附录A. EQ2.15到EQ2.16的推导（1）

梯度定义：设 $\mathbf{x}=(x_1,x_2,\ldots,x_N)$，$y=f(\mathbf{x})$ 则

$$\nabla_{\mathbf{x}}y=\nabla_{\mathbf{x}}f(\mathbf{x})=\left(\frac{\partial f(\mathbf{x})}{\partial x_1},\frac{\partial f(\mathbf{x})}{\partial x_2},\ldots,\frac{\partial f(\mathbf{x})}{\partial x_N}\right)$$

由此，对第三项，有 $\mathbf{P}\mathbf{w}^T=\sum_{i=0}^{N-1}p_i w_i$

$$\nabla_{\mathbf{w}}\left(\mathbf{P}\mathbf{w}^T\right)=\left(\frac{\partial\sum_{i=0}^{N-1}p_i w_i}{\partial w_0},\frac{\partial\sum_{i=0}^{N-1}p_i w_i}{\partial w_1},\ldots,\frac{\partial\sum_{i=0}^{N-1}p_i w_i}{\partial w_{N-1}}\right)$$

$$=(p_0,p_1,\ldots,p_{N-1})=\mathbf{P}$$

附录A. EQ2.15到EQ2.16的推导（2）

对第二项，有 $\mathbf{w}\mathbf{R}\mathbf{w}^T=\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}w_i r_{ij} w_j$ ，以对 w_0 的求导为例：

$$\begin{cases} a.\ \ i=0, j\neq 0\text{时} & \sum_{j=1}^{N-1} r_{0j}w_j \\ b.\ \ i\neq 0, j=0\text{时} & \sum_{i=1}^{N-1} r_{i0}w_i \\ c.\ \ i=0, j=0\text{时} & 2w_0 r_{00} \\ d.\ \ i\neq 0, j\neq 0\text{时} & 0 \end{cases}$$

注意到$\mathbf{R}$的对称性，若$i=j$，则$r_{i0}=r_{0j}$。所以，

$$a+b+c+d=2\sum_{i=0}^{N-1}r_{i0}w_i=2\mathbf{w}\mathbf{R}(:,0)$$

$$\therefore\ \ \nabla_{\mathbf{w}}\mathbf{w}\mathbf{R}\mathbf{w}^T=\left[2\mathbf{w}\mathbf{R}(:,0), 2\mathbf{w}\mathbf{R}(:,1),\ldots,2\mathbf{w}\mathbf{R}(:,N-1)\right]=2\mathbf{w}\mathbf{R}\text{。}$$

Return

附录B. 误差曲线是抛物线的说明

已知 $y=\sum_{i=0}^{N-1}w_i x_i$，$E\left[\varepsilon^2\right]=E\left[(d-y)^2\right]$

考察 w_I 的改变对 $E\left[\varepsilon^2\right]$ 的影响：

$$y=\sum_{i=0}^{N-1}w_i x_i=w_I x_I+\sum_{\substack{i=0\\i\neq I}}^{N-1}w_i x_i=w_I x_I+C$$

所以 $E\left[\varepsilon^2\right]=E\left[(d-y)^2\right]=E\left[(d-w_I x_I-C)^2\right]$

$$=E\left[x_I^2\right]w_I^2-E\left[2(d-C)x_I\right]w_I+E\left[(d-C)^2\right]$$

即$E\left[\varepsilon^2\right]$是关于$w_I$ 的一元二次方程。

Return

附录C. 对复合全微分的说明

设 z 是 $y_i, i=1,\cdots,N$ 的函数，而 y_i 又是 x 的函数，即

$$z=f_0(y_1,y_2,\ldots,y_N),\qquad y_i=f_i(x), i=1,2,\ldots N$$

则，z 对 x 的导数为

$$\frac{dz}{dx}=\frac{\partial z}{\partial y_1}\frac{\partial y_1}{\partial x}+\frac{\partial z}{\partial y_2}\frac{\partial y_2}{\partial x}+\cdots+\frac{\partial z}{\partial y_N}\frac{\partial y_N}{\partial x}$$

$$=\sum_{i=1}^{N}\frac{\partial z}{\partial y_i}\frac{\partial y_i}{\partial x}$$

Return

TC辨识网络

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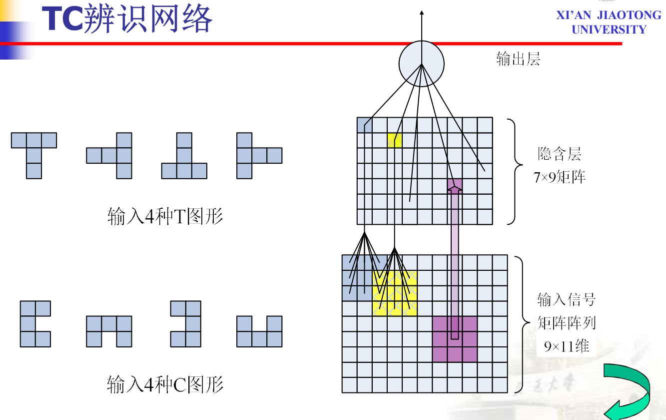

解决T-C辨识的神经网络

TC辨识结果

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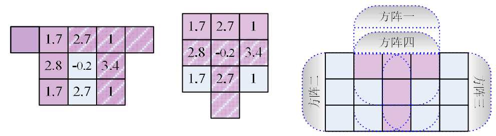
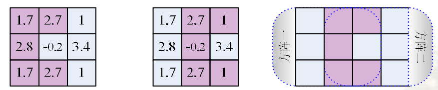


T图形对应隐含层四个神经元输出1 辨识T图形的四个关键方阵

辨识错误的图形有8种辨识正确率为98.4%



C图形对应隐含层两个神经元输出1 辨识C图形的两个关键方阵

理想运算放大器

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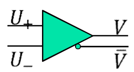
- 差模电压增益$K_d = \infty$
- 差模输入电阻$r_d = \infty$
- 输出电阻$r_o = \infty$
- 带宽$\Delta f = \infty$



- 输入电流$I_i = 0$

$$I_i = \frac{U_+ - U_-}{r_d} = 0$$

- $U_+ = U_-$

反向加法器

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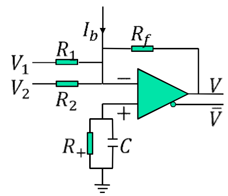
$$\frac{V_1 - U_-}{R_1} + \frac{V_2 - U_-}{R_2} + I_b = \frac{U_- - V}{R_f}$$

$$U_+ = U_-$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + I_b = -\frac{V}{R_f}$$

$$w_1 x_1 + w_2 x_2 - \theta$$



微分方程的稳定状态

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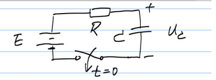
Hopfield 迭代解微分方程.

便笺标题 200

1. 由环路电压方程



$$RC\frac{du_c}{dt} + u_c = E$$

解方程. $u_c = E + A\cdot e^{-\frac{1}{RC}t}$.

A与初始条件有关。 当 $u_c(0)=0$ 时 $A=-E$.

$t\to\infty$ 时 $u_c = E$

2. 三要素法. $f(\infty) + [f(0_+) - f(\infty)]e^{-\frac{t}{\tau}}$.

用计算机迭代解微分方程

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3. 迭代 $\frac{du_c}{dt} = \frac{E-u_c}{RC}$

$RC=\tau$ 为电路时间常数。

$$\frac{u_c(t+\Delta t) - u_c(t)}{\Delta t} = \frac{E - u_c(t)}{\tau}$$

$$\Rightarrow u_c(t+\Delta t) = \frac{\Delta t E + (\tau - \Delta t)u_c(t)}{\tau}.$$

附录A. EQ3.28的推导

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$$E = \frac{1}{2}\left(x - \sum_{i=0}^{3} 2^i V_i\right)^2 - \frac{1}{2}\sum_{i=0}^{3} 2^{2i} V_i (V_i - 1)$$

$$= \frac{1}{2}x^2 - \underline{x\sum_{i=0}^{3} 2^i V_i} + \underline{\underline{\frac{1}{2}\left(\sum_{i=0}^{3} 2^i V_i\right)^2 - \frac{1}{2}\sum_{i=0}^{3} 2^{2i} V_i^2}} + \underline{\frac{1}{2}\sum_{i=0}^{3} 2^{2i} V_i}$$

$$= -\sum_{i=0}^{3}\left(2^i x - 2^{2i-1}\right)V_i + \underline{\underline{\frac{1}{2}\left(\sum_{i=0}^{3} 2^i V_i\right)\left(\sum_{j=0}^{3} 2^j V_j\right) - \frac{1}{2}\sum_{i=0}^{3} 2^{2i} V_i^2}}$$

$$= \frac{1}{2}\sum_{j=0}^{3}\sum_{\substack{i=0 \\ i\neq j}}^{3} 2^{(i+j)} V_i V_j - \sum_{i=0}^{3}\left(2^i x - 2^{2i-1}\right)V_i$$

Return

附录B. 对TSP双下标的说明

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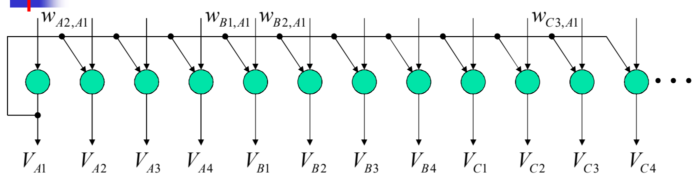


$$w_{Xi,Yj} = -\alpha\delta_{XY}\left(1-\delta_{ij}\right) - \beta\delta_{ij}\left(1-\delta_{XY}\right) - \gamma - \lambda d_{XY}\left(\delta_{j,i+1} + \delta_{j,i-1}\right)$$

$$w_{A2,A1} = -\alpha - \gamma$$

$$w_{B1,A1} = -\beta - \gamma$$

$$w_{B2,A1} = -\gamma - \lambda d_{AB}$$

$$w_{C3,A1} = -\gamma$$

Return

附录C. 对自联想记忆的说明

假设现在输入的是标准样本 $\mathbf{X}_0$，那么

$$
\begin{aligned}
\mathbf{Y} &= \operatorname{sgn}(\mathbf{W}\mathbf{X}_0) \\
&= \operatorname{sgn}\left[\left(\sum_{s=0}^{M-1}\mathbf{X}_s\mathbf{X}_s^T - \mathbf{I}\right)\mathbf{X}_0\right] \\
&= \operatorname{sgn}\left[\sum_{s=0}^{M-1}\mathbf{X}_s\mathbf{X}_s^T\mathbf{X}_0 - M\mathbf{X}_0\right] \\
&= \operatorname{sgn}\left[\mathbf{X}_0\mathbf{X}_0^T\mathbf{X}_0 + \sum_{s=1}^{M-1}\mathbf{X}_s\mathbf{X}_s^T\mathbf{X}_0 - M\mathbf{X}_0\right] \\
&= \operatorname{sgn}\left[120\mathbf{X}_0 + \sum_{s=1}^{7}\mathbf{X}_s\mathbf{X}_s^T\mathbf{X}_0 - 8\mathbf{X}_0\right] \\
&= \operatorname{sgn}\left[112\mathbf{X}_0 + \sum_{s=1}^{7}\mathbf{X}_s\mathbf{X}_s^T\mathbf{X}_0\right]
\end{aligned}
$$

Return

附录D. 对双向联想记忆能量变化的说明

设从 t 时刻到 $t+1$ 时刻，系统由 $\mathbf{B}$ 联想 $\mathbf{A}$，则

$$E(t) = -\mathbf{A}^T(t)\mathbf{W}\mathbf{B}(t) + \mathbf{A}^T(t)\boldsymbol{\theta} + \mathbf{B}^T(t)\boldsymbol{\mu}$$

$$E(t+1) = -\mathbf{A}^T(t+1)\mathbf{W}\mathbf{B}(t+1) + \mathbf{A}^T(t+1)\boldsymbol{\theta} + \mathbf{B}^T(t+1)\boldsymbol{\mu}$$

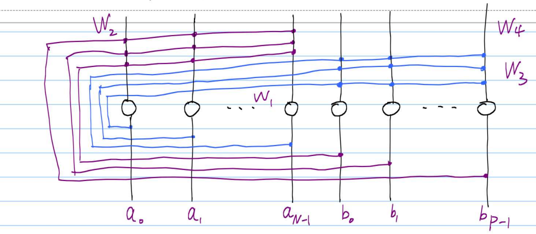
已知 $\mathbf{B}(t+1) = \mathbf{B}(t)$，令 $\Delta\mathbf{A} = \mathbf{A}(t+1) - \mathbf{A}(t)$ 可得：

$$
\begin{aligned}
\Delta E &= E(t+1) - E(t) \\
&= -\left[\mathbf{A}^T(t+1) - \mathbf{A}^T(t)\right]\mathbf{W}\mathbf{B} + \left[\mathbf{A}^T(t+1) - \mathbf{A}^T(t)\right]\theta \\
&= -\Delta\mathbf{A}^T\mathbf{W}\mathbf{B} + \Delta\mathbf{A}^T\theta
\end{aligned}
$$

Return

BAM的能量函数

BAM 能量函数



Hopfield 能量函数 $E = -\frac{1}{2}V^TWV - V^TI$

其中 $V = [a_0, a_1, \cdots a_{N-1}, b_0\ b_1\ \cdots\ b_{P-1}]^T = [A^T\ \ B^T]^T$

BAM的能量函数

BAM中: $E = -\frac{1}{2}\begin{bmatrix}A\\B\end{bmatrix}^T\overline{W}\begin{bmatrix}A\\B\end{bmatrix} - \begin{bmatrix}A\\B\end{bmatrix}^T \cdot [\underbrace{\theta_0 \cdots \theta_{N-1}}_{\vec{\theta}^T}\ \underbrace{\mu_0 \cdots \mu_{P-1}}_{\vec{\mu}^T}]^T$

将 $\overline{W}$ 分块: $\overline{W} = \begin{bmatrix} W_1\ (N\times N) & W_2\ (N\times P) \\ W_3\ (P\times N) & W_4\ (P\times P) \end{bmatrix}$ $(N+P)\times(N+P)$

其中: $W_1 = 0_{N\times N}$. A向量元素之间的反馈。

W_2 $(N\times P)$ B向量对A向量的反馈 $W_2 = W$

W_3 $(P\times N)$ A : : : B : : : : : : $W_3 = W^T$

$W_4 = 0_{P\times P}$ B向量元素之间的反馈.

BAM的能量函数

$$\because \quad E = -\frac{1}{2}\begin{bmatrix} A^T & B^T \end{bmatrix}\begin{bmatrix} 0 & W \\ W^T & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} - A^T\vec{\theta} - B^T\vec{\mu}$$

$$= -\frac{1}{2}\begin{bmatrix} \underset{(1\times N)}{B^T W^T} & \underset{(1\times P)}{A^T W} \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} - A^T\vec{\theta} - B^T\vec{\mu}$$

$$= -\frac{1}{2}\left[B^T W^T A + A^T W B \right] - A^T\vec{\theta} - B^T\vec{\mu}$$

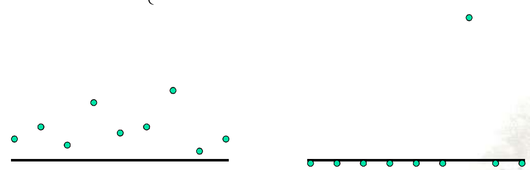
$$= -A^T W B - A^T\vec{\theta} - B^T\vec{\mu}$$

[证完]

附录A. MAXNET算法的解释(1)

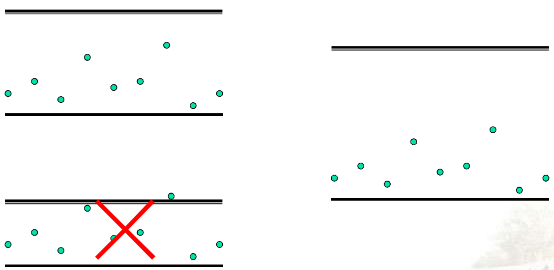
$$y_j(t+1) = f_t\left(y_j(t) - \varepsilon \sum_{\substack{k=1 \\ k\neq j}}^{M} y_k(t) \right) \quad j = 1, 2, \cdots, M$$

$$f_t(\alpha) = \begin{cases} C\alpha & \alpha > 0 \\ 0 & \alpha \le 0 \end{cases} \quad C \ge 1$$

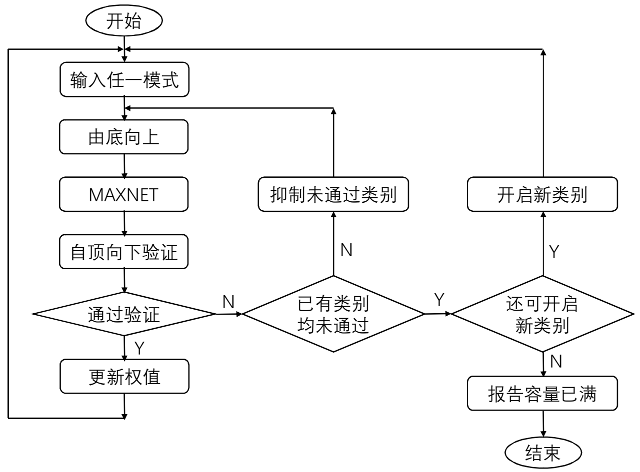


Return

附录A. MAXNET算法的解释(2)



Return



ART基本原理示意流程图

附录B. EQ4.30到EQ4.32的推导

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$$|\mathbf{R}| = \left[\sum_{j=0}^{N-1} r_j^2\right]^{\frac{1}{2}} = \frac{\left[\sum_{j=0}^{N-1}(u_j + cp_j)^2\right]^{\frac{1}{2}}}{|\mathbf{U}| + c|\mathbf{P}|} = \frac{\left[\sum_{j=0}^{N-1}(u_j^2 + 2cu_j p_j + c^2 p_j^2)\right]^{\frac{1}{2}}}{1 + c|\mathbf{P}|}$$

$$= \frac{\left[\sum_{j=0}^{N-1} u_j^2 + 2c\sum_{j=0}^{N-1} u_j p_j + c^2 \sum_{j=0}^{N-1} p_j^2\right]^{\frac{1}{2}}}{1 + c|\mathbf{P}|} = \frac{\left[|\mathbf{U}|^2 + c^2|\mathbf{P}|^2 + 2c\sum_{j=0}^{N-1} u_j p_j\right]^{\frac{1}{2}}}{1 + c|\mathbf{P}|}$$

$$\because \quad \sum_{j=0}^{N-1} u_j p_j = \langle \mathbf{U}, \mathbf{P} \rangle = |\mathbf{P}||\mathbf{U}|\cos(\mathbf{U}, \mathbf{P})$$

$$\therefore \quad |\mathbf{R}| = \frac{\left[1 + c^2|\mathbf{P}|^2 + 2c|\mathbf{P}|\cos(\mathbf{U}, \mathbf{P})\right]^{\frac{1}{2}}}{1 + c|\mathbf{P}|}$$

Return