Optimal Sharing and Fair Cost Allocation of Community Energy Storage

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***Abstract*—This paper studies an energy storage (ES) sharing model which is cooperatively invested by multiple buildings for harnessing on-site renewable utilization and grid price arbitrage. To maximize the economic benefits, we jointly consider the ES sizing, operation, and cost allocation via a coalition game formulation. Particularly, we study a fair *ex-post* cost allocation based on *nucleolus* which addresses fairness by minimizing the minimal dissatisfaction of all the players. To overcome the exponential computation burden caused by the implicit characteristic function, we employ a constraint generation technique to gradually approach the unique *nucleolus* by leveraging the sparse problem structure. We demonstrate both the fairness and computational efficiency of the method through case studies, which are not provided by the existing *Shapley approach* or *proportional method*. Particularly, only a small fraction of characteristic function (less than 1% for 20 buildings) is required to achieve the cost allocation versus the exponential information required by *Shapley approach*. Though there exists a minor increase of computation over the *proportional method*, the proposed method can ensure fairness while the latter fails in some cases. Further, we demonstrate both the building-wise and community-wise economic benefits are enhanced with the ES sharing model over the individual ES (IES) model. Accordingly, the overall *value*[1] of ES is considerably improved (about 1.83 times).**

***Index Terms*—Energy storage sharing, coalition game, cost allocation, nucleolus, fairness.**

I. Introduction

ENERGY storage (ES) is a key technology to advance a sustainable, flexible, and reliable energy system [1]. In consumer premise, ES can generate stacked economic benefits by synchronizing local volatile renewable supply with non-shiftable demand and responding to grid price variations [2]. Though the benefits are profound and the governments around the globe have set mandatory goals, the commercial deployment of ES is still largely impeded by the high capital cost [3], [4].

In recent years, *sharing economy* has manifested in transportation and housing systems [5]. Naturally, such sharing-oriented solution is penetrating energy systems for bringing in new technologies like ES [5], [6]. For example, multiple users can cooperatively invest and share a central ES [7]–[9]. The enhanced benefits of shared ES over individual ES (IES) are comprehensive. *First*, the utilization of ES can be increased by exploiting the complementary feature of users. *Second*, duplicate installation cost can be avoid, which accounts for 20%-50%[2] of the total capital cost. *Third*, additional benefit from economy of scale can be embraced, i.e., purchase bulks of ES at wholesale price. *Last but not the least*, it can relieve the space issue for individuals. The augmented benefits of ES sharing are clear, however how to reap them relies on a comprehensive business model.

A. Related Works

In the literature, a number of works have studied ES sharing models from the perspective of sizing and operation.

For ES sizing, [10] proposed an analytical approach by using a Markovian fluid model to capture the stochastic user demands where the ES was used as backup to ensure user demands under grid capacity limits. Reference [11] studied the optimal sizing of shared ES with the objective to facilitate photovoltaic (PV) utilization so as to minimize consumer cost.

The operation of cooperative ES sharing generally addresses the charging and discharging coordination among different users. Typically, [8] proposed a credit-based distributed algorithm to manage a central ES shared by a group of cost-aware households. The discharging rate of the shared ES is dynamically allocated among the households by their credits which characterize their accumulated stored energy in the shared ES. Reference [12] proposed an ES sharing model for minimizing the reputation-weighted energy cost of consumers, which are characterized by the proportions of their renewable injection into the shared ES over the historical time periods. Similarly, [13], [14] studied the control of a shared

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[1]The proportion of total electricity bill reduction relative to the ES capital cost.

[2]A Tesla Powerwall costs 7,600$ before installation. However, accounting for the installation cost, a rough estimate of the Tesla Powerwall cost $9,600-$15,600 for a full system installation.

ES for optimizing the total weighted cost of a group of homes which reflects their agreements on cost-saving priorities. Reference [15] studied a shared ES model working as an energy provider to serve consumers with elastic demand. The operation of ES is managed by an aggregator obligated to minimize the total electricity bill for consumers. Particularly, a marginal service price model was deduced to charge the consumers. Reference [16] studied the optimal operation of a solar-plus-storage system shared across multiple consumers for social welfare or profit maximization. Generally, these works studied the *ad hoc* operation of a shared ES for some specified objectives. They hardly addressed the optimal ES sizing and capital cost allocation among the participants which are important for practice.

Besides, the sizing and operation of cooperative ES sharing are mostly addressed separately in the literature, however they should be jointly considered so as to justify the high ES capital cost and maximize the economic benefits. Moreover, it is essential to study the fair cost allocation (i.e., ES capital cost) as the users are self-interested and independent stakeholders. This paper works towards such objectives. Particularly, we study an ES sharing model that is cooperatively invested and shared by multiple users to harness the economic benefits of grid price arbitrage as well as local renewable integration. One close work is [17] which studied the similar sharing paradigm of ES, and an analytical fair cost allocation formula based on *core* was identified. However, that work only considered grid price arbitrage for ES which is generally not enough to justify the high ES capital cost, and the results (i.e., ES operation, sizing, and cost allocation) are restricted to the two-period market setting and cannot be extended to the case with time-dependent renewable generation. Exceptionally, [18], [19] studied an ES sharing model that accounts for both price arbitrage and local renewable integration. Whereas those works focused on a different sharing paradigm in which a third-party leads the ES sharing among its "consumers" and pursues profit maximization. Another related work that shares the similar structure of ours by including the ES sizing, operation and cost allocation is [20]. However, that work studied the ES sharing across multiple electricity retailers. Besides, the cost allocation was rule-based and did not address the fairness[3] due to the computational challenges.

B. Main Contributions

This paper studies a cooperative ES sharing model among multiple buildings, each of which seeks economic benefits from local renewable integration and grid price arbitrage. Our main contributions are as follows.

- We formulate the optimal ES sharing integrating optimal sizing, operation and cost allocation as a coalition game.
- We address the fair *ex-post* cost allocation for ES sharing based on *nucleolus*. In particular, we employ a constraint generation technique [21] to overcome the exponential computation burden caused by the implicit characteristic function.

[3]In this paper, we refer to fairness as satisfaction and we interchangeably use the word "fairness" and "satisfaction".

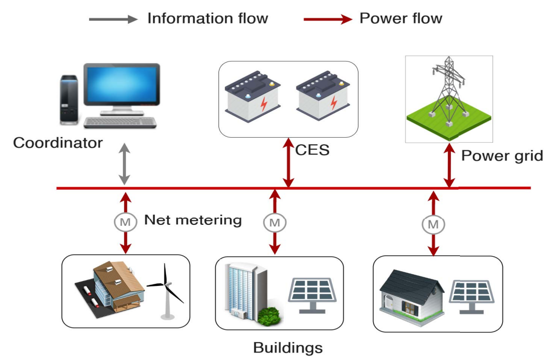


Fig. 1. The configuration of a community energy storage (CES).

- We demonstrate the superiority of the cost allocation over the existing *Shapley approach* and *proportional method* by providing both fairness and computational efficiency through case studies. Particularly, only a small fraction of the characteristic function (less than 1% for 20 buildings) is required to achieve the cost allocation.
- We show the enhanced economic benefits of the ES sharing model over the IES model both at the building-wise and community-wise. Specifically, the ES sharing model yields higher cost reduction to each committed building as well as the whole community. Accordingly, the overall *value* of ES is considerably improved (about 1.83 times).

The remainder of this paper. In Section II, we present the coalition game formulation. In Section III, we study the fair cost allocation. In Section IV, we study the cost allocation and economic benefits of the ES sharing model through case studies. In Section V, we briefly conclude this paper.

II. Coalition Game Formulation of ES Sharing

A. The Configuration of ES Sharing

Fig. 1 shows the configuration of the ES sharing model studied in this paper. We consider a grid-connected community composed of multiple buildings with on-site renewable generation (i.e., wind and solar power). Wherein the buildings cooperatively invest and share a community ES (CES) to harness renewable utilization and grid price arbitrage. The buildings can charge the CES with local renewable generation or the procured electricity from the grid. Conversely, they can discharge it to supply their demand when required. Besides, we allow the buildings to sell energy (i.e., renewable generation or discharged energy from the ES) back to the grid. As the ES is shared by multiple buildings, we assume a central coordinator obligated to coordinate their charging and discharging behaviors of the buildings. Supportively, a net metering is attached to each building for monitoring the energy flow. Particularly, different buildings can charge and discharge simultaneously as the central coordinator only cares about the net power flow. In other word, if one building charges and another building discharges, there would exist some cancellations. However, each individual building can not charge or discharge at the same time due to the physical limits.

We study the optimal ES sharing model that encompasses the optimal sizing, operation, and *ex-post* cost allocation. Considering the computation burden of long-term planning (i.e., sizing), we project the problem on a daily basis and study the problem in a discretized-time framework.

B. Main Assumptions

We clarify our main assumptions as below.

($A1$) The purchase price from the grid is much higher than the selling price to the grid.

($A2$) The buildings do not share (or trade) energy with each other through the shared ES.

($A3$) We only consider non-elastic demand and the demand response of buildings is not included.

Remark 1: We impose ($A2$) with the purpose of studying the enhanced economic benefits from sharing ES capacity. However, the problem formulation and solution can be readily extended to allow energy sharing among the users, which is expected to further increase the overall economic benefits.

C. Coalition Game Formation

We formulate the problem as a coalition game [22]. Coalition game is a branch of game theory that studies the cooperative behaviors of a group of rational agent, which accords with our settings. Following the standard terminologies, we label the building players by $\mathcal{N} := \{1, 2, \dots, N\}$. We study the problem over the discretized time slots $\mathcal{T} := \{0, 1, \dots, T-1\}$.

(i) Coalitions: For a community composed of N buildings, we refer to any subset of buildings $\mathcal{S} \subseteq \mathcal{N}$ cooperatively sharing an ES as an ES coalition or sub-coalition $\mathcal{S}$.

(ii) Coalition Value and Characteristic Function: Coalition value $\nu(\mathcal{S})$ quantifies the economic benefit of an ES coalition $\mathcal{S}$. We indicate a N-player coalition game by $(\mathcal{N}, \nu)$, where function $\nu : 2^N \rightarrow \mathbb{R}$ represents the characteristic function of coalition game that assigns *value* $\nu(\mathcal{S})$ to each sub-coalition $\mathcal{S} \subseteq \mathcal{N}$. Particularly, the number of sub-coalitions $\mathcal{S} \subseteq \mathcal{N}$ grows exponentially with the scale, i.e., $O(2^N)$.

In this paper, we characterize the *value* of an ES coalition $\nu(\mathcal{S})$ by the total cost: the total electricity bills plus the ES capital cost. When we use a collection of representative scenarios to capture the patterns of renewable generation and building demand, the coalition value $\nu(\mathcal{S})$ can be characterized by a two-stage stochastic optimization problem that couples the optimal ES sizing and operation:

$$(\boldsymbol{\mathcal{P}}):\ \nu(\mathcal{S}) = \min_{E_{\mathcal{S}}, P_{\mathcal{S}} \geq 0} c(\boldsymbol{x}_{\mathcal{S}}) + \sum_{\omega \in \Omega} \rho_\omega g(\boldsymbol{x}_{\mathcal{S}}, \zeta_\omega) \qquad (\mathcal{P}_U)$$

$$g(\boldsymbol{x}_{\mathcal{S}}, \zeta_\omega) = \min_{\boldsymbol{y}_i^\omega, i \in \mathcal{S}} g\big(\boldsymbol{x}_{\mathcal{S}}, \boldsymbol{y}^\omega, \zeta_\omega\big) \qquad (\mathcal{P}_U)$$

$$\text{subject to: } \boldsymbol{y}_i^\omega \in \mathcal{Y}_i^\omega,\ i \in \mathcal{S}. \tag{1a}$$

$$\sum_{i \in \mathcal{S}} e_{i,t}^{b,\omega} \leq E_{\mathcal{S}}, t \in \mathcal{T}. \tag{1b}$$

$$\sum_{i \in \mathcal{S}} p_{i,t}^{\mathrm{ch},\omega} \leq P_{\mathcal{S}}, t \in \mathcal{T}. \tag{1c}$$

$$\sum_{i \in \mathcal{S}} p_{i,t}^{\mathrm{dis},\omega} \leq P_{\mathcal{S}}, t \in \mathcal{T}. \tag{1d}$$

where i and t are building and time indices. $\boldsymbol{x}_{\mathcal{S}} = (E_{\mathcal{S}}, P_{\mathcal{S}})$ denotes the ES capacity: energy capacity $E_{\mathcal{S}}$ (in kWh) and power capacity $P_{\mathcal{S}}$ (in kW). ω, ζ_ω, Ω and p_ω represent scenario indices, scenario realizations, scenario collection and scenario probabilities. $\boldsymbol{y}_i^\omega$ denotes the operating strategy for building i under scenarios ω, which includes the charging/discharging of the ES and energy trading with the grid. Accordingly, $\mathcal{Y}_i^\omega$ indicates the set of admissible strategies.

- The first-stage objective ($\mathcal{P}_U$) captures the total cost which is equal to the ES capital cost $c(\boldsymbol{x}_{\mathcal{S}})$ plus the weighted operation cost $\sum_{\omega \in \Omega} p_\omega g(\boldsymbol{x}_{\mathcal{S}}, \zeta_\omega)$. For the ES capital cost, we capitalize on an amortized price model [19], [23]: $c(\boldsymbol{x}_{\mathcal{S}}) = k_p P_{\mathcal{S}} + k_e E_{\mathcal{S}}$. k_p ($s\$/kWh$) and k_e ($s\$/kW$) are the amortized ES capacity price which are obtained according to the projected ES price 100€/kWh and 300€/kW by 2025 [24].
- The second-stage objective ($\mathcal{P}_L$) characterizes the optimal operation cost $g(\boldsymbol{x}_{\mathcal{S}}, \boldsymbol{y}^\omega, \zeta_\omega)$ for each scenario ω, which is subject to the ES capacity $\boldsymbol{x}_{\mathcal{S}}$, the scenario realization ζ_ω, and the building operating strategies $\boldsymbol{y}^\omega = [y_i^\omega], \forall i \in \mathcal{S}$. The operation cost consists of electricity purchase cost and the revenue of selling energy to the grid. We use $c_t^{\mathrm{g}+}$, $c_t^{\mathrm{g}-}$ and $c^{g,\max}$ to indicate the purchase, selling and demand charge price. $p_{i,t}^{\mathrm{g}+,\omega}$ and $p_{i,t}^{\mathrm{g}-,\omega}$ denote the procured and sold energy of building i over period t. $p_i^{g,\max,\omega}$ characterizes the peak demand over the billing cycle. We have the operation cost:

$$g(\boldsymbol{x}, \boldsymbol{y}^\omega, \zeta_\omega) = \sum_{i \in \mathcal{S}} \left\{ \sum_{t \in \mathcal{T}} \left(c_t^{\mathrm{g}+} p_{i,t}^{\mathrm{g}+,\omega} - c_t^{\mathrm{g}-} p_{i,t}^{\mathrm{g}-,\omega} \right) + c^{g,\max} p_i^{g,\max,\omega} \right\} \tag{2}$$

- We define the operation strategy of building i as $\boldsymbol{y}_i^\omega = [p_{i,t}^{\mathrm{ch}}, p_{i,t}^{\mathrm{dis}}, e_{i,t}^{\mathrm{b},\omega}, p_{i,t}^{\mathrm{g}+,\omega}, p_{i,t}^{\mathrm{g}-,\omega}, p_i^{g,\max,\omega}]$ which includes the charging and discharging energy: $p_{i,t}^{\mathrm{ch}}, p_{i,t}^{\mathrm{dis}}$, the stored energy: $e_{i,t}^{\mathrm{b},\omega}$, the procured and sold energy from/to the grid: $p_{i,t}^{\mathrm{g}+,\omega}, p_{i,t}^{\mathrm{g}-,\omega}$ and the peak demand over the billing cycle: $p_i^{g,\max,\omega}$. Correspondingly, the set of admissible operation strategies $\mathcal{Y}_i^\omega$ is constituted by

$$0 \leq p_{i,t}^{\mathrm{ch}} \leq p^{\mathrm{ch},\max}, \tag{3a}$$

$$0 \leq p_{i,t}^{\mathrm{dis}} \leq p^{\mathrm{dis},\max}, \tag{3b}$$

$$e_{i,t+1}^{\mathrm{b},\omega} = e_{i,t}^{\mathrm{b},\omega} + p_{i,t}^{\mathrm{ch},\omega} \eta^{\mathrm{ch}} - p_{i,t}^{\mathrm{dis}} / \eta^{\mathrm{dis}}, \tag{3c}$$

$$e_{i,t}^{\mathrm{b},\omega} \geq 0, \tag{3d}$$

$$p_{i,t}^{\mathrm{g}+,\omega} - p_{i,t}^{\mathrm{g}-,\omega} = p_{i,t}^{\mathrm{ch},\omega} - p_{i,t}^{\mathrm{dis},\omega} + p_{i,t}^{\mathrm{d},\omega} - p_{i,t}^{\mathrm{r},\omega}, \tag{3e}$$

$$p_{i,t}^{\mathrm{g}+,\omega}, p_{i,t}^{\mathrm{g}-,\omega} \leq p_i^{g,\max,\omega}, \tag{3f}$$

$$p_{i,t}^{\mathrm{ch},\omega} p_{i,t}^{\mathrm{dis},\omega} = 0, \tag{3g}$$

$$p_{i,t}^{\mathrm{g}+,\omega} p_{i,t}^{\mathrm{g}-,\omega} = 0,\ \forall t \in \mathcal{T} \tag{3h}$$

where constraints (3a)-(3b) model the charging and discharging rate limits. Constraint (3c) tracks the stored energy for each building subject to the charging and discharging efficiency $\eta^{\mathrm{ch}}, \eta^{\mathrm{dis}}$. In this formulation, we do not consider energy sharing, thus each building can not over deplete its stored energy as imposed by constraint (3d). Constraint (3e) models the instantaneous balance of supply and demand of each building, with $p_{i,t}^{\mathrm{d},\omega}$ and $p_{i,t}^{\mathrm{r},\omega}$ denoting the non-elastic demand and local renewable generation. Constraint (3f) captures peak demand over the billing cycle. Particularly, the complementary constraints (3g)-(3h) enforce the physical limits of non-simultaneous charging (purchasing) and discharging (selling).

Note that problem $(\mathcal{P})$ is non-linear and non-convex due to the presence of complementary constraints (3g)-(3h), making it computationally intractable with *off-the-shelf* solvers. However, considering the ES efficiency, i.e., $\eta^{\mathrm{ch}}, \eta^{\mathrm{dis}} < 1$, and the grid price setting, i.e., $c_t^{\mathrm{g}+} > c_t^{\mathrm{g}-}$, the complementary constraints can be relaxed without affecting the optimal solution. We refer the readers to an illustrative proof in **Appendix** A [25]. With constraints (3g)-(3h) relaxed, there only exist linear constraints. Besides, we note that the two-stage problem has a min-min structure, making it possible to be converted to the single-stage convex problem (4) that can be tacked by some existing commercial solvers (e.g., CPLEX).

$$(\boldsymbol{\mathcal{P}'}) : \ \nu(\mathcal{S}) = \min c(\boldsymbol{x}_{\mathcal{S}}) + \sum_{\omega\in\Omega} \rho_\omega g(\boldsymbol{x}, \boldsymbol{y}^\omega, \zeta_\omega)$$
$$\text{subject to: } (1\text{b}) - (1\text{d}).\ (3\text{a}) - (3\text{g}),\ \forall i \in \mathcal{S}.$$
$$\text{var: } E_{\mathcal{S}}, P_{\mathcal{S}}, \boldsymbol{y}_i^\omega,\ \forall i \in \mathcal{S}. \tag{4}$$

D. ES Coalition Game Properties

In this part, we study the characteristics of the ES coalition game. We have the following main result.

Theorem 1: The ES coalition game $(\mathcal{N}, \nu)$ is subadditive, i.e., $\nu(\mathcal{S}^1 \cup \mathcal{S}^2) \le \nu(\mathcal{S}^1) + \nu(\mathcal{S}^2),\ \forall \mathcal{S}^1, \mathcal{S}^2 \subseteq \mathcal{N}, \mathcal{S}^1 \cap \mathcal{S}^2 = \emptyset$.

Remark 2: We defer the proof to **Appendix** B [25]. **Theorem** 1 implies that it won't be worse off for two disjoint groups of buildings to merge and share a single ES. Fundamentally, the overall economic benefits can be enhanced through merging. Therefore the buildings within a community are inclined to form a grand ES coalition $\mathcal{N}$ to maximize economic benefits.

III. Cost Allocation Based on Nucleolus

In Section II-D, we have proved the enhanced overll benefits of the ES sharing model over IES model. However, the building-wise gains relies on the *ex-post* cost allocation, which is supposed to be fair to ensure stable cooperation: all participants are satisfied and have no motivations to deviate or disrupt the cooperation.

There exist multiple *solution concepts* regarding fair cost allocation of coalition game [26]. One prominent one is *core* [22], [27]. Normally, computing *core* corresponds to a NP-complete linear programming (LP) that depends on the entire characteristic function. For example, for a N-player coalition game, it generally requires to solve a LP problem with $(2^N - 1)$ constraints associating with $O(2^N - 1)$ coalition *value*. This is a non-trivial task due to the exponential computation burden. Besides, the existence and uniqueness of *core* is another general concern for practice [22].

Another primary *solution concept* is *Shapley* value with the main idea of distributing the co-created *value* by the players' marginal contributions. One main advantage of *Shapley* over *core* is the existence and uniqueness. However, it also suffers computation intensity from the entire characteristic function.

Nucleolus is another essential *solution concept* which pursues fairness by minimizing the dissatisfaction of all players [28]. Reasonably, a cost allocation can be viewed as fair if all players are satisfied (i.e., the maximum dissatisfaction of all players is non-positive). Besides, the nonempty and uniqueness property of *nucleolus* is preferable, motivating us to study the cost allocation of ES sharing based on *nucleolus*. Nevertheless, the computation challenge is yet to be addressed as computing *nucleolus* is as or even more difficult than the other *solution concepts*.

A. Definitions

We first introduce the main definitions.

Definition 1 (Cost Allocation): We use vector $\boldsymbol{x} \in \mathcal{R}^N$ to denote a cost allocation of coalition game $(\mathcal{N}, \nu)$, where the entry x_i denotes the allocation to player i.

Definition 2 (Imputation): An imputation $\boldsymbol{x} \in \mathbb{R}^N$ is a cost allocation for a grand coalition $(\mathcal{N}, \nu)$ which is both efficient and individually rational. We have the set of imputations:

$$\mathcal{I} = \left\{\boldsymbol{x} \in \mathbb{R}^N : x(\mathcal{N}) = \nu(\mathcal{N}) \text{ and } \nu(\{i\}) \ge x_i, \forall i \in \mathcal{N}\right\}$$

where we define $x(\mathcal{S}) = \sum_{i\in\mathcal{S}} x_i, \forall \mathcal{S} \subseteq \mathcal{N}$.

Definition 3 (Core): Core refers to the imputation that no subsets of players has incentives to deviate from the grand coalition $(\mathcal{N}, \nu)$. The set of core is defined as

$$\mathcal{C} = \{\boldsymbol{x} \in \mathcal{I} : x(\mathcal{S}) \le \nu(\mathcal{S}), \forall \mathcal{S} \subseteq \mathcal{N}\}.$$

Definition 4 (Lexographical Order): Assume two N-dimensional vectors $\boldsymbol{y}, \boldsymbol{z} \in \mathbb{R}^N$ with elements arranged in non-increasing order, i.e., $y_i \ge y_j$ and $z_i \ge z_j$ if $i < j$. We claim vector $\boldsymbol{y}$ is lexographically smaller than vector $\boldsymbol{z}$, i.e., $\boldsymbol{y} \prec_{\text{lex}} \boldsymbol{z}$, if $\exists k < N$ that $y_i = z_i, \forall i < k$ and $y_k < z_k$ (if $k = 1$, we have $y_1 < z_1$).

Definition 5 (Excess of Coalition): For a given cost allocation $\boldsymbol{x} \in \mathbb{R}^{|\mathcal{S}|}$, the excess of coalition $\mathcal{S}$ is defined as

$$e(\boldsymbol{x}, \mathcal{S}) = x(\mathcal{S}) - \nu(\mathcal{S})$$

For a coalition game characterized by cost minimization, the *excess of coalition* $e(\boldsymbol{x}, \mathcal{S})$ can be interpreted as the *dissatisfaction* of coalition $\mathcal{S}$ with the cost allocation $\boldsymbol{x}$. Since there exists a group of players, the *dissatisfaction* of players regarding a specified cost allocation $\boldsymbol{x} \in \mathbb{R}^N$ correspond to a *excess of coalition* vector. To address this issue, *nucleolus* is defined by the lexographical order of *excess of coalition* vectors. More specifically, *nucleolus* is the *imputation* with the lexicographically minimal *excess of coalition* vector. The formal definition is given below.

Definition 6 (Nucleolus): For a coalition game $(\mathcal{N}, \nu)$, let $O(\boldsymbol{x}) \in \mathbb{R}^{2^N-1}$ be the *excess of the coalition* vector for cost allocation $\boldsymbol{x} \in \mathbb{R}^N$ (imputation) arranged in a non-increasing order, i.e., $O_i(\boldsymbol{x}) \ge O_j(\boldsymbol{x}), \forall i < j$, then a cost allocation $\boldsymbol{x} \in \mathbb{R}^N$

is the nucleolus if we have

$$O(\boldsymbol{x}) \prec_{\text{lex}} O(\boldsymbol{x}'), \forall \boldsymbol{x}' \in \mathcal{I}\backslash\{\boldsymbol{x}\}.$$

B. An Algorithm to Find the Nucleolus

From the definition, we note that *nucleolus* is always nonempty and unique. Particularly, *nucleolus* will locate in the *core* if the latter is nonempty [21]. However, computing *nucleolus* is nontrivial as it corresponds to searching for the lexicographically minimal *excess of coalition* vector. More specifically, we are required to solve a sequence of lexicographically minimization problems, i.e., first identifying the set of cost allocations $\boldsymbol{X}_1$ that minimize $O_1(\boldsymbol{x})$, and then minimize $O_2(\boldsymbol{x})$ over $\boldsymbol{X}_1$, where $O_i(\boldsymbol{x})$ denotes the i-th entry of the *excess of coalition* vector $O(\boldsymbol{x})$ (see **Definition** 6). These episodes are carried forward until the unique *nucleolus* is approached. Particularly, we note that the entire characteristic function is required in each episode to compute the *excess of coalition* vector. This is computationally intensive for ES coalition game as the characteristic function is implicit and characterized by stochastic optimization problems. For example, for an ES coalition with 20 buildings, we are required to solve the stochastic optimization problem (4) $2^{20} - 1$ (more than 10^7) times to identify the characteristic function.

To overcome the computational challenges, this section develops an algorithm to search for *nucleolus* of the ES coalition game by employing a constraint generation technique [21]. The main idea is spurred by the uniqueness of *nucleolus* and the underlying sparse structure of problem. Specifically for a N-player coalition game, the cost allocation corresponds to solving a sequence of LPs (i.e., lexicographically minimization) with N decision variables subject to $2^N - 1$ linear constraints. This implies at most N of $2^N - 1$ constraints are *binding* at the optima. Intuitively, if the N *binding* constraints are known *a priori*, only N coalition *value* are actually required. Indeed, the essential idea of the proposed cost allocation is to identify such critical constraints using constraint generation technique.

To be noted, the algorithm to search for *nucleolus* is constituted by multiple episodes. We first starts with computing the *least core* [26]. After that a sequence of lexiographically minimization problems are solved. In each episode, we capitalize on a constraint generation technique to identify the dissatisfied coalitions with the current cost allocation. The algorithm will terminate until we encounter the unique *nucleolus*. In the subsequent, we introduce the details.

1) Computing Least Core: The *least core* of coalition game $(\mathcal{N}, \nu)$ is the solutions of the LP problem [26]:

$$\begin{aligned} z^* = \min\ & z \\ \text{subject to: } & x(\mathcal{N}) = \nu(\mathcal{N}) \\ & x(\mathcal{S}) - \nu(\mathcal{S}) \leq z,\ \forall \mathcal{S} \subseteq \mathcal{N}\backslash\{\emptyset, \mathcal{N}\}. \end{aligned} \quad (5)$$

Problem (5) is equivalent to $z^* = \min_{\boldsymbol{x}\in\mathcal{I}} \max_{\mathcal{S}\subseteq\mathcal{N}\backslash\{\emptyset,\mathcal{N}\}} e(\boldsymbol{x}, \mathcal{S})$. We note that *least core* is the set of *imputations* with the minimum maximum *excess of coalition*, i.e., $\min_{\boldsymbol{x}\in\mathcal{I}} O_1(\boldsymbol{x})$. Therefore, the *least core* includes *nucleolus* based on the definition.

An example: assume a 2-player coalition game with the *excess of coalition* vectors $O(\boldsymbol{x}) = [5, 3, 3]$, $O(\boldsymbol{y}) = [5, 4, 2]$, and $O(\boldsymbol{z}) = [6, 4, 1]$ for the cost allocation $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$. We can figure out the *least core* $\boldsymbol{x}, \boldsymbol{y}$ and the unique *nucleolus* $\boldsymbol{x}$ because we have $O_1(\boldsymbol{x}) = O_1(\boldsymbol{y}) \leq O_1(\boldsymbol{z})$ and $O_2(\boldsymbol{x}) < O_2(\boldsymbol{y})$.

Clearly, *least core* is well-defined and can be used to narrow the search scope of *nucleolus*. Nevertheless, computing *least core* by solving problem (5) is computationally intensive as it requires the entire characteristic function: $\nu : 2^N \to \mathbb{R}$ which corresponds to the *value* of all sub-coalitions $\mathcal{S} \subseteq \mathcal{N}$. To overcome the computational challenge, we capitalize on a constraint generation technique to gradually approach the *least core* instead of solving problem (5) all at once. The main idea contains three steps: *i)* solve the relaxed problem (6) corresponding to a subset of coalitions $\mathcal{F}_1$ (e.g., start with $\mathcal{F}_1 = \{\{1\}, \{2\}, \ldots, \{N\}\}$); *ii)* identify the most "violated" sub-coalition (i.e., maximum *excess of coalition*) with the obtained cost allocation; *iii)* add the identified coalition to $\mathcal{F}_1$. This process is repeated until no "violated" sub-coalitions with the obtained cost allocation exists. This indicates the *least core* defined in problem (5) is approached. We defer the implementation of constraint generation technique later.

$$z^{1,*} = \begin{bmatrix} \min z \\ \text{subject to:} \\ \quad x(\mathcal{N}) = \nu(\mathcal{N}) \\ \quad x(\mathcal{S}) - \nu(\mathcal{S}) \leq z,\ \forall \mathcal{S} \in \mathcal{F}_1\backslash\{\emptyset, \mathcal{N}\} \end{bmatrix}. \quad (6)$$

2) Lexicographically Optimization: Intuitively, if the *least core* is unique, the *nucleolus* is found. However, that is not the usual case and we usually have to carry on to identify the unique *nucleolus* by solving a sequence of lexicographically optimization problems. For example, minimize $O_2(\boldsymbol{x})$ over the *least core* $\boldsymbol{x} \in \{\boldsymbol{x}|O_1(\boldsymbol{x}) = z^{1,*}\}$ and so forth. Considering the general case, we introduce the problem of minimizing $O_k(\boldsymbol{x})$ over $\{\boldsymbol{x}|O_j(\boldsymbol{x}) = z^{j,*}, \forall j = 1, 2, \ldots, k-1\}$ at episode k. Akin to computing *least core*, we capitalize on the constraint generation technique to overcome the computation burden by executing the three steps. Slightly different, we have the relaxed problem (7) with blocks of *binding* constraints indicated by Γ_j. The interpretation is that at each episode k, we solve the lexicographically optimization $\min_{\boldsymbol{x}} O_k(\boldsymbol{x})$ within the scope of $\{\boldsymbol{x} \in \mathbb{R}^n|O_j(\boldsymbol{x}) = z^{j,*}, \forall j = 1, 2, \ldots, k-1\}$.

$$z^{k,*} = \begin{bmatrix} \min\ z \\ \text{subject to: } x(\mathcal{S}) - \nu(\mathcal{S}) \leq z, \\ \qquad \forall \mathcal{S} \in \mathcal{F}_k \backslash \cup_{j\leq k-1} \Gamma_j. \\ \quad z^{j,*} = x(\mathcal{S}) - \nu(\mathcal{S}), \\ \qquad \forall \mathcal{S} \in \Gamma_j, j = 1, 2, \ldots, k-1. \\ \quad x(\mathcal{N}) = \nu(\mathcal{N}). \end{bmatrix}. \quad (7)$$

3) Constraint Generation: This part introduces the implementation of the constraint generation to identify the most "violated" sub-coalition for 1) and 2).

Suppose we have a cost allocation $\boldsymbol{x}^{k,*}$ (the solution of problem (6) or (7) with a specific subset of coalitions $\mathcal{F}_k$). The constraint generation technique requires to identify the most "dissatisfied" or "violated" sub-coalition (the subset of players) not included in $\mathcal{F}_k$. This requires to identify the sub-coalition with the maximum *excess of coalition* regarding the cost allocation $\boldsymbol{x}^{k,*}$ within the remaining coalitions $\mathcal{N}\backslash\mathcal{F}_k$. To address such issue, we define some binary variables $s_i \in \{0, 1\}, \forall i \in \mathcal{N}$ to indicate whether player i is in the identified sub-coalition or not. Thus we can interchangeably indicate a coalition by $\mathcal{S}^j \subseteq \mathcal{N}$ or a binary vector $\boldsymbol{s}^j = \{s_1^j, s_2^j, \ldots, s_N^j\}$, where we have $s_i^j = 1$ if player i is in coalition $\mathcal{S}^j$, otherwise $s_i^j = 0$. Thus, we can formulate the problem as

$$c^* = \max_{\mathcal{S}} \sum_{j\in\mathcal{S}} x_j^{k,*} - z^{k,*} - \nu(\mathcal{S})$$

$$\text{subject to: } 1 \leq \sum_{i\in\mathcal{N}} s_i \leq N-1. s_i \in \{0,1\},\ \forall i \in \mathcal{N}. \quad \text{(8a)}$$

$$\sum_{\{i|s_i^j=0\}} s_i + \sum_{\{i|s_i^j=1\}} (1-s_i) \geq 1,\ \forall j|\mathcal{S}^j \in \mathcal{F}^k \quad \text{(8b)}$$

where constraint (8a) is imposed to exclude the empty coalition $\emptyset$ and grand coalition $\mathcal{N}$. Constraint (8b) enforces the exclusion of coalitions $\mathcal{F}_k$.

We note that problem (8) requires the explicit characteristic function $\nu(\mathcal{S})$. However, for the ES coalition game, the characteristic function $\nu(\mathcal{S})$ is characterized by the stochastic optimization problem (4). To address such issue, we blend the problem as

$$\begin{aligned} c^* = \max\ & \sum_{j\in\mathcal{S}} x_j^{k,*} - z^{k,*} - \left(c(\boldsymbol{x}_{\mathcal{S}}) + \sum_{\omega\in\Omega} \rho_\omega g(\boldsymbol{x}_{\mathcal{S}}, \zeta_\omega) \right) \\ \text{subject to: }\ & (1b)-(1d).\ (3a)-(3e),\ (3g)-(3h),\ \forall i \in \mathcal{N}. \\ & p_{i,t}^{g+,\omega} - p_{i,t}^{g-,\omega} \geq s_i p_{i,t}^{b,\omega} + p_{i,t}^{d,\omega} - p_{i,t}^{r,\omega}, \\ & p_{i,t}^{g+,\omega}, p_{i,t}^{g-,\omega} \leq s_i P^{g,\max} \\ & 1 \leq \sum_{i\in\mathcal{N}} s_i \leq N-1. \quad s_i \in \{0,1\},\ \forall i \in \mathcal{N}. \\ & \sum_{\{i|s_i^j=0\}} s_i + \sum_{\{i|s_i^j=1\}} (1-s_i) \geq 1,\ \forall j|\mathcal{S}^j \in \mathcal{F}^k. \\ \text{var: }\ & E_{,\mathcal{S}}, P_{\mathcal{S}}, \boldsymbol{s}, \boldsymbol{y}_i^\omega, \forall i \in \mathcal{N} \end{aligned} \quad (9)$$

where $P^{g,\max}$ indicates the maximum trading energy with the grid of each building over single period. The objective of problem (9) characterizes the *excess of coalition* for an ES coalition. Particularly, we use the combinatorial constraints $p_{i,t}^{g+,\omega} - p_{i,t}^{g-,\omega} \geq s_i p_{i,t}^{b,\omega} + p_{i,t}^{d,\omega} - p_{i,t}^{r,\omega}$ to uniformly capture the load balance for the buildings in or out of the sub-coalition. Specifically, for the buildings in the identified sub-coalition, we have $s_i = 1$ and the procured electricity from the grid is at least to satisfy the building demand, otherwise we have $s_i = 0$ and the load balance constraints are relaxed.

Problem (9) is a mixed-integer linear programming (MILP) with $O(N)$ binary variables, which can be handled by some existing solvers like CPLEX for moderate scales. However, if the scale is very large and solving problem (9) directly becomes computationally intensive, we would need to find some other ways to handle the problem. As aforementioned, with the (most) "dissatisfied" sub-coalition $\mathcal{S}^*$ regarding the current cost allocation proposal $\boldsymbol{x}^{k,*}$ obtained, our next step is to added it to $\mathcal{F}_k$ (i.e., $\mathcal{F}_k := \mathcal{F}_k \cup \{\mathcal{S}^*\}$) and adjust the cost allocation accordingly.

Algorithm 1: Search for Nucleolus of ES Coalition Game Based on Constraints Generation

Initialize: $\mathcal{N} := \{1, 2, \ldots N\}$: building participants.
Output : $\boldsymbol{x} \in \mathbb{R}^N$: cost allocation for the buildings.

```
1  Initialize: k → 1, F_1 = {{1}, ..., {N}},  STOP: = false;
2  while !STOP do
3      Solve prolem (7) (or (6) if k = 1) and obtain the
         solution x^{k,*} and z^{k,*};
4      if the solution is unique then
5          STOP: = true;
6          break;
7      end
8      Solve problem (9) to identify the most "dissatisfied"
         sub-coalition S* and the corresponding excess of
         coalition c^{*,k} with the current cost allocation x^{k,*};
9      if c^{k,*} > 0 then
10         Add the identified sub-coalition: F_k := F_k ∪ {S*};
11     else
12         Identify the active and binding constraints Γ_k;
13         F_{k+1} := F_k;
14         k := k + 1;
15     end
16 end
```

The procedures of identifying the most "violated" sub-coalition and adjusting the cost allocation are alternated until no "violated" sub-coalition is found, i.e., the optimal value of problem (8) is non-positive ($c^* \leq 0$). This implies the optimal solution of problem (6) or (7) is approached, i.e., the *least core* or the cost allocation for $\min O_k(\boldsymbol{x})$ has been identified.

We display the main procedures to search for the *nucleolus* of the ES coalition game in **Algorithm** 1. Particularly, we clarify three main points regarding the algorithm. *First*, the algorithm includes two-loops: *outer-loop* and *inner-loop*. The *outer-loop* associates with the lexicographically optimization indicated by the episode k. Whereas the *inner-loop* iteratively solve the lexicographically optimization by employing constraints generation technique. In *inner-loop*, we alternatively identify the most "violated" coalition and update the cost allocation. The *inner-loop* will terminate until no "violated" sub-coalition is found (i.e., $c^* \leq 0$ for problem (9)), which indicates the lexicographically optimization has been solved. *Second*, there are two crucial steps when switching from the *inner-loop* to the *outer-loop*: *i)* at the end of each *inner-loop*, the *active* or *binding* constraints are required to be identified (line 12). This can be achieved by checking the inequality constraints of problem (6) or (7). Specifically, with the obtained solution $\boldsymbol{x}^{k,*}, z^{k,*}$, we identify the *active* or *binding* constraints Γ_k for next computing epoch by checking the equality $z^{k,*} = \nu(\mathcal{S}) - \boldsymbol{x}(\mathcal{S}),\ \forall \mathcal{S} \in \mathcal{F}_k \backslash \cup_{i\leq k-1} \Gamma_i$; *ii)* the subset of coalitions is copied for the next computing epoch $k+1$ (line 13). *Third*, the overall algorithm will terminate until the solution of lexicographically optimization is unique (line 4-7).

IV. Case Study

This section reports the numeric results. We first study the fairness and computational efficiency of the cost allocation based on *nucleolus*. We then investigate the enhanced economic benefits of the ES sharing model over the IES model.

A. Simulation Setup

We set up the case studies based on the real building demand profiles [29] and renewable generation profiles (i.e., wind and solar power) [30] for one year (i.e., 365 scenarios). To account for the complementary feature of energy use in buildings, multiple types of buildings (e.g., office, hotel, school, hospital and restaurant) are considered. Considering the large number of scenarios lead to high computation cost, we choose $S = 10$ representative scenarios to capture the patterns of renewable generations and building demands, respectively. The ES charging and discharging efficiency is set as $\eta^{\text{ch}}, \eta^{\text{dis}} = 0.9$. For the amortized ES capital price, we assume an annual interest rate $r = 0.06$ and ES lifetime $L = 10$ years. We study the problem on a daily circle with the time equally discretized into $T = 24$ time slots, corresponding to a decision interval of $\Delta = 1$h. We refer to the time-of-use electricity price of Singapore: $c_t^{\text{g}+} = 0.1271$ s\$kWh (off-peak 23:00-7:00) and $c_t^{\text{g}+} = 0.2085$ s\$kWh(peak 8:00-22:00) and demand charge $c^{g,\max} = 0.1335$ s\$kW (we set selling price as $c_t^{\text{g}-} = 0$). The maximum trading power with the grid is set as $P^{g,\max} = 10^3$ kW for each building.

B. Fairness and Computational Efficiency

This part evaluates the fairness and computational efficiency of the cost allocation for ES sharing (**Algorithm** 1). We consider five ES coalition of different scales: $N := \{3, 5, 8, 10, 20\}$. We compare the cost allocation based on *nucleolus* with *proportional method* [20] and *Shapley approach* [22]. The *proportional method* is empirical and easy to compute whereas *Shapley approach* is more sophisticated but computationally intensive.

- *Proportional method:* distributes the ES capital cost among the buildings based on their proportions of operation cost (electricity bill) reduction (no ES as *baseline*).
- *Shapley approach:* distribute the payoff among the buildings by their marginal contributions to the ES coalition.

As aforementioned, we evaluate a cost allocation to be fair if all the players are satisfied. In this paper, we assume all the building players are profit-oriented and have no other preferences, therefore it is reasonable to quantify their *dissatisfaction* (*satisfaction*) by their allocated cost. To account for the group of players, we focus on the minimum *excess of coalition* which can be used as an indicator of minimum *dissatisfaction* (DSAT) over all the buildings regarding the cost allocation. Specifically, we have

$$\text{DSAT} = \min_{\mathcal{S} \subseteq \mathcal{N}, \mathcal{S} \neq \emptyset, \mathcal{N}} e(\boldsymbol{x}, \mathcal{S}) \tag{10}$$

Clearly, all players are satisfied (i.e., fair) if the minimum *excess of coalition* is non-positive (DSAT ≤ 0). Moreover, we prefer a cost allocation with a more negative DSAT.

TABLE I
Cost Allocations of Different Methods

Scale (N)	Build.	**Proportional** $\times 10^2$(s\$)	**Shapley** $\times 10^2$ (s\$)	**Nucleolus** $\times 10^2$ (s\$)
3	B1	2.63	2.64	2.69
	B2	4.46	4.49	4.44
	B3	2.40	2.37	2.38
	DSAT	-5.58(Y)	-6.37(Y)	-5.80(Y)

Scale (N)	Build.	**Proportional** $\times 10^2$(s\$)	**Shapley** $\times 10^2$(s\$)	**Nucleolus** $\times 10^2$ (s\$)
5	B1	2.57	2.57	2.59
	B2	4.43	4.43	4.48
	B3	2.30	2.27	2.26
	B4	6.66	6.72	6.75
	B5	4.72	4.68	4.60
	DSAT	-2.00(Y)	-6.73(Y)	-10.18(Y)

Scale (N)	Build.	**Proportional** $\times 10^2$(s\$)	**Shapley** $\times 10^2$ (s\$)	**Nucleolus** $\times 10^2$ (s\$)
8	B1	2.61	2.59	2.64
	B2	4.44	4.40	4.43
	B3	2.32	2.28	2.27
	B4	6.59	6.62	6.60
	B5	4.66	4.64	4.61
	B6	5.82	5.79	5.77
	B7	7.54	7.56	7.54
	B8	6.69	6.79	6.82
	DSAT	8.34(N)	-0.64(Y)	-4.98(Y)

Scale (N)	Build.	**Proportional** $\times 10^2$(s\$)	**Shapley** $\times 10^2$ (s\$)	**Nucleolus** $\times 10^2$ (s\$)
10	B1	2.62	2.58	2.61
	B2	4.44	4.44	4.43
	B3	2.22	2.30	2.22
	B4	6.68	6.59	6.64
	B5	4.60	4.68	4.59
	B6	5.74	5.79	5.75
	B7	7.51	7.48	7.50
	B8	6.77	6.71	6.82
	B9	6.44	6.45	6.46
	B10	7.97	7.96	7.97
	DSAT	10.49(N)	-1.48(Y)	-4.85(Y)

Note: Y: satisfied N: not satisfied

For each scale, we apply the three cost allocation methods to achieve the *ex-post* cost allocation across the building participants. For notation, the buildings are labeled by B1-B20 with the allocated cost displayed in TABLE I (the results for $N = 20$ are omitted due to space limits). First of all, we note that the total cost for each scale are the same regardless of the *ex-post* cost allocation mechanism used. This is caused by the same optimization problem (4) we rely on to compute both the *Shalepy* and *propertional* allocations. However, there exist some differentials regarding the allocated cost to each building under the different cost allocation mechanisms, which lead to the different DSAT as indicated in the last row of each table. Particularly, we find that for the scales $N := \{3, 5\}$, all the three cost allocation mechanisms can ensure fairness as indicated by the negative DSAT, whereas for the larger scales $N := \{8, 10\}$, the fairness is only ensured by the proposed method and *Shapley approach* not the *proportional method*. This demonstrate that the proposed method and

TABLE II
COMPUTATION OF DIFFERENT METHODS

Scale (N)	**Proportional Computation**	**Shapley Computation**	**Nucleolus Computation**
3	4	7	8 (4)
5	6	31	13 (7)
8	9	255	24 (15)
10	11	1023	37 (26)
20	21	$> 10^7$	88 (67)
N	$N+1$	$2^N - 1$	$N+1+\bar{K}(\bar{K})$

Shapley approach can ensure *fairness* whereas *proportional method* may fail in some cases.

For the computational efficiency, we quantify the computation cost of different methods by the fraction of characteristic function required (i.e., the number of coalition *value* computed by solving problem (4)). The computation cost for different scales ($N := \{3, 5, 8, 10, 20\}$) are presented in TABLE II. Particularly, for the *nucleolus*, we start with the singleton and grand coalitions (i.e., $\mathcal{F}_1$), and we record the number of constraint generations (i.e., $\bar{K}$) performed in the execution. First of all, we observe that *Shapley approach* shows the highest computation cost with the entire characteristic function ($2^N - 1$ coalition *value*) required. On the contrary, *proportional method* is most efficient and only requires $N+1$ coalition *value* to achieve the cost allocation (N corresponds to computing the cost for each building with no ES and 1 corresponds to computing the total cost of grand coalition). Notably, we observe the computation cost with the *nucleolus* is slightly higher than the *proportional method* but significantly lower than the *Shapley approach*. It's noteworthy that for $N = 10$, only 2.54% (26/1023) of the characteristic function is required, and when the scale is increased to $N = 20$, the computation burden is reduced to less than 1% ($88/10^7$). This demonstrates the superior computational efficiency of *nucleolus* over the *Shapley approach*. Therefore, we conclude that the *nucleolus* outperforms *Shapley approach* and *proportional method* by providing both computation efficiency and fairness.

C. Economic Benefits of ES Sharing

In this part, we study the enhanced economic benefits of the ES sharing model (referred to CES model) over the IES model. For the IES model, each building invests private ES separately where the optimal ES sizing and operation are obtained by solving problem (4) with $N = 1$. Particularly, for the CES model, we consider two settings: without energy sharing (CES) and with energy sharing (CES + Share). For the CES + Share model, we can follow the previous notations and formulations but replace (3d) with $\sum_{i=1}^{N} e_{i,t}^{\mathrm{b},\omega} \geq 0, \forall t \in \mathcal{T}$ which indicates the stored energy injected by the different buildings are commonly owned. Using the scale with $N = 5$ and $N = 10$ buildings as examples, we study both the building-wise (B1-B10) and community-wise (Com.) economic benefits with the two ES models. The building-wise economic benefits with the CES and CES + Share model are calculated based on the *ex-post* cost allocation of *nucleolus*. The community-wise economic benefits represent the overall economic benefits

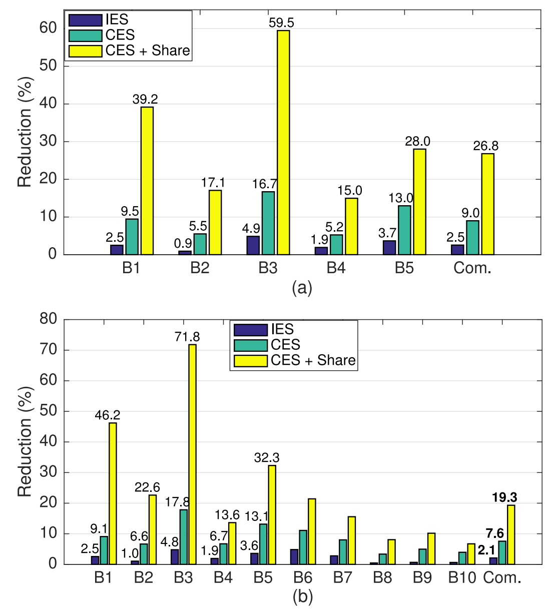


Fig. 2. The building-wise (B1-B10) and community-wise (Com.) cost reduction with the IES and CES model: (a) $N = 5$. (b) $N = 10$ (no ES as *baseline*).

for all the buildings. Using the cost with no ES as *baseline*, the building-wise (B1-B10) and community-wise (Com.) economic benefits can be quantified by the cost reduction as shown in Fig. 2. We see the CES model yields higher percentage of cost reduction to each committed building and the whole community over the IES model. Taking the case with $N = 5$ as an example [Fig. 2(a)], the cost of B3 (i.e., electricity bill plus ES capital cost) is cut off by 16.7% with the CES model versus 4.9% with IES model, and the overall cost is reduced by 9.0% versus 2.5%. This implies the CES model can enhance both the building-wise and community-wise economic benefits over the IES model. Besides, we note that the CES + Share model can enhance the economic benefits significantly further. For example, the cost reduction for B3 is up to 59.5% with the CES + Share model. We see the similar results with the scale $N = 10$. Further, by comparing the results with $N = 5$ and $N = 10$, we find that B1-B5 (appear in both scales) all gain higher cost reduction with the larger coalition (i.e.,$N = 10$) (the marginal decrease of B1 is caused by computing accuracy). This demonstrates that by forming a large ES sharing coalition, the economic benefits of the building participants can be further enhance, however this may require a more powerful central coordinator for coordination.

Further, we study the average *value* of ES (VoS) with the different ES models (i.e., IES, CES, CES + Share). The VoS is defined as the proportion of operation cost reduction over the ES capital cost, representing the average return on investment (ROI):

$$\mathrm{VoS} = \left[\bar{x}^{\mathrm{Opex}} - x^{\mathrm{Opex}}\right] / x^{\mathrm{Cap}}$$

where x^{Opex} and x^{Cap} denote the optimal operation cost and capital cost yield by an ES model. $\bar{x}^{\mathrm{Opex}}$ represents the operation cost with no ES. Intuitively, there exists potential to

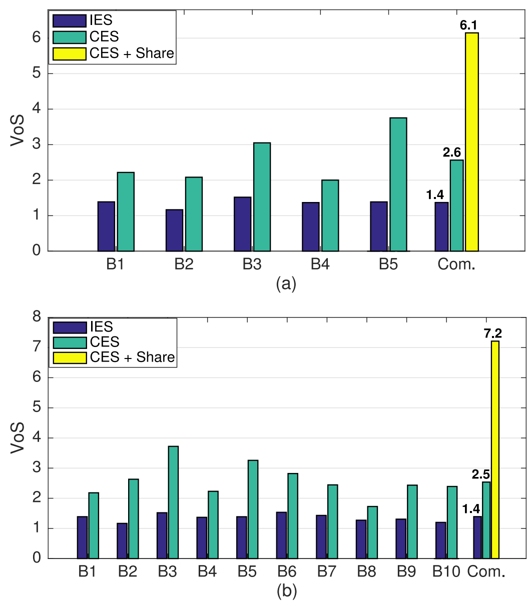


Fig. 3. The building-wise (B1-B10) and community-wise (Com.) VoS with the IES and CES model: (a) $N = 5$. (b) $N = 10$.

invest ES if and only if VoS > 1. Moreover, we would prefer an ES model with a higher VoS. In this part, we study the VoS for both individual buildings and the whole community. Particularly, the ES capital cost allocated to each building with the CES model can be obtained by the total allocated cost minus the electricity bill (obtained by solving problem (4)). For the CES + Share model, we only study the VoS for the whole community due to the lack of ES capital cost for individual buildings. As with CES + Share model, the cost of each building consists of the electricity bill, energy trading cost, and ES capital cost. However, the latter two parts can not be distinguished from the cost allocation. Similarly, we use the case with $N = 5$ and $N = 10$ as examples and we present the results in Fig. 3. First, we observe that the VoS for each building is apparently increased with the CES model over the IES model. Overall, the VoS for the whole community is about 1.8 times with the CES model than the IES model. Besides, by comparing the results with $N = 5$ and $N = 10$, we find that a large ES sharing coalition also favors the VoS for individual buildings (B1-B5). Notably, we see the CES + Share model can significantly increase the overall VoS for the community than the CES model. This is reasonable that allowing the buildings to share their surplus local renewable generation can reduce the over grid purchase for supplying the building demand.

V. Conclusion

This paper studied a cooperative energy storage (ES) business model based on the sharing mechanism. To maximize the economic benefits of ES, we studied the problem by integrating the optimal planning (i.e., ES sizing), operation, and fair *ex-post* cost allocation via a coalition game formation, thus yielding higher economic benefits to each building and the whole community over the individual energy storage (IES) model. Particularly, the fair *ex-post* cost allocation was achieved based on *nucleolus* which ensures fairness by minimizing the dissatisfaction of all players. To handle the exponential computation burden, we applied the constraint generation technique to gradually approach the unique *nucleolus* considering the sparse problem structure, which was demonstrated with both computation efficiency and fairness. Further, through the case studies, we found that by enabling energy sharing through the shared ES, the economic benefits of ES can be further enhanced for the buildings with surplus local renewable generation. As the commercial deployment of ES is currently impeded by the high capital cost, this work can work as an example how business model designs can benefit the practice of ES technologies. Currently, we do not consider the degradation of ES capacity caused by the charging and discharging circles due to the complexity of quantification, however it seems an interesting work to incorporate the recently developed convex rainflow cycle-based model [31] to address that issue in the future.

References

[1] F. J. De Sisternes, J. D. Jenkins, and A. Botterud, "The value of energy storage in decarbonizing the electricity sector," *Appl. Energy*, vol. 175, pp. 368–379, Aug. 2016.

[2] Y. Yu, Z. Cai, and Y. Huang, "Energy storage arbitrage in grid-connected micro-grids under real-time market price uncertainty: A double-Q learning approach," *IEEE Access*, vol. 8, pp. 54456–54464, 2020.

[3] D. S. Mallapragada, N. A. Sepulveda, and J. D. Jenkins, "Long-run system value of battery energy storage in future grids with increasing wind and solar generation," *Appl. Energy*, vol. 275, Oct. 2020, Art. no. 115390.

[4] W. Hu, Z. Chen, and B. Bak-Jensen, "Optimal operation strategy of battery energy storage system to real-time electricity price in Denmark," in *Proc. IEEE PES Gen. Meeting*, Minneapolis, MN, USA, 2010, pp. 1–7.

[5] H. Heinrichs, "Sharing economy: A potential new pathway to sustainability," *GAIA Ecol. Perspect. Sci. Soc.*, vol. 22, no. 4, pp. 228–231, 2013.

[6] P. Lombardi and F. Schwabe, "Sharing economy as a new business model for energy storage systems," *Appl. Energy*, vol. 188, pp. 485–496, Feb. 2017.

[7] B. P. Koirala, R. A. Hakvoort, E. C. van Oost, and H. J. van der Windt, "Community energy storage: Governance and business models," in *Consumer, Prosumer, Prosumager: How Service Innovations Will Disrupt the Utility Business Model*. London, U.K.: Academic, 2019, pp. 209–234.

[8] H. Zhu and K. Ouahada, "Credit-based distributed real-time energy storage sharing management," *IEEE Access*, vol. 7, pp. 185821–185838, 2019.

[9] B. P. Koirala, E. van Oost, and H. van der Windt, "Community energy storage: A responsible innovation towards a sustainable energy system?" *Appl. Energy*, vol. 231, pp. 570–585, Dec. 2018.

[10] I. S. Bayram, M. Abdallah, A. Tajer, and K. A. Qaraqe, "A stochastic sizing approach for sharing-based energy storage applications," *IEEE Trans. Smart Grid*, vol. 8, no. 3, pp. 1075–1084, May 2015.

[11] I. Kim and D. Kim, "Optimal capacity of shared energy storage and photovoltaic system for cooperative residential customers," in *Proc. Int. Conf. Inf. Commun. (ICIC)*, Hanoi, Vietnam, 2017, pp. 293–297.

[12] T. AlSkaif, A. C. Luna, M. G. Zapata, J. M. Guerrero, and B. Bellalta, "Reputation-based joint scheduling of households appliances and storage in a microgrid with a shared battery," *Energy Build.*, vol. 138, pp. 228–239, Mar. 2017.

[13] J. Yao and P. Venkitasubramaniam, "Optimal end user energy storage sharing in demand response," in *Proc. IEEE Int. Conf. Smart Grid Commun. (SmartGridComm)*, Miami, FL, USA, 2015, pp. 175–180.

[14] J. Yao and P. Venkitasubramaniam, "Privacy aware stochastic games for distributed end-user energy storage sharing," *IEEE Trans. Signal Inf. Process. Netw.*, vol. 4, no. 1, pp. 82–95, Mar. 2018.

[15] W. Zhang, W. Wei, L. Chen, B. Zheng, and S. Mei, "Service pricing and load dispatch of residential shared energy storage unit," *Energy*, vol. 202, Jul. 2020, Art. no. 117543.

[16] A. Fleischhacker, H. Auer, G. Lettner, and A. Botterud, "Sharing solar PV and energy storage in apartment buildings: Resource allocation and pricing," *IEEE Trans. Smart Grid*, vol. 10, no. 4, pp. 3963–3973, Jul. 2019.

[17] P. Chakraborty, E. Baeyens, K. Poolla, P. P. Khargonekar, and P. Varaiya, "Sharing storage in a smart grid: A coalitional game approach," *IEEE Trans. Smart Grid*, vol. 10, no. 4, pp. 4379–4390, Jul. 2019.

[18] D. Zhao, H. Wang, J. Huang, and X. Lin, "Pricing-based energy storage sharing and virtual capacity allocation," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Paris, France, 2017, pp. 1–6.

[19] D. Zhao, H. Wang, J. Huang, and X. Lin, "Virtual energy storage sharing and capacity allocation," *IEEE Trans. Smart Grid*, vol. 11, no. 2, pp. 1112–1123, Mar. 2020.

[20] J. Liu, X. Chen, Y. Xiang, D. Huo, and J. Liu, "Optimal planning and investment benefit analysis of shared energy storage for electricity retailers," *Int. J. Elect. Power Energy Syst.*, vol. 126, Mar. 2021, Art. no. 106561.

[21] Å. Hallefjord, R. Helming, and K. Jørnsten, "Computing the nucleolus when the characteristic function is given implicitly: A constraint generation approach," *Int. J. Game Theory*, vol. 24, no. 4, pp. 357–372, 1995.

[22] W. Saad, Z. Han, M. Debbah, A. Hjorungnes, and T. Basar, "Coalitional game theory for communication networks," *IEEE Signal Process. Mag.*, vol. 26, no. 5, pp. 77–97, Sep. 2009.

[23] P. Harsha and M. Dahleh, "Optimal management and sizing of energy storage under dynamic pricing for the efficient integration of renewable energy," *IEEE Trans. Power Syst.*, vol. 30, no. 3, pp. 1164–1181, May 2015.

[24] H. Pandžić, "Optimal battery energy storage investment in buildings," *Energy Build.*, vol. 175, pp. 189–198, Sep. 2018.

[25] Y. Yang, G. Hu, and C. J. Spanos, "Optimal sharing and fair cost allocation of community energy storage," 2020. Accessed: May 24, 2021. [Online]. Available: https://arxiv.org/abs/2010.15455.

[26] A. S. Schulz and N. A. Uhan, "Approximating the least core value and least core of cooperative games with supermodular costs," *Discrete Optim.*, vol. 10, no. 2, pp. 163–180, 2013.

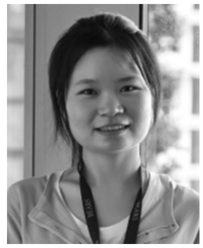
[27] D. B. Gillies, "Solutions to general non-zero-sum games," in *Contributions to the Theory of Games*, vol. 4. Princeton, NJ, USA: Princeton Univ. Press, 1959, pp. 47–85.

[28] D. Schmeidler, "The nucleolus of a characteristic function game," *SIAM J. Appl. Math.*, vol. 17, no. 6, pp. 1163–1170, 1969.

[29] *OpenEI*. Accessed: Oct. 29, 2020. [Online]. Available: https://openei.org/datasets/files/961/pub/

[30] *Measurement and Instrumentation Data Center (MIDC), NREL Transforming Energy*. Accessed: Oct. 29, 2020. [Online]. Available: https://midcdmz.nrel.gov/apps/daily.pl?site=NWTC&start=20010824&yr=2020&mo=1&dy=28

[31] Y. Shi, B. Xu, Y. Tan, D. Kirschen, and B. Zhang, "Optimal battery control under cycle aging mechanisms in pay for performance settings," *IEEE Trans. Autom. Control*, vol. 64, no. 6, pp. 2324–2339, Jun. 2019.



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