy storage will make the prosumers more sensitive to the network charge price changes and impact the P2P market.

Index Terms—Bi-level optimization, network charge, peer-to-peer (P2P) transaction, Stackelberg game, transmission loss.

I. INTRODUCTION

DRIVEN by the technology advances and the pressure to advance low-carbon society, power systems are experiencing the steady increase of distributed energy resources (DERs), such as home batteries, roof-top solar panels, and on-site wind

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turbines, etc. [1]. As a result, the traditional centralized energy management is being challenged as the DERs on the customer side are beyond the control of power systems operator. In this context, peer-to-peer (P2P) energy trading has emerged as a promising mechanism to account for the DERs [2]. P2P aims for a consumer-centric electricity market which allows the consumers with DERs (i.e., prosumer) to trade energy surplus or deficiency mutually [3], [4]. The vision of P2P is to empower the prosumers to achieve the balance of supply and demand autonomously and economically by leveraging their complementary and flexible generation and consumption. P2P energy trading is beneficial to both the power grid operator and the prosumers. Specifically, P2P can bring monetary value to the prosumers by allowing them to sell surplus local renewable generation to their neighbors or vice verse [5]. P2P also favors the power grid operation in term of reducing the cost of generation and transmission expansion to account for the yearly increasing demand as well as reducing transmission loss by driving local self-sufficiency [6].

Due to the widespread prospect, P2P energy trading mechanism has raised extensive interest from the research community. A large body of works has made efforts in addressing the matching of supply and demand bids for prosumers with customized preferences or interests. This is usually termed market clearing mechanisms. The mechanisms in discussion are diverse and plentiful, which can be broadly categorized by optimization-based approaches [7], [8], auction schemes [6], [9], and bilateral contract negotiations [10], [11]. Quite a few of comprehensive and systematic reviews have documented those market clearing mechanisms, such as [5], [12]–[14]. On top of that, many works has discussed the trust, secure, and transparent implementation of P2P markets by employing the well-known blockchain technology (see [15], [16] for examples).

The above studies are mainly focused on the business models of energy trading in virtual layer and in the shoes of prosumers. Whereas the energy transaction in a P2P market require the delivery of power in physical layer taken by the power grid operator who is responsible for securing the transmission capacity constraints and compensating the transmission loss. In this regard, the effective interaction between the prosumers making energy transaction in virtual layer and the power grid operator delivering the trades in physical layer is essential for the successful deployment of P2P market scheme. The interaction requires to secure the economic benefit of prosumers in the P2P

market as well as ensure the operation feasibility of power grid operator. This has been identified as a remaining key issue for P2P transaction [17].

The idea to allow the grid operator to impose some grid-related cost (usually known as network charge) on the prosumers for P2P transaction has been advocated as a promising tool to bridge this interaction. Network charge is reasonable and natural considering many aspects. First of all, network charge is necessary for the power grid to attribute the network investment cost and the transmission loss [18]. In traditional power systems where customers trade energy with the power grid, such cost has been internalized in the electricity price, it is therefore natural to pass the similar cost of P2P to the prosumers via some price mechanisms. Besides, network charge can work as a means to shape the P2P market so as to ensure the feasible operation of physical layer taken by the grid operator [19]. Network charge is often charged by trades, therefore it can be used to guide the energy trading behaviors of prosumers in the P2P market. As a result, several recent works have relied on network charge to account for the grid-related cost and shape the P2P markets, such as [11], [18], [20]. Specifically, [20] has involved network charge in developing a decentralized P2P market clearing mechanism. The work [18] comparatively simulated three network charge models (i.e., unique model, electrical distance based model, and zonal model) on shaping the P2P market. The work [11] has relied on a network charge model to achieve *ex-post* transmission loss allocations across the prosumers. These works have demonstrated the effectiveness of network charge in shaping P2P market. In addition, network charge can work as a tool to attribute grid-related cost and transmission loss which are actually taken by the grid operator. However, the existing works have mainly focused on studying how the network charge will affect the behaviors of prosumers in a P2P market instead of studying how the network charge price to be designed which couples the grid operator and the prosumers acting as independent stakeholders and playing different roles.

To fill the gap, this paper aims to study an optimal network charge mechanism that jointly considers the power grid operator who provides transmission service and the prosumers who make energy transaction in a P2P market. Considering that the power grid operator and prosumers are independent stakeholders and the power systems show a hierarchical structure, we model the interaction between the power grid operator and prosumers as a Stackelberg game. Specifically, the grid operator as a *leader* first decides on the optimal network charge price to trade off the network charge revenue and the transmission loss considering the network constraints, and then the prosumers as *followers* optimize their energy management (i.e., consuming, storing and trading) for maximum economic benefits. Our main contributions are:

- *C1):* We propose a Stackelbeg game model to account for the interaction between the power grid operator imposing network charge price and the prosumers making energy transaction in a P2P market. The distributed renewable generators and energy storage (ES) devices on the prosumer side are considered. We prove that
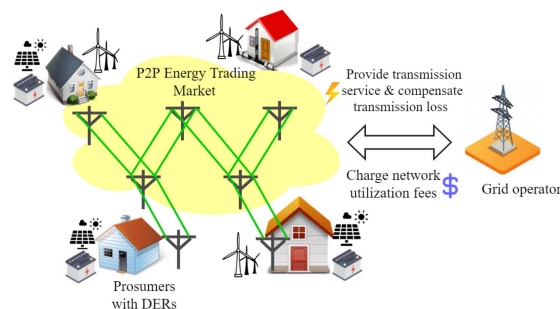

Fig. 1.   Interaction between the grid operator and a P2P energy trading market.

the Stackelberg game admits an *equilibrium* network charge price.

- *C1):* To deal with the computational challenges of obtaining the network charge price, we convert the bi-level optimization problem yield by the Stackelberg game to a single-level mixed-integer quadratic programming (MIQP) by exploring the problem structures. The method can handle a reasonable scale of prosumers efficiently.
- *C2):* By simulating the IEEE bus systems, we demonstrate that the network charge mechanism is favorable as it can benefit both the grid operator and the prosumers for empowering the P2P market. Moreover, it can provide *near-optimal* social welfare. In addition, we imply that the presence of ES will make the prosumers more sensitive to the network charge price changes.

The rest of this paper is as: in Section II, we present the Stackelberg game formulation; in Section III, we propose a single-level conversion method; in Section IV, we examine the proposed network charge mechanism via case studies; in Section V, we conclude this paper and discuss the future work.

II. PROBLEM FORMULATION

Fig. 1 shows the general hierarchical interaction between the grid operator and a P2P energy trading market. By providing transmission service and compensating transmission loss for empowering P2P trading, the grid operator plays the leading role by deciding the network charge price. In response, the prosumers with DERs (e.g., solar panels, wind turbines, ES, etc.) in the P2P market will decide their optimal energy management (i.e., consuming, storing and trading) for maximum economic benefits. In this paper, we assume the grid operator and prosumers are independent stakeholder and are both profit-oriented, expecting to maximizing their own profit via interactions. For the grid operator, the profit is evaluated by the network charge revenue minus the cost of transmission loss (this paper only focuses on the service of grid operator for empowering P2P market. The interactions between the power grid operator and the pursumers regarding other possible business are not included). For the prosumers, the profit is defined as the utility (i.e., satisfaction) of consuming certain amount of energy minus the network charge payment for P2P energy trading. In this paper, we assume

the prosumers are cooperative and the P2P energy trading price in internal. In other words, we assume the prosumers in the P2P market as a whole while negotiating the network price with the power grid operator. The objective of this paper is to determine the optimal network charge price that can achieve the equilibrium between the grid operator and the prosumers.

A. Network Charge Model

How P2P transaction to be charged is still an open question. One way in extensive discussion is based on the electrical distance and the volume of transaction [18], [20]. This paper focuses on the same network charge model. Specifically, if prosumer i buys p_{ij} [kW] units of power from prosumer j over an electrical distance of d_{ij} [km], the network charge is

$$T(p_{ij}) = \gamma d_{ij} p_{ij} \tag{1}$$

where γ [s\$/(kW $\cdot$ km)] is the network charge price broadcast by the grid operator and represents the network utilization fee for per unit of energy transaction over per unit of electrical distance.

The electrical distance is determined by the electrical network topology. For a given electrical network, there are several ways to measure the electrical distances [21]. One often used for network charge billing is *Power Transfer Distance Factor* (PTDF) (see [11], [18], [20] for examples). We therefore use the PTDF for measuring the electrical distances. For an electrical network characterized by transmission lines $\mathcal{L}$, the electrical distance between any trading peers i, j based on PTDF is defined as

$$d_{ij} = \sum_{\ell \in \mathcal{L}} |\text{PTDF}_{\ell,ij}| \tag{2}$$

where $\text{PTDF}_{\ell,ij}$ represents the PTDF of prosumer i, j related to transmission line $\ell \in \mathcal{L}$, which characterizes the estimated power flow change of line ℓ caused by per unit of energy transaction between prosumer i and j. The power flow changes are often estimated through DC power flow sensitivity analysis. In the following, we only present the main calculation equations and an illustration example for interpretation. The details to draw the calculation equations can refer to [22].

Assume an electrical network with N buses and L transmission lines characterized by a nodal acceptance matrix $\mathbf{B} = [B_{ij}]$, i.e.,

$$B_{ij} = \begin{cases} \sum_{k=1}^{N} \frac{1}{x_{ik}}, & \text{if } j = i. \\ -\frac{1}{x_{ij}}, & \text{if } j \neq i. \end{cases}$$

where x_{ij} represents the reactance of the line connecting bus i and bus j. To calculate PTDF, we first obtain the sub-matrix $\mathbf{B}_r$ by eliminating the row and column related to the reference bus r of $\mathbf{B}$. Without losing any generality, we specify bus N as the reference bus, we thus have $\mathbf{B}_r = \mathbf{B}[1 : N - 1, 1 : N - 1]$ and the reverse $\mathbf{X}_r = \mathbf{B}_r^{-1}$. By setting *zero* row and column for the reference bus $r = N$, we have the augmented matrix

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_r & \mathbf{0} \\ 0 & 0 \end{pmatrix}$$
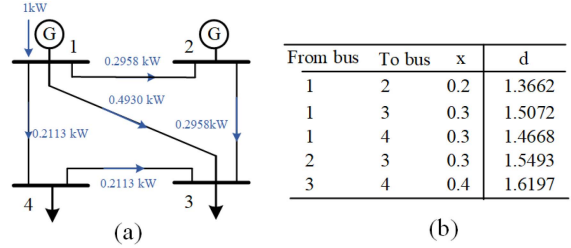

Fig. 2.   (a) Power flow changes across the network if bus 1 transmits 1 kW power to bus 3 based on DC power flow analysis. (b) The electrical distances based on PTDF for the 5-bus system.

Based on matrix $\mathbf{X}$, the PTDF is calculated as

$$\text{PTDF}_{\ell,ij} = \frac{X_{mi} - X_{mj} - X_{ni} + X_{nj}}{x_\ell} \tag{3}$$

where $X_{mi}, X_{mj}, X_{ni}, X_{nj}$ represent the elements of matrix $\mathbf{X}$ at row m, n and column i, j, ℓ is the transmission line connecting bus m and bus n.

An illustration example: For each pair of buses, PTDF can be interpreted as the total absolute power flow changes across the network caused by per unit of power transmitted from one bus to the other. We use a 5-bus system shown in Fig. 2 to show the interpretation. Suppose bus 1 transmits 1 kW power to bus 3, the estimated power changes across the electrical network based on DC power flow analysis are shown in Fig. 2(a). Meanwhile, by using the above method, we have the electrical distances based on PTDF across the buses shown in Fig. 2(b) (column $\mathbf{d}$). For example, the electrical distance between bus 1 and bus 3 is $d_{13} = 1.5072$ and we exactly have the total power flow changes $0.2958 + 0.4930 + 0.2113 + 0.2958 + 0.2113 = 1.5072$ if bus 1 transmits 1 unit (kW) of power to bus 3. This implies that the PTDF for each pair of buses can be interpreted as the total power flow changes across the electrical network caused by per unit of energy trading between them.

B. Stackelberg Game Formulation

As discussed, the interaction between the grid operator and the prosumers shows a hierarchical structure. This corresponds well to a Stackelberg game in which the power grid behaves as a *leader* and the prosumers are *followers*. Before the formulation, we first define the main notations in Table I. In this paper, we interchangeably use power and energy for they have been unified by the decision interval.

1) Leader: As a leader, the power grid optimizes the network charge price γ to trade off network charge revenue and the transmission loss considering the transmission network constraints. Network charge revenue is calculated by (1) and the power transmission loss is consolidated by the DC power flow [23]. We have the problem for the power grid operator:

$$\min_{\mathbf{x}_U} \text{Profit} = \sum_t \sum_i \sum_j \left(T(p_{ij,t}^+) + T(p_{ij,t}^-) \right) / 2 \qquad (P_U)$$

$$- \rho \sum_t \sum_{(i,j) \in \mathcal{L}} b_{ij} (\theta_{i,t} - \theta_{j,t})^2$$

TABLE I
MAIN NOTATIONS

Notation	Definition
i, j	Prosumer/bus index.
t	Time index.
γ	Network charge price.
$p_{ij,t}^+ / p_{ij,t}^-$	Traded (buy or sell) energy between prosumer i, j.
$\theta_{i,t}$	Phase angle at bus i.
$\mathcal{P}_{i,t}$	Consumed or generated power of prosumer i.
$P_{i,t}$	Injected power at bus i.
$p_{i,t}^{\mathrm{ch}} / p_{i,t}^{\mathrm{dis}}$	Charged/discharged power of prosumer i's ES.
$e_{i,t}$	Stored energy of prosumer i's ES.
$U_{i,t}(\cdot)$	Utility function of prosumer i.
$T(\cdot)$	Network charge billing.
$F_{ij}^{\max}$	Network transmission capacity of line $(i,j) \in \mathcal{L}$.
$p_{i,t}^{\mathrm{r}}$	Renewable generation of prosumer i.
$C_{ij}^{\max}$	Max. trading power between prosumer i, j.
$e_i^{\min} / e_i^{\max}$	Min./max. stored energy of prosumer i's ES.
$\mathcal{P}_{i,t}^{\min} / \mathcal{P}_{i,t}^{\max}$	Min./max. consumption/generation of prosumer i.

$$\text{s.t.} \quad \gamma_{\min} \le \gamma \le \gamma_{\max}. \tag{4a}$$

$$\mathbf{B}\boldsymbol{\theta}_t = \mathbf{P}_t, \forall t. \tag{4b}$$

$$\theta_{r,t} = 0, \forall t. \tag{4c}$$

$$|(\theta_{i,t} - \theta_{j,t})b_{ij}| \le F_{ij}^{\max}, \forall (i,j) \in \mathcal{L}, t. \tag{4d}$$

$$P_{i,t} = \sum_j p_{ij,t}^- - \sum_j p_{ij,t}^+, \forall i, t. \tag{4e}$$

$$\mathbf{P}^{\min} \le \mathbf{P}_t \le \mathbf{P}^{\max}, \forall t. \tag{4f}$$

where the decision variables for the power grid operator are $\mathbf{x}_U = [\gamma, \theta_{i,t}], \forall i, t$, including the network charge price and the optimal phase angles. We use the $\mathbf{P}_t = [P_{i,t}], \forall i$ to denote the power injections at the buses and b_{ij} denotes the admittance of the line connecting bus i and bus j. We have $\gamma_{\min}, \gamma_{\max} > 0$ characterize the range of network charge price. We use the term $\rho \sum_{(i,j) \in \mathcal{L}} b_{ij}(\theta_{i,t} - \theta_{j,t})^2 = \rho \sum_{(i,j) \in \mathcal{L}} P_{ij,t}^2 / b_{ij}$ related to the DC power flows $[P_{ij,t}], \forall (i,j) \in \mathcal{L}$ to consolidate the transmission loss across the network and ρ is the transmission loss cost coefficient [23]. Constraints (4b) represent the DC power flow equations. Constraints (4c) specify the phase angle of reference bus r. Constraints (4d) model the transmission line capacity limits. In this paper, we use the DC power flow model to account for the transmission constraints and transmission loss. Whereas the proposed framework can be readily extended to AC power flow model by replacing (4b)–(4d) with a DistFlow [24] or modified DistFlow [25] model. The nonconvex AC power flow model can be further convexified into a second-order cone program (SOCP) or a semi-definite program (SDP). Then the proposed method of this paper is still applicable though with increased problem complexity.

2) Followers: As follower, the prosumers respond to the network charge price γ both by regulating flexible energy use, alternating P2P transaction and charging/discharging their ES (if existed) for economic benefit. Due to the presence of DERs, a prosumer could be a consumer or a producer. We use $U_{i,t}(\mathcal{P}_{i,t})$ to represent the utility of prosumer i, which could be the satisfaction of a customer for consuming $\mathcal{P}_{i,t}$ units of energy or the cost

of a producer for generating $\mathcal{P}_{i,t}$ units of energy. We also involve the possible ES devices in the formulation. A prosumer with an ES can dynamically charge or discharge its device to enhance economic benefit. As discussed, we assume the prosumers are cooperative in the P2P market and the energy trading price is internalized and not discussed in this paper. We formulate the optimal energy management for the prosumers as a centralized optimization problem. The motivation for a cooperative formation is that may existing works have proved that could make all prosumers better off with suitable *ex-post* profit allocation mechanisms (see [26]–[28] for examples). Since the network charge is measured by the absolute traded power, we distinguish the purchased power and sold power between prosumer i and prosumer j by $p_{ij,t}^+$ and $p_{ij,t}^-$. For each trade, the network charge is equally shared by the seller and the buyer. The centralized optimal energy management problem for the prosumers is formulated as follows.

$$\max_{\mathbf{x}_L} \text{Profit} = \sum_t \sum_i U_{i,t}(\mathcal{P}_{i,t}) \tag{P_L}$$
$$- \sum_t \sum_i \sum_j \left(T(p_{ij,t}^+) + T(p_{ij,t}^-) \right) / 2$$

$$\text{s.t.} \quad p_{ij,t}^+ = p_{ji,t}^-, \forall i, j, t. \tag{5a}$$

$$0 \le p_{ij,t}^+ \le C_{ij}^{\max}, \forall i, j, t. \tag{5b}$$

$$0 \le p_{ij,t}^- \le C_{ij}^{\max}, \forall i, j, t. \tag{5c}$$

$$\mathcal{P}_{i,t} \le p_{i,t}^{\mathrm{r}} + p_{i,t}^{\mathrm{dis}} - p_{i,t}^{\mathrm{ch}} + \sum_j p_{ij,t}^+ - \sum_j p_{ij,t}^-, \forall i, t. \tag{5d}$$

$$\mathcal{P}_{i,t}^{\min} \le \mathcal{P}_{i,t} \le \mathcal{P}_{i,t}^{\max}, \forall i, t. \tag{5e}$$

$$e_{i,t+1} = e_{i,t} + p_{i,t}^{\mathrm{ch}}\eta - p_{i,t}^{\mathrm{dis}}/\eta, \forall i, t. \tag{5f}$$

$$0 \le p_{i,t}^{\mathrm{ch}} \le P_i^{\mathrm{ch,max}}, \forall i, t. \tag{5g}$$

$$0 \le p_{i,t}^{\mathrm{dis}} \le P_i^{\mathrm{dis,max}}, \forall i, t. \tag{5h}$$

$$e_i^{\min} \le e_{i,t} \le e_i^{\max}, \forall i, t. \tag{5i}$$

where the decision variables are $\mathbf{x}_L = [p_{ij,t}^+, p_{ij,t}^-, \mathcal{P}_{i,t}, p_{i,t}^{\mathrm{ch}}, p_{i,t}^{\mathrm{dis}}, e_{i,t}], \forall i, t$, including P2P trading, flexible energy consumption, and ES operation. Constraints (5a) model the consistence of energy transaction between each pair of seller and buyer. Since the transmission loss is compensated by the power grid operator, we can assume the amount of energy that prosumer i buys from prosumer i equals that prosumer j sells to prosumer i. Constraints (5b)–(5c) impose the transaction limits between the trading peers. Constraints (5d) ensure the load balance of each prosumer. Particularly, we use inequality to capture the case that some renewable is curtailed. Constraints (5e) characterize the energy consumption or generation flexibility of prosumers. Constraints (5f) track the energy state of prosumers' ES with $\eta \in (0, 1)$ denoting the charging/discharging efficiency. Constraints (5g)–(5h) impose the ES charging, discharging and capacity limits. In this paper, we only focus on the energy trading among the prosumers. For the case where prosumers also trade
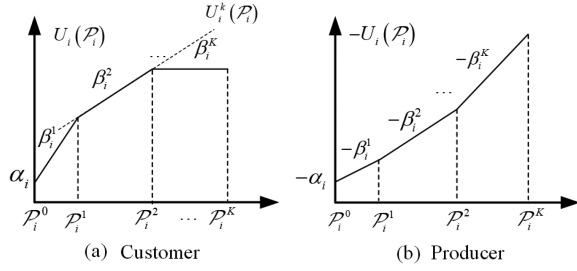
Fig. 3. (a) Piece-wise linear (PWL) utility function for a consumer $\nabla U_i(\mathcal{P}_i) \geq 0$. (b) Piece-wise linear (PWL) utility function for a producer $\nabla U_i(\mathcal{P}_i) \leq 0$.

energy with grid, the model can be easily extended by adding the cost or revenue related to such energy trading in the objective.

3) Piece-Wise Linear Utility Function: This paper employs concave piece-wise linear (PWL) utility functions to capture the prosumers' energy consumption and generation flexibility. The concern is that PWL functions are universal and can approximate all types of utility functions, such as quadratic and logarithmic [29]. We may obtain the PWL utility functions by linearizing non-linear utility functions or directly learn it from data [30]. Due to the presence of DERs, the prosumer could be a consumer in energy deficiency or a producer with energy surplus. We uniformly formulate such two cases by PWL functions but with opposite sign of the slopes as shown in Fig. 3(a) and (b). Specifically, we have non-negative slopes (i.e., $\nabla U_i(\mathcal{P}_i) \geq 0$) for a consumer and non-positive slopes (i.e., $\nabla U_i(\mathcal{P}_i) \leq 0$) for a producer (time t is omitted). As shown in Fig. 3, a PWL utility function composed of K segments can be characterized by the transition points and slopes: $\mathcal{P}_i^k$ and β_i^k, $k = 1, 2, \ldots, K$. The linear function associated with the k-th segment can be described as

$$U_i^k(\mathcal{P}_i) = \alpha_i + \sum_{\ell=1}^{k-1} \beta_i^\ell \left(\mathcal{P}_i^\ell - \mathcal{P}_i^{\ell-1}\right) + \beta_i^k \left(\mathcal{P}_i - \mathcal{P}_i^{k-1}\right), \forall i, k.$$
(6)

where α_i is the constant component of prosumer i's utility function, which could represent the satisfaction level of a prosumer for consuming zero unit of energy or the start-up generation cost for a producer. It is easy to note that we have $U_i(\mathcal{P}_i) = U_i^k(\mathcal{P}_i)$ if $\mathcal{P}_i \in [\mathcal{P}_i^{k-1}, \mathcal{P}_i^k]$.

For the proposed Stackelberg game, we have the following results regarding the existence of *equilibrium*.

Theorem 1: The Stackelberg game (P_U)–(P_L) admits an equilibrium.

Proof: We have the lower-level problem (P_L) compact and convex with any given network charge price γ. This implies that the optimal solution for the lower-level problem (P_L) always exists and can be expressed by $\mathbf{x}_L(\gamma)$. By substituting the closed-form solution $\mathbf{x}_L(\gamma)$ (if explicitly available) into the upper-level problem (P_U) and by expressing the phase angle decision variables $\boldsymbol{\theta}$ with the power flows determined by the lower-level problem solution $\mathbf{x}_L(\gamma)$, we can conclude a single-level optimization problem for the Stackelberg game subject to the only bounded decision variables $\gamma \in [\gamma^{\min}, \gamma^{\max}]$,

which will yield at least one optimal solution. This implies that the proposed Stackelberg game (P_U)–(P_L) admits at least one Stackelberg *equilibrium*. ∎

Remark 1: The existence of the Stackelberg *equilibrium* implies that the proposed model can yield a network charge price that achieves the equilibrium between the grid operator and the prosumers.

III. METHODOLOGY

Note that the optimal network charge price is the *equilibrium* of the Stackelberg game (P_U)–(P_L) which corresponds to a bi-level optimization. Bi-level optimization is generally NP-hard and difficult to solve [31]. This section proposes to convert the bi-level problem to single-level to accommodate a reasonable scale of prosumers by exploring the problems structures. The key idea is to replace the convex lower-level problem by its Karush-Kuhn-Tucker (KKT) conditions [32]. To achieve the conversion, we first restate the lower-level problem (P_L) as

$$\max_{\mathbf{x}_L} \text{Profit} = \sum_t \sum_i u_{i,t} - \sum_t \sum_i \sum_j T(p_{ij,t}^+) \qquad (\text{P}'_L)$$

$$\text{s.t.} \quad 0 \leq p_{ij,t}^+ \leq C_{ij}^{\max} : \qquad \underline{\nu}_{ij,t}, \overline{\nu}_{ij,t} \geq 0, \forall i, j, t. \quad (7a)$$

$$\mathcal{P}_{i,t} \leq p_{i,t}^r + p_{i,t}^{\text{dis}} - p_{i,t}^{\text{ch}} + \sum_j p_{ij,t}^+ - \sum_j p_{ji,t}^+ :$$
$$\mu_{i,t} \geq 0, \forall i, t. \quad (7b)$$

$$\mathcal{P}_{i,t}^{\min} \leq \mathcal{P}_{i,t} \leq \mathcal{P}_{i,t}^{\max} : \qquad \underline{\sigma}_{i,t}, \overline{\sigma}_{i,t} \geq 0, \forall i, t. \quad (7c)$$

$$e_{i,t+1} = e_{i,t} + p_{i,t}^{\text{ch}} \eta - p_{i,t}^{\text{dis}}/\eta : \quad \mu_{i,t}^e \in \mathbb{R}, \forall i, t. \quad (7d)$$

$$0 \leq p_{i,t}^{\text{ch}} \leq P_i^{\text{ch,max}} : \qquad \underline{\mu}_{i,t}^{\text{ch}}, \overline{\mu}_{i,t}^{\text{ch}} \geq 0, \forall i, t. \quad (7e)$$

$$0 \leq p_{i,t}^{\text{dis}} \leq P_i^{\text{dis,max}} : \qquad \underline{\mu}_{i,t}^{\text{dis}}, \overline{\mu}_{i,t}^{\text{dis}} \geq 0, \forall i, t. \quad (7f)$$

$$e_i^{\min} \leq e_{i,t} \leq e_i^{\max} : \qquad \underline{\mu}_{i,t}^e, \overline{\mu}_{i,t}^e \geq 0, \forall i, t. \quad (7g)$$

$$u_{i,t} \leq U_i^k(\mathcal{P}_{i,t}) : \qquad \delta_{i,k,t} \geq 0, \forall i, k, t. \quad (7h)$$

where the decision variable $\mathbf{P}^- = [p_{ij,t}^-], \forall i, j, t$ are dropped by the substitution $p_{ji,t}^- = p_{ij,t}^+, \forall i, j, t$. Some auxiliary variables $u_{i,t}$ are introduced to relax the non-smooth prosumer utility functions. Since the utility functions are concave, it is easy to prove the equivalence of (P'_L) to (P_L). We besides claim the dual variables for the constraints on right-hand side.

We next draw the KKT conditions for the reformulated lower-level problem (P'_L). We first have the first-order optimality conditions:

$$4\partial\mathcal{L}/\partial\mathcal{P}_{i,t} = \mu_{i,t} - \underline{\sigma}_{i,t} + \overline{\sigma}_{i,t} - \sum_k \delta_{i,k,t} \nabla U_{i,t}^k(\mathcal{P}_{i,t}) = 0$$
(8a)

$$\partial\mathcal{L}/\partial p_{ij,t}^+ = \gamma d_{ij} - \underline{\nu}_{ij,t} + \overline{\nu}_{ij,t} - \mu_{i,t} + \mu_{j,t} = 0 \quad (8b)$$

$$\partial\mathcal{L}/\partial u_{i,t} = -1 + \sum_k \delta_{i,k,t} = 0 \quad (8c)$$

$$\partial\mathcal{L}/\partial p_{i,t}^{\text{ch}} = \mu_{i,t} - \mu_{i,t}^e \eta - \underline{\mu}_{i,t}^{\text{ch}} = 0 \quad (8d)$$

$$\partial\mathcal{L}/\partial p_{i,t}^{\text{dis}} = -\mu_{i,t} + \mu_{i,t}^e/\eta - \underline{\mu}_{i,t}^{\text{dis}} + \overline{\mu}_{i,t}^{\text{dis}} = 0 \quad (8e)$$

$$\partial L / \partial e_{i,t} = -\mu_{i,t}^{\mathrm{e}} + \mu_{i,t-1}^{\mathrm{e}} - \underline{\mu}_{i,t-1}^{\mathrm{e}} + \overline{\mu}_{i,t-1}^{\mathrm{e}} = 0, \forall t > 1 \tag{8f}$$

where we use $\mathcal{L}$ to denote the Lagrangian function of (P′$_L$). Based on (6), we have $\nabla U_{i,t}^k(\mathcal{P}_{i,t}) = \beta_i^k$ which represents the slope of the prosumer i's utility function at the k-th segment.

In addition, we have the complementary constraints for the inequality constraints (7a)–(7c) and (7e)–(7h). Using (7d) as an example, we have the complementary constraints

$$\mu_{i,t}\left(\mathcal{P}_{i,t} - p_{i,t}^{\mathrm{r}} - p_{i,t}^{\mathrm{dis}} + p_{i,t}^{\mathrm{ch}} - \sum_j p_{ij,t}^{+} + \sum_j p_{ji,t}^{+}\right) = 0,$$

$$p_{i,t}^{\mathrm{r}} + p_{i,t}^{\mathrm{dis}} - p_{i,t}^{\mathrm{ch}} + \sum_j p_{ij,t}^{+} - \sum_j p_{ji,t}^{+} - \mathcal{P}_{i,t} \geq 0,$$

$$\mu_{i,t} \geq 0, \forall i, t. \tag{9}$$

The general way to handle the non-linear complementary constraints such as (9) is to introduce binary variables to relax the constraints (see [33] for an example). This could be problematic for problem (P′$_L$) due to the large number of inequality constraints. To deal with the computational challenges, we make use of the linear programming (LP) structure of problem (P′$_L$). For a LP, we have the *strong duality* and the *complementary constraints* are interchangeable (see [34], Ch4, pp. 147 for detailed proof). Therefore, we use the strong duality condition for problem (P′$_L$) to replace the complementary constraints. We have the strong duality for following problem (P′$_L$).

$$-\sum_t \sum_i \sum_j \overline{\nu}_{ij,t} C_{ij}^{\max} - \sum_t \sum_i \mu_{i,t} p_{i,t}^{\mathrm{r}} + \sum_t \sum_i \underline{\sigma}_{i,t} \mathcal{P}_{i,t}^{\min}$$

$$-\sum_t \sum_i \overline{\sigma}_{i,t} \mathcal{P}_{i,t}^{\max} - \sum_t \sum_i \overline{\mu}_{i,t}^{\mathrm{ch}} P_i^{\mathrm{ch,max}} - \sum_t \sum_i \overline{\mu}_{i,t}^{\mathrm{dis}} P_i^{\mathrm{dis,max}}$$

$$+\sum_t \sum_i \underline{\mu}_{i,t}^{\mathrm{e}} e_i^{\min} - \sum_t \sum_i \overline{\mu}_{i,t}^{\mathrm{e}} e_i^{\max} - \sum_t \sum_i \sum_k \delta_{i,k,t} U_i^k(0)$$

$$= \sum_t \sum_i \sum_j T(p_{ij,t}^{+}) - \sum_t \sum_i u_{i,t} \tag{10}$$

Note that the strong duality (10) can be used to eliminate the large number of non-linear complementary constraints but requires to tackle the bi-linear terms related to the network charge calculations: $T(p_{ij,t}^{+}) = \gamma d_{ij} p_{ij,t}^{+}$. To handle such bi-linear terms, we discretize the network charge price and convert the non-linear terms into mixed-integer constraints. We define an auxiliary variable Z by

$$Z = \sum_t \sum_i \sum_j d_{ij} p_{ij,t}^{+}$$

We thus have the total network charge for P2P transaction

$$\sum_t \sum_i \sum_j T(p_{ij,t}^{+}) = \gamma Z \tag{11}$$

We discretize the admissible network charge price range $[\gamma_{\min}, \gamma_{\max}]$ into L levels $\{\gamma_1, \gamma_2, \ldots, \gamma_L\}$ with an equal interval $\Delta\gamma = (\gamma_{\max} - \gamma_{\min})/L$. Accordingly, we introduce the binary variables $\mathbf{x} = [x_\ell]$, $\ell = 1, 2, \ldots, L$ to indicate which levels are selected, i.e., we have $\mathbf{x}_\ell = 1$ if the network charge price γ_ℓ is selected and otherwise $\mathbf{x}_\ell = 0$. Since only one network

charge price can be selected, we thus have

$$\gamma Z = \sum_{\ell=1}^{L} x_\ell \gamma_\ell Z \tag{12}$$

$$\sum_{\ell=1}^{L} x_\ell = 1, x_\ell \in \{0, 1\} \tag{13}$$

Note that the network charge calculations (12)–(13) involve the product of binary variable x_ℓ and continuous variable Z. This can be equivalently expressed by the integer algebra

$$-Mx_\ell \leq Y_\ell \leq Mx_\ell \tag{14}$$

$$-M(1 - x_\ell) \leq Z - Y_\ell \leq M(1 - x_\ell) \tag{15}$$

where we have $\gamma Z = \sum_{\ell=1}^{L} \gamma_\ell Y_\ell$ and M is a sufficiently large positive constant.

By plugging $\sum_t \sum_i \sum_j T(p_{ij,t}^{+}) = \gamma Z = \sum_{\ell=1}^{L} \gamma_\ell Y_\ell$ in (10), and by replacing the lower level problem (P′$_L$) with KKT conditions, we have the single-level mixed-integer quadratic programming (MIQP)

$$\max_{\mathbf{x}_U, \mathbf{x}_L, \lambda} \text{Profit} = \sum_{\ell=1}^{L} \gamma_\ell Y_\ell - \rho \sum_{(i,j) \in \mathcal{L}} b_{ij}(\theta_i - \theta_j)^2 \tag{P}$$

s.t. (7a) − (7h). Primal constraints

$$\left.\begin{array}{l} (8a) - (8f). \\ (10), (13) - (15). \end{array}\right\} \text{KKT conditions}$$

where $\lambda = [\underline{\nu}, \overline{\nu}, \mu, \underline{\sigma}, \overline{\sigma}, \mu^{\mathrm{e}}, \underline{\mu}^{\mathrm{ch}}, \overline{\mu}^{\mathrm{ch}}, \underline{\mu}^{\mathrm{dis}}, \overline{\mu}^{\mathrm{dis}}, \underline{\mu}^{\mathrm{e}}, \overline{\mu}^{\mathrm{e}}, \delta]$ are dual decision variables. Note that the single-level conversion favors computation as the number of binary variables is only determined by the granularity of network charge discretization (i.e., L) and independent of the scale of prosumers, making it possible to accommodate a reasonable scale of prosumers.

IV. CASE STUDIES

In this section, we examine the network charge mechanism via simulations. We first use IEEE 9-bus system to evaluate the solution quality of the method, the existence of *equilibrium* network charge price, and the social welfare. We further examine the results on larger electrical networks including IEEE 39-bus, IEEE 57-bus, and IEEE 118-bus systems to demonstrate the scalability. Particularly, we also compare the results with and without ES in the case studies.

A. Simulation Set-ups

We set up the case studies by rescaling real building load profiles [35] and renewable generation profiles (i.e., wind and solar) [36]. To capture the demand flexibility, we set the lower and upper prosumer demand as $\mathcal{P}_{i,t}^{\min} = 0$ and $\mathcal{P}_{i,t}^{\max} = demand\,profile_{i,t} + 30$ kW (we focus on flexible demand in this paper). For each time period t, we uniformly generate the slopes of prsumer PWL utility functions in $\beta_{i,t}^k \in [0, 1]$ with $K = 2$ or 3 segments (we only consider customers in the following studies and the producers can be included by setting $\beta_{i,t}^k \in [-1, 0]$ if exist). We set the constant components of PWL

TABLE II
SIMULATION SET-UPS

Param.	Definition	Value
T	Network charge billing periods	24
$\alpha_{i,t}$	Proumer PWL utility constant	0
$\beta_{i,t}^k$	Prosumer PWL utility slopes	$[0,1]$
K	Prosumer PWL utility segments	2 or 3
$[\gamma^{\min}, \gamma^{\max}]$	Network charge price range	$[0,1]$ s\$/(kW·km)
$\Delta\gamma$	Network charge price discretization	0.02 s\$/(kW·km)
L	Network charge discretization levels	51
$e_i^{\min}/e_i^{\max}$	Min./max. energy state of ES	0/60 kW h
$P_i^{\mathrm{ch/dis,max}}$	Max. charging/discharging power	50 kW
η	Charging/discharging efficiency	0.9
ρ	Transmission loss cost coefficient	0.01

TABLE III
MARKET CONFIGURATIONS FOR COMPARISONS

Market	ES	P2P	Network charge
No P2P			
Free P2P		✓	
Social P2P		✓	Internalized
Optimal P2P		✓	✓
No P2P + ES	✓		
Free P2P + ES	✓	✓	
Social P2P + ES	✓	✓	Internalized
Optimal P2P + ES	✓	✓	✓

Fig. 4. IEEE 9-bus system with 9 prosumers (P1-P9).

Fig. 5. Grid profit w.r.t. network charge price γ for IEEE 9-bus system: (a) P2P + No ES. (b) P2P + With ES. (γ_L: minimum network charge price for the grid to attribute transmission loss. γ_{opt}: optimal network charge price for maximum grid profit. γ_U: maximum network charge price that the prosumers would take.).

utility function as $\alpha_{i,t} = 0$ for all customers. Correspondingly, we equally divide the ranges of prosumer demand $[\mathcal{P}_{i,t}^{\min}, \mathcal{P}_{i,t}^{\max}]$ into $K = 2$ or 3 segments to obtain the PWL utility function transition points $\mathcal{P}_{i,t}^k$. We simulate the P2P market for 24 periods with a decision interval of one hour. The settings for the above parameters and the prosumers' ES are gathered in Table II. Particularly, we set the range of network charge price as $\gamma^{\min} = 0$ and $\gamma^{\max} = 1.0$ s\$/(kW·km) and the discretization interval as $\Delta\gamma = 0.02$ s\$/(kW·km) based on the simulation results in Section IV-B, which suggest such settings are expected to provide solutions with sufficiently high accuracy. The electrical distances based on PTDF are calculated by the method in Section II-A.

In this paper, we refer to the P2P market with the proposed network charge as *Optimal P2P*. The network charge price is obtained by solving problem (P) with the *off-the-shelf* solvers. In the following studies, we compare *Optimal P2P* with *No P2P* (P2P transaction is forbidden), *Free P2P* (P2P transaction is allowed without any network charge) and *Social P2P* (P2P transaction is determined by maximizing the social profit which is the sum of grid operator profit and prosumer profit defined in (P_U) and (P_L)). Note that the network charge with *Social P2P* will be internalized as the grid operator and the prosumers are unified as a whole. For *Free P2P*, the grid operator has no manipulation on the P2P market and the optimal transaction can be determined by directly solving the lower-level problem (P_L) with $\gamma = 0$ (To ensure the uniqueness of solution, we set a sufficiently small network charge price γ). For the cases without ES, we set $e_i^{\max} = P_i^{\mathrm{ch,max}} = P_i^{\mathrm{dis,max}} = 0$ and for the cases with ES, we assume an ES with the configurations of Table II for each prosumer. The market configurations for comparisons
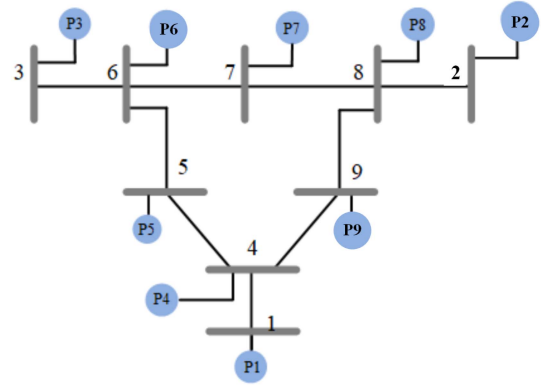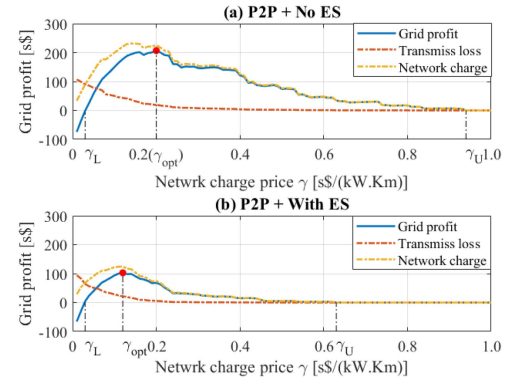
are shown in Table III. We highlight *Optimal P2P* and *Optimal P2P + ES* as our main focus.

B. IEEE 9-bus System

We first use IEEE 9-bus system with 9 prosumers shown in Fig. 4 to evaluate the network charge mechanism. By solving problem (P), we have the optimal network charge price $\gamma_{\mathrm{opt}} = 0.2$ s\$/(kW·km) (No ES) and $\gamma_{\mathrm{opt}} = 0.12$ s\$/(kW·km) (With ES). To verify the solution quality, we compare the obtained solutions with that identified from simulating the range of network charge price $\gamma \in [0,1]$ s\$/(kW·km) with an incremental of $\Delta\gamma = 0.01$ s\$/(kW·km). For each simulated network charge price, we evaluate the grid profit defined in (P_U) and display their changes w.r.t. the network charge price in Fig. 5 (we also evaluate and display the network charge and transmission loss in the figures for later analysis). The simulated optimal network charge price can be identified from where the grid profit is maximized, which are $\gamma_{\mathrm{opt}} = 0.2$ s\$/(kW·km) (No ES) and $\gamma_{\mathrm{opt}} = 0.12$ s\$/(kW·km) (With ES). This is in line with the obtained optimal network charge from solving the single-level problem. This demonstrates the solution quality of the proposed method. By further examining the simulation results, we can draw or verify the following results.
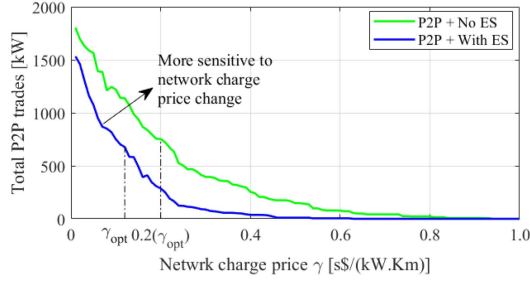
Fig. 6. Total P2P trades w.r.t. network charge price γ for IEEE 9-bus system (γ_{opt}: optimal network charge price).

1) The Network Charge Model Admits an Equilibrium Network Charge Price: From Fig. 5(a) (No ES) and Fig. 5(b) (With ES), we see that the grid profit first approximately increases and then drops after reaching the optimal network charge price γ_{opt}. Since for any given network charge price γ, there exists an optimal energy management strategy for the prosumers (i.e., there exists an optimal solution for the lower level problem (P_L)), we imply that γ_{opt} is the *equilibrium* network charge price. This demonstrates the existence of *equilibrium* for the proposed Stackelberg game, which is in line with Theorem 1.

Besides, we can imply that there exists a minimal network charge price for the grid operator to attribute the cost of transmission loss. Such minimal network charge price occurs where the network charge revenue equals the cost of transmission loss (i.e., *zero* grid profit). Specifically, we have the minimal network charge price $\gamma_L = 0.03 \, \mathrm{s\$/(kW \cdot km)}$ both with and without ES for the tested case. In addition, there exists a maximal network charge price that the prosumers are willing to take, which are $\gamma_U = 0.94 \, \mathrm{s\$/(kW \cdot km)}$ (No ES) and $\gamma_U = 0.6 \, \mathrm{s\$/(kW \cdot km)}$ (With ES). When the network charge price exceeds the maximal price, no P2P transaction actually happens (we have *zero* grid profit, network charge and transmission loss after γ_U). In addition, we imply that the prosumers would be willing to take lower network charge price if they have ES. This is reasonable as the prosumers can use ES to shift surplus renewable generation for future use in addition to trade in the P2P market. This can be further perceived from Fig. 6 where we have simulated the total P2P trades w.r.t. the network charge price both with and without ES. The results show that less trades are made with ES compared with No ES for any given network charge price. Moreover, the total trades drop faster w.r.t. the increase of network charge price with ES. This implies that the deployment of ES will make the prosumers more sensitive to the network charge price and impact the P2P market.

2) The Network Charge Can Benefit the Grid Operator While Securing the Prosumers' Profit: We have concluded that the network charge mechanism can provide positive profit to the grid operator. An interesting question to ask is how the economic benefit of P2P is shared between the grid operator and the prosumers with the network charge. To answer that question, we use *No P2P* as the base and evaluate the increased profit for the grid operator and the prosumers, which can be viewed as their benefits harvested from the P2P market. We compare the results with *Optimal P2P* and *Free P2P*. For *Optimal P2P*, we
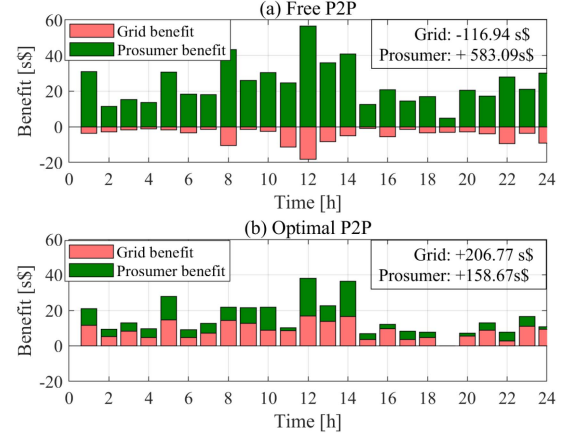

Fig. 7. Benefits of the grid operator and prosumers with *Optimal P2P* and *Free P2P* for IEEE 9-bus system (*No P2P* as benchmark).
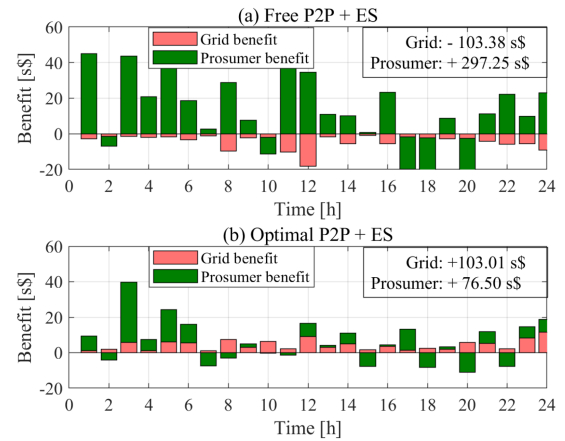

Fig. 8. Benefits of the grid operator and prosumers with *Free P2P + ES* and *Optimal P2P + ES* for IEEE 9-bus system (*No P2P* as benchmark).

impose the optimal network charge price $\gamma_{\mathrm{opt}} = 0.2 \, \mathrm{s\$/(kW \cdot km)}$ (No ES) and $\gamma_{\mathrm{opt}} = 0.12 \, \mathrm{s\$/(kW \cdot km)}$ (With ES). For *Free P2P*, we set the sufficiently small network charge price $\gamma = 1e - 7 \, \mathrm{s\$/(kW \cdot km)}$ to ensure the uniqueness of solution as mentioned. We evaluate the benefits for grid operator and prosumers over the 24 periods and display the results in histograms in Fig. 7 (No ES) and Fig. 8 (With ES). We imply that when there is no network charge (i.e., *Free P2P* and *Free P2P + ES*), the prosumers can gain considerable benefit from the P2P market. However, the grid operator is faced with the cost of transmission loss which are 116.94 s\$(No ES) and 103.38 s\$(With ES). Comparatively, the network charge mechanism are able to provide positive benefit to both the grid operator and the prosumers, i.e., 206.77 s\$vs. 158.67 s\$(*Optimal P2P + No ES*) and 103.01 s\$vs. 76.50 s\$(*Optimal P2P + With ES*). This implies the network charge mechanism can benefit the grid operator for empowering P2P market while securing the prosumers' profit. Specifically, the benefit of P2P is almost equally shared by the grid operator and the prosumers, i.e., 56.58% vs. 43.42% (No ES) and 57.38% vs. 42.62% (With ES).
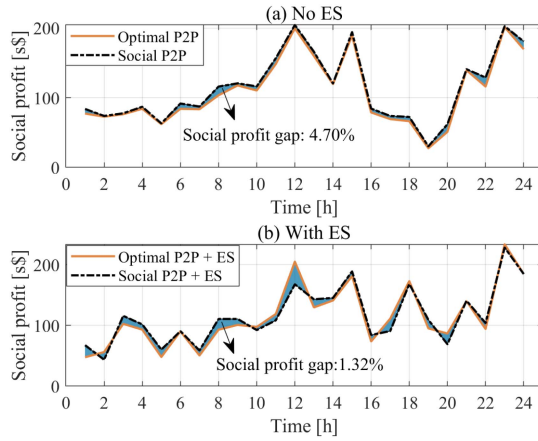
Fig. 9. Social profit over 24 periods for IEEE 9-bus system: (a) No ES. (b) With ES (Positive shaded area is the social optimality gap of *Optimal P2P* over *Social P2P*).

3) The Network Charge Provides Near-Optimal Social Welfare: Social welfare is one of the most important measures to be considered for market design. For the concerned P2P market involving the grid operator and the prosumers, *social welfare* (i.e., social profit) is the total grid and prosumers' profit, which is evaluated as Social profit $= \sum_t \sum_i U_{i,t}(\mathcal{P}_{i,t}) - \rho \sum_t \sum_{(i,j) \in \mathcal{L}} b_{ij}(\theta_{i,t} - \theta_{j,t})^2$. In this part, we examine the social profit of P2P market with the network charge mechanism. To identify the social optimality gap, we compare *Optimal P2P* with *Social P2P*. We evaluate the social profit over the 24 periods and display the results in curves in Fig. 9(a) (No ES) and Fig. 9(b) (With ES). To distinguish the social optimality gap, we fill the differences of curves between *Optimal P2P* and *Social P2P* in blue. Note that the *positive area* can be interpreted as the social optimality gap of *Optimal P2P* over *Social P2P*. We have the social optimality gap 4.70% (No ES) and 1.32% (With ES). Note that though we observe a larger shaded area with ES (Fig. 9(b)) over No ES (Fig. 9(a)), the accumulated area in positive is smaller with the former. Considering the social optimality gap, we imply that the network charge mechanism can provide *near-optimal* social welfare.

We further study how the social profit is affected by the network charge price. Similarly, we simulate the range of network charge price $\gamma \in [0,1]\,\mathrm{s\$/(kW \cdot km)}$ with an incremental of $\Delta\gamma = 0.01\,\mathrm{s\$/(kW \cdot km)}$. For each simulated network charge price, we evaluate the accumulated social profit over the 24 periods and display the results in Fig. 10. We observe that the social profit first increases w.r.t. the network charge price and then drops after the social optima is reached. Besides, we see that though the optimal network charge price does not coincide with the social optima, the social optimality gap is quite small, which are only 4.70% (No ES) and 1.32% (With ES) in accordance with the previous analysis.

4) The Network Charge Favors Localized Transaction and Curbs Long Distancing Transaction: In this part, we study how the network charge mechanism shapes the P2P markets. We compare *Optimal P2P* with *Free P2P* both with and without ES on the prosumer side. For each market, we calculate the
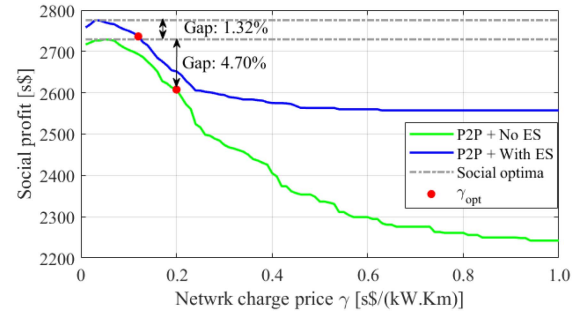

Fig. 10. Social profit w.r.t. network charge price for IEEE 9-bus system (*Optimal P2P + With ES:* social optimality gap 1.32%. *Optimal P2P + No ES:* social optimality gap 4.70%).
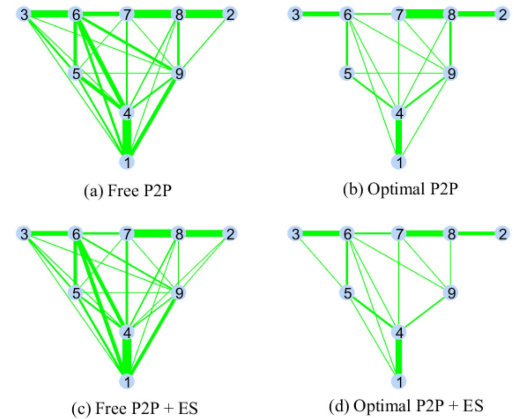

Fig. 11. Total P2P transaction over the 24 periods across the prosumers for IEEE 9-bus system (line thickness represents the amounts of transaction).

aggregated transaction (in kW) for the trading peers over the 24 periods and visualize the P2P transaction in Fig. 11. The circles with IDs indicate the prosumers and the line thickness represents the amounts of P2P transaction. We observe that the network charge has an obvious impact on the trading behaviors of prosumers. Specifically, by comparing Fig. 11(a) (No ES) and Fig. 11(b) (With ES), we notice that the network charge dose not affect the transaction between the prosumers in close proximity (e.g., 3–6, 7–8, 2–8, 1–4) but obviously discourages the long distancing transaction (e.g., 4–6, 1–6, 1–9). This is reasonable as the network charge counts on the electrical distance. Therefore, the network charge mechanism favors localized transaction and curbs long distancing transaction. This is celebrated considering the transmission losses of long distancing power transfer. For the case with ES, we draw the same conclusion.

C. IEEE 39-bus, 57-bus and 118-bus Systems

We further evaluate the performance of the network charge mechanism on IEEE 39-bus, 57-bus, and 118-bus systems. We follow the same simulation set-ups in Section IV-A and compare the different P2P markets in Table III. We report the results for the different markets and bus systems in Table IV. We group the results by *Grid-wise*, *Prosumer-wise* and *System-wise*. For *Grid-wise*, we study the total transmission loss, network charge revenue and the grid profit. For *Prosumer-wise*, we are concerned

TABLE IV
OUTCOMES OF DIFFERENT P2P MARKETS

System	Market	Grid-wise			Prosumer-wise		System-wise
		Transmission loss $\times 10^2$ [s\$]	Network charge $\times 10^2$ [s\$]	Grid profit $\times 10^2$ [s\$]	Prosumer profit $\times 10^2$ [s\$]	Total transaction $\times 10^2$ [kW h]	Social profit $\times 10^2$ [s\$]
9-bus	No P2P	0	0	0	22.42	0	22.42
	Free P2P	1.17	0	-1.17	28.25	17.43	27.10
	Social P2P	–	–	–	–	–	27.35
	Optimal P2P	**0.19**	**2.26**	**2.07**	**24.00**	**7.82**	**26.08**
	No P2P+ES	0	0	0	25.57	0	25.57
	Free P2P+ES	1.03	0	-1.03	28.54	15.57	27.51
	Social P2P+ES	–	–	–	–	–	27.87
	Optimal P2P+ES	**0.28**	**1.22**	**0.94**	**26.56**	**7.64**	**27.50**
39-bus	No P2P	0	0	0	110.86	0	110.86
	Free P2P	3.27	0	-3.27	151.03	112.64	147.76
	Social P2P	–	–	–	–	–	148.15
	Optimal P2P	**0.55**	**14.79**	**14.24**	**123.44**	**52.33**	**137.68**
	No P2P+ES	0	0	0	124.69	0	124.69
	Free P2P + ES	2.62	0	-2.62	154.38	107.69	151.76
	Social P2P + ES	–	–	–	–	–	152.18
	Optimal P2P+ES	**0.60**	**11.26**	**10.65**	**134.07**	**45.17**	**144.73**
57-bus	No P2P	0	0	0	191.63	–	191.63
	Free P2P	65.52	0	-65.52	26.20	185.61	196.53
	Social P2P	–	–	–	–	–	232.05
	Optimal P2P	**4.72**	**22.40**	**17.68**	**205.94**	**61.16**	**223.61**
	No P2P + ES	0	0	0	212.50	0	212.50
	Free P2P + ES	66.20	0	-66.20	272.92	177.23	206.75
	Social P2P + ES	–	–	–	–	–	240.19
	Optimal P2P+ES	**5.39**	**18.49**	**13.10**	**222.52**	**52.45**	**235.62**
118-bus	No P2P	0	0	0	427.68	0	427.68
	Free P2P	111.20	0	-111.20	567.38	367.09	455.64
	Social P2P	–	–	–	–	–	530.84
	Optimal P2P	**5.33**	**46.97**	**41.63**	**46.76**	**149.59**	**509.18**
	No P2P + ES	0	0	0	474.84	0	474.84
	Free P2P + ES	177.47	0	-177.47	603.83	391.05	426.35
	Social P2P + ES	–	–	–	–	–	557.79
	Optimal P2P + ES	**5.46**	**37.43**	**31.97**	**505.38**	**128.27**	**537.35**

with the total prosumer profit and total P2P transaction. For *System-wise*, we evaluate the social profit (i.e., the total grid and prosumers' profit). Note that the results for IEEE 9-bus system are also included for completeness. The results associated with *Optimal P2P* and *Optimal P2P + ES* have been highlighted in bold as our main focus.

Overall, we draw similar conclusions for the large bus systems as before. Specially, the network charge mechanism provides positive profit both to the power grid and the prosumers as reported by *Optimal P2P* and *Optimal P2P + ES*. Whereas *Free P2P* and *Free P2P + ES* only please the prosumers with considerable profit increase over *No P2P* and will displease the power grid operator considering the uncovered transmission loss (i.e., negative grid profit). This demonstrates the necessity of network charge to enable the successful deployment of P2P market in the existing power systems from the perspective of economic benefit.

Besides, the network charge mechanism is favorable considering the benefits of P2P shared by the grid operator and the prosumers. Similarly, using *No ES* as the benchmark, we define the profit increase of grid operator and prosumers as their benefits. Based on the results in Table IV, we report the benefits of grid operator and the prosumers from the P2P market in Fig. 12(a) (No ES) and 12(b) (With ES). Notably, we see that the
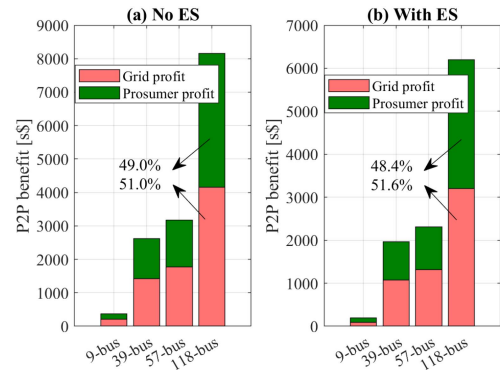
Fig. 12. Benefits of grid operator and the prosumers with *Optimal P2P:* (a) No ES. (b) With ES (*No P2P* as benchmark).

grid operator and the prosumers achieve almost equal benefits from the P2P market with all cases. For example, the benefits for the grid operator and prosumers are about 51.0% vs. 49.0% (No ES) and 51.6% vs. 48.4% (With ES) for the tested IEEE-118 bus systems. This is celebrated to achieve the equilibrium between the power grid and the prosumers.

In addition, we also verify the capability of network charge in shaping the P2P markets. For the tested bus
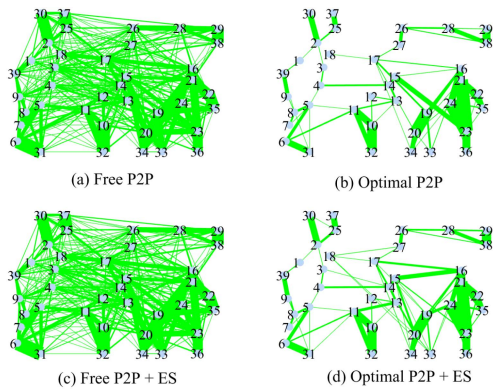
Fig. 13. Total P2P transaction over the 24 periods across the prosumers for IEEE 39-bus system (line thickness represents the amount of transaction).
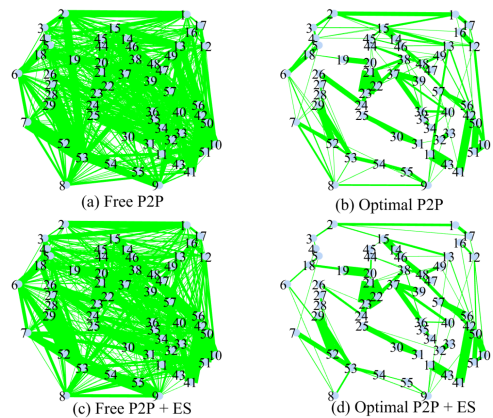

Fig. 14. Total P2P transaction over the 24 periods across the prosumers for IEEE 57-bus system (line thickness represents the amount of transaction).
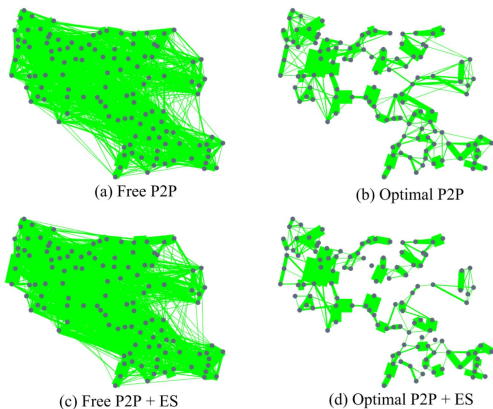

Fig. 15. Total P2P transaction over the 24 periods across the prosumers for IEEE 118-bus system (line thickness represents the amount of transaction).

systems, we compare the P2P transaction of *Free P2P* and *Optimal P2P* both with and without ES. Similarly, we visualize the total P2P transaction over the 24 periods across the trading peers in Fig. 13 (IEEE 39-bus), Fig. 14 (IEEE 57-bus), Fig. 15 (IEEE 118-bus). The circles with IDs indicate prosumers located at the buses and line thickness represents the amounts of transactions. We can perceive the
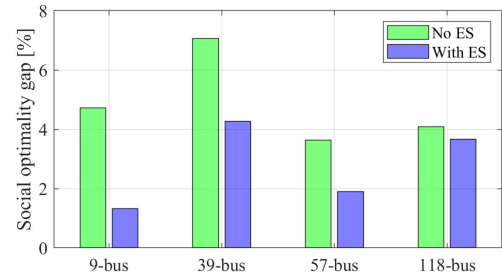

Fig. 16. Social optimality gap of *Optimal P2P*.

obvious impact of network charge on the trading behaviors of prosumers. Specifically, when there is no network charge and the grid operator has no manipulation on the P2P market, the prosumers trade mutually regardless of the electrical distances, leading to massive long distancing transaction. This is not expected considering the high transmission loss and the possible network violations taken by the grid operator. Comparatively, the network charge mechanism favors localized transaction and discourages long distancing transaction, yielding much lower transmission loss as reported in Table IV (Column 3). More importantly, the network charge can ensure the network constraints.

Last but not the least, the network charge mechanism favors social welfare. By examining the *System-wise* performance indicated by social profit in Table IV (Column 8), we find that *Optimal P2P* and *Optimal P2P + ES* provide social profit quite close to *Social P2P*. The social optimality gap is less than 7% (No ES) and 5% (With ES) as reported in Fig. 16,

V. CONCLUSION AND FUTURE WORKS

This paper discussed the integration of the P2P market scheme into the existing power systems by considering the interaction of grid operator providing transmission service and the prosumers behaving in a P2P market. We used network charge as a means for the grid operator to attribute grid-related cost (i.e., transmission loss) and ensure network constraints. We formulated the hierarchical interaction as a Stackelberg game where the grid operator decides on the optimal network charge price to trade off the network charge revenue and the transmission loss considering network constraints, and the prosumers involved in a P2P market optimize their energy management (i.e., consuming, storing and trading). We proved the Stackelberg game admits an *equilibrium* network charge price and proposed a solution method to obtain the *equilibrium* price. By simulating the IEEE bus systems, we demonstrated that the network charge mechanism is favorable as it can *i)* benefit the grid operator while securing the prosumers' profit, and *ii)* provide *near-optimal* social welfare. In addition, we found that the presence of ES will impact the P2P market.

In this paper, we have studied the optimal network charge with deterministic supply and demand, and found that the network charge is effective in shaping the trading behaviors of prosumers in the P2P market. Some future works include: 1) designing optimal network charge price considering uncertain supply and

demand; 2) using the P2P + network charge as a tool to design demand response mechanism.

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