Original Article

Institution of MECHANICAL ENGINEERS
Journal of ENGINEERING TRIBOLOGY

Proc IMechE Part J:
J Engineering Tribology
1–8

DOI: 10.1177/13506501211038106
journals.sagepub.com/home/pij
SAGE

Lubricating oil deterioration modeling and remaining useful life prediction based on hidden semi-Markov modeling

Ying Du[1], Chaoqun Duan[2] and Tonghai Wu[3]

Abstract

Lubricating oil, which carries information about machine's health condition, is of great importance to the performance of machines in the full life cycle. The main purpose of oil deterioration modeling and its remaining useful life prediction is to determine the exact time that the lubricating oil has degraded and it is no longer able to maintain its functions. Generally, lubricating oil deterioration can be partially detected by condition monitoring based on wear debris analysis, and thus can be categorized into three states. In our paper, vector data, which contain wear debris concentration and carry information about the state of lubricating oil, are obtained by an on-line visual ferrograph sensor from a four-ball tester at regular sampling epochs. The oil's state process is described by a hidden semi-Markov model, and its sojourn times in each state are assumed to be Erlang distributed. A vector autoregressive method based on time series modeling is presented to obtain residual observations, which are regarded as the observable process of oil information in the hidden semi-Markov model framework. The unknown parameters of the hidden semi-Markov model are then estimated by using expectation-maximization algorithm. Afterward, a Bayesian updating approach is presented to derive the explicit formulas of the conditional reliability and mean residual life. To validate the proposed approach, a real case study of lubricating oil deterioration is demonstrated and a comparison with the hidden Markov model is given to illustrate the effectiveness of the new developed remaining useful life prediction approach for lubricating oil.

Keywords

Oil deterioration, wear debris analysis, hidden semi-Markov model, remaining useful life prediction, reliability modeling

Date received: 23 October 2020; accepted: 14 July 2021

Introduction

Lubricating oil, which reflects the system performance in the full life cycle through its properties,[1] should be monitored and replaced appropriately to maintain the service life of machines in a good condition.[2,3] Oil wear analysis and lubrication quality analysis have been widely accepted and performed to describe oil deterioration and machine failure in practical operations[4–7]. Wear particles in lubricated tribo-systems can cause wear increment and accelerate oil deterioration. The concentration of wear particles in oil-system can be used to evaluate machine health[8], and can be regarded as a degraded feature to illustrate deterioration degree of lubricating oil as well[9]. According to the "Bathtub Curve" with three stages: a run-in stage, a normal stage, and a severe stage, wear conditions of tribo-pairs can be characterized by the concentration or the quantity of wear particles in lubricating oil. Furthermore, the degraded features are the indicator of tribological behaviors and performance of lubricating oil. Therefore, the wear debris concentration (an index representing the particle covered area) is considered as the feature to characterize oil deterioration in this research. The oil deterioration process is slow, and the actual deterioration degree is extremely difficult to be examined. The main objective in this research is to propose an approach for modeling the oil deterioration and estimating the residual life for future system decision-makings.

Oil analysis using wear debris obtained from condition monitoring (CM) has been applied in practice for many years, and can be used for actual health condition estimation of operating machines[8]. Nevertheless, there is little research work on lubricating oil deterioration assessment

[1]School of Aeronautics, Northwestern Polytechnical University, China
[2]School of Mechatronic Engineering and Automation, Shanghai University, China
[3]Key Laboratory of Education Ministry for Modern Design and Rotor-Bearing System, Xi'an Jiaotong University, China

Corresponding author:
Chaoqun Duan, School of Mechatronic Engineering and Automation, Shanghai University, Shanghai 200444, China.
Email: chaoqun.duan@hotmail.com

and remaining useful life (RUL) prediction using statistical approaches. To our knowledge, RUL prediction was investigated for the lubricating oil under the condition of particle contamination using physical models based on particle filtering[10], as well as for the rotating machinery based on fractal theory and entropy theory, etc[11–14]. Statistical models have been widely applied for fault prognostics and health management based on the actual status extracting from CM data[15,16], such as the proportional hazard model[4,17,18], hidden Markov model (HMM)[19,20], hidden semi-Markov model (HSMM)[21,22], etc[23]. However, the research works on oil deterioration characterization and its RUL prediction using statistical models are very limited. In previous works, a three-state HMM was proposed to illustrate oil deterioration process and implement oil RUL prediction[19], but sojourn times in each unobservable state of HMM are assumed to follow exponential distributions, which are not realistic with the actual oil deterioration process. HSMM is proved to be more powerful and efficient for real deterioration modeling without the unrealistic assumption[24]. Jiang et al.[25] proposed a parameter estimation method for a HSMM with multi-variate observations. Dong and He [26] proposed a HSMM-based methodology for diagnosis and prognosis, and the unknown model parameter estimation was implemented by the forward–backward training algorithm. Khaleghei and Makis [21] presented a 3-state HSMM for degradation modeling and RUL estimation of a deteriorating system, in which sojourn times of each state were assumed to follow a two-phase Erlang distribution. It is shown that the HSMM provides more precise RUL prediction than the HMM in real cases[27]. Thus, an HSMM for early fault detection and RUL estimation for lubricating oil can be applied. Meanwhile, sojourn times in both the hidden states are considered to be Erlang distributed, which can cover more actual situations of lubricating oil deterioration. It is necessary to measure oil deterioration by monitoring wear debris information, and to evaluate oil condition by employing the HSMM. Based on the built HSMM, RUL prediction for lubricating oil can be presented, where the mean residual life (MRL) can be obtained and updated via the new collected observations[28].

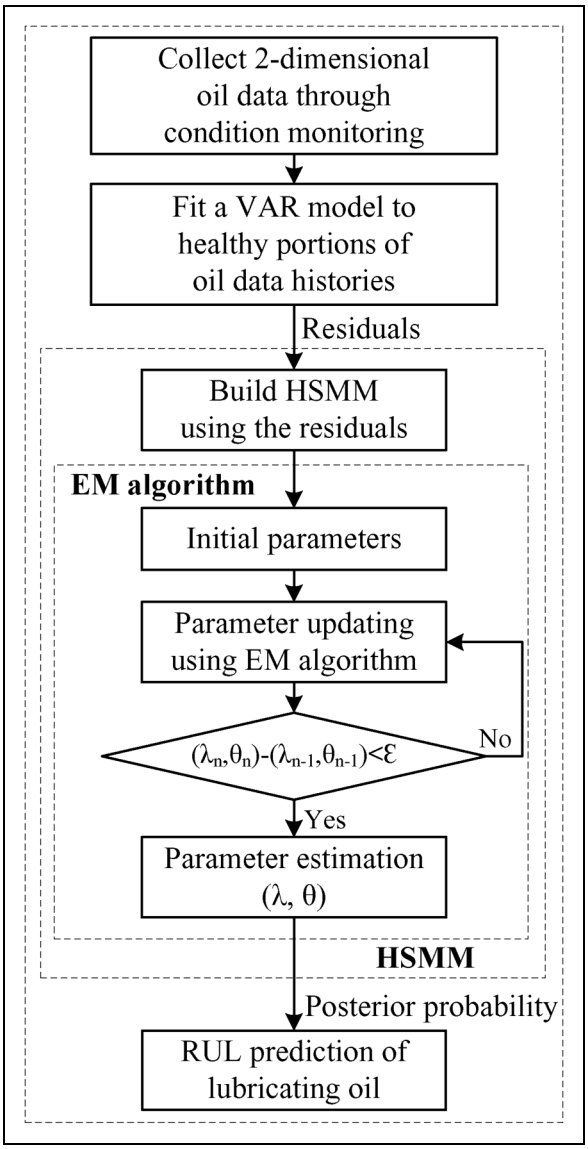

Figure 1. HSMM-based scheme for RUL prediction of lubricating oil. HSMM: hidden semi-Markov model; RUL: remaining useful life.

In this paper, a scheme for oil deterioration and RUL prediction based on HSMM is presented and illustrated in Figure 1, and the main contributions are summarized in Table 1. The remainder of this paper is arranged as follows. In the section "Condition monitoring and oil data pre-processing", condition-based monitoring system is presented, and a vector autoregressive (VAR) method based on time series modeling is proposed to fit the two-dimensional oil data obtained at regular sampling epochs, in order to obtain the residual observations for hidden semi-Markov modeling. In the section "HSMM development and parameter estimation", the obtained residual observations are considered as the observation process in the HSMM framework. An expectation-maximization (EM) algorithm is used to estimate the unknown parameters of HSMM framework. In the section "Bayesian RUL prediction of lubricating oil", RUL prediction for

Table 1. Summary of main contributions.

No.	Contributions
1	Pre-processing development for two-dimensional oil data based on a VAR model. Residual observations are obtained for the HSMM framework.
2	Model development for lubricating oil deterioration based on a three-state HSMM. Erlang distributions are applied to model the sojourn times in each hidden state.
3	Estimation of unknown HSMM parameters based on EM algorithm.
4	RUL prediction for lubricating oil based on a Bayesian updating approach.

HSMM: hidden semi-Markov model; RUL: remaining useful life; VAR: vector autoregressive; . EM: expectation-maximization.

lubricating oil is provided by the Bayesian updating approach. Finally, the conclusions are summarized in the "Conclusions" section.

Condition monitoring and oil data pre-processing

Condition monitoring for lubricating oil and data collection

The real two-dimensional CM data for lubricating oil consisting of small wear particle concentration and large wear particle concentration were collected by an on-line visual ferrograph (OLVF) sensor from a four-ball tester[9]. The experimental equipment is shown in Figure 2, and the oil deterioration process can be simulated and obtained. For more details on the experiment procedure, see Du et al.[9]

The CM data for lubricating oil were collected every $\Delta = 4$ minutes until suspensions or failures. A filter was applied after the sampling of OLVF sensor to ensure that CM data can clearly represent the wear condition at each sampling epoch. The number of suspension histories is $M = 16$, which is defined that the data ends when the oil can still fulfil its functions. The number of failure histories is $N = 11$, which is defined that the data ends with an observable failure, and it is better to replace the oil on the purpose of wear reduction and condition recovery. The total number of the recorded histories of lubricating oil is $M + N = 27$. Figure 3 shows one of the typical data histories of lubricating oil.

Residual observation computation based on VAR model

A VAR model based on time series modeling, was firstly employed to pre-process the data histories[29], where the oil was in good or healthy conditions. The residual observations are obtained and considered as observation process for establishing the HSMM. For the $M + N = 27$ histories, Let $\{z_1^i, z_2^i, \ldots, z_{t_i}^i, i = 1, 2, \ldots, N + M\}$ denote the good and healthy portions of each oil data history, which are assumed to follow a VAR model. The model's standard form is given by

$$\left(Z_n - \mu_0\right) - \sum_{r=1}^{p} \Phi_r\left(Z_{n-r} - \mu_0\right) = \varepsilon_n, \ n \in \mathbf{Z} \quad (1)$$

where $p \in \mathbf{N}$ is the model order, $\mu_0 \in \mathbf{R}^2$ is the mean model parameter, $\mathbf{C} \in \mathbf{R}^{2\times2}$ is the covariance model parameter, $\Phi_r \in \mathbf{R}^{2\times2}$ are the autocorrelation matrices, and ε_n are i.i.d. $N_2(\mathbf{0}, \mathbf{C})$. All these parameters are unknown that need to be estimated.

Set $\mu = \mu_0 - \sum_{r=1}^{p} \Phi_r\mu_0$, the VAR model can be re-written as $Z_n = \mu + \sum_{r=1}^{p} \Phi_r Z_{n-r} + \varepsilon_n, \ n \in \mathbf{Z}$. Then, the regression representation $\mathbf{W} = \mathbf{VA} + \mathbf{E}$ can be obtained for all the healthy portion of the $M + N$ lubricating oil data histories, where $\mathbf{W} = [z_{t_{M+N}}^{M+N}, \ldots, z_{p+1}^{M+N}, \ldots, z_{t_1}^1, \ldots, z_{p+1}^1]'$, $\mathbf{A} = [\mu, \Phi_1, \ldots, \Phi_p]'$, $\mathbf{E} = [\varepsilon_{p+1}^1, \ldots, \varepsilon_{t_1}^1, \ldots, \varepsilon_{p+1}^{N+M}, \ldots, \varepsilon_{t_{N+M}}^{N+M}]'$, and

$$\mathbf{V} = \begin{bmatrix} 1 & z_p^1 & \cdots & z_1^1 \\ 1 & \vdots & \vdots & \vdots \\ 1 & z_{t_1-1}^1 & \cdots & z_{t_1-p}^1 \\ 1 & \vdots & \vdots & \vdots \\ 1 & z_p^{N+M} & \cdots & z_1^{N+M} \\ 1 & \vdots & \vdots & \vdots \\ 1 & z_{t_{N+M}-1}^{N+M} & \cdots & z_{t_{N+M}-p}^{N+M} \end{bmatrix}$$

Using the method of least squares and by searching the table of Chi-square distributions ($\alpha = 0.05$, and the degree of freedom is 2), the parameter estimates of the VAR model are obtained (for explicit computation, see Du et al.[4] and Meng et al.[19]),

$$\hat{p} = 4, \hat{\mu} = \begin{pmatrix} 0.9623 \\ 0.2453 \end{pmatrix}, \hat{\mathbf{C}} = \begin{pmatrix} 0.3319 & 0.0114 \\ 0.0114 & 0.1824 \end{pmatrix}$$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.2524 & -0.0705 \\ 0.2267 & -0.1905 \end{pmatrix}, \hat{\Phi}_2 = \begin{pmatrix} 0.1936 & -0.0946 \\ 0.4151 & -0.0159 \end{pmatrix}$$

$$\hat{\Phi}_3 = \begin{pmatrix} -0.0057 & 0.0079 \\ -0.0059 & 0.0004 \end{pmatrix}, \hat{\Phi}_4 = \begin{pmatrix} 0.0232 & -0.0165 \\ 0.0259 & -0.0084 \end{pmatrix}$$

Then, with the estimates $\hat{\gamma} = (\hat{\mu}, \hat{p}, \hat{\Phi}_1, \hat{\Phi}_2, \hat{\Phi}_3, \hat{\Phi}_4, \hat{\mathbf{C}})$, the residual process $(Y_{n\Delta})$ of lubricating oil can be defined by

$$Y_{n\Delta} := \begin{cases} Z_n - E_{\hat{\gamma}}(Z_n|Z_1, Z_2, \ldots, Z_{n-1}), & n \le \hat{p} \\ Z_n - \left(\hat{\mu} + \sum_{r=1}^{\hat{p}} \Phi_r Z_{n-r}\right), & n > \hat{p} \end{cases} \quad (2)$$

For one of the lubricating oil data histories, the calculated residual observations are illustrated in Figure 4. The normality test (Henze-Zirker multivariate normality test) with $\alpha = 0.05$ is performed for oil data histories with healthy and unhealthy portions, to ensure that the calculated residual observations are multivariate normal distributed (for more details, see[29]). The results in both healthy and unhealthy oil data histories are larger than the significance level α, as illustrated in Table 2. It indicates that the residual observations follow normal distributions, and p-values in the healthy and unhealthy portions agree with the assumption. Normality test is necessary on account that HSMM requires the observations to be normal distributed.

In the next section, the obtained residuals will be regarded as the observation process to establish the HSMM with three macro-states, and the hidden states and observation parameters will be estimated by using an EM algorithm.

HSMM development and parameter estimation

The oil deterioration condition is assumed to be modeled as a semi-Markov process $\{X_t : t \in \mathbf{R}_+\}$ with the macro-state space $\chi = \{1, 2\} \cup \{3\}$, which is continuous and time

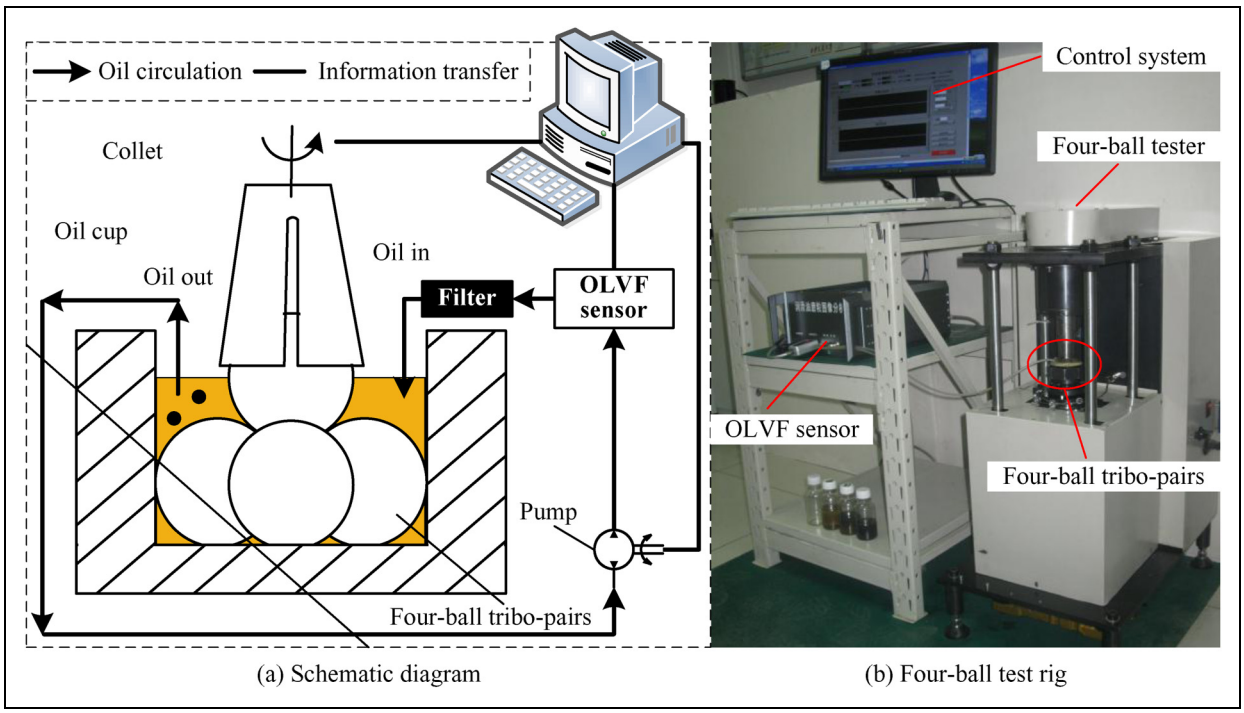

Figure 2. Oil deterioration monitoring system using a four-ball tester.

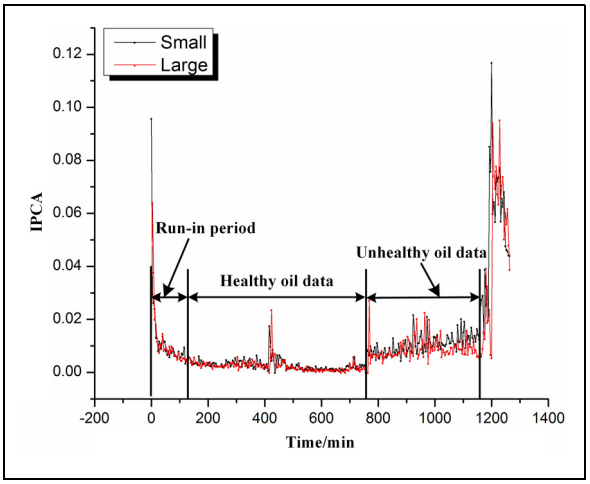

Figure 3. A typical oil data history[19].

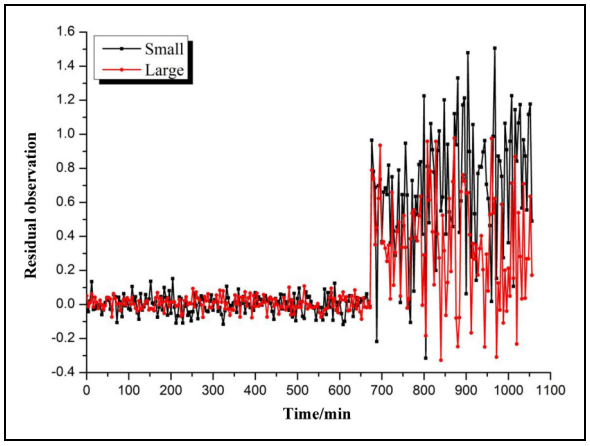

Figure 4. Residual observation process calculated from one of the oil data histories.

homogeneous. Meanwhile, sojourn times in both the state 1 and state 2 follow Erlang distributions with parameters λ_1 and λ_2, respectively[21]. State 1 represents that the operational condition of lubricating oil is healthy or "good", state 2 represents that the oil is in the condition of unhealthy or warning, and state 3 represents that the oil fails and cannot maintain its performances any more. In general, state 1 and state 2 are always unobservable, and state 3 is observable. We assume that, at the very beginning of the service life, lubricating oil is working in a good or healthy state, i.e., $P(X_0 = 1) = 1$, and it can transition from healthy (state 1) to unhealthy (state 2) with the probability p_{12} or to failure (state 3) with the probability p_{13}. The degradation process is slow and irreversible (i.e., $p_{23} = 1$). Assuming that sojourn times in both state 1 and state 2 follow a two-phase Erlang distribution with parameters $(2, \lambda_i)$, we enlarge the state space to $\psi = \{1, 2, 3, 4, 5\}$ as illustrated in Figure 5, where states $\{1, 2\}$ indicate that the oil is under the condition of a healthy state, states $\{3, 4\}$ represent that the oil is working in an unhealthy or warning state, and state $\{5\}$ denotes the observable failure of lubricating oil. Jiang[30] showed that the two-phase Erlang distribution is sufficient to model the degradation process with nonconstant failure rate under reasonable computational time. It can well balance the computational time and the prognostic accuracy in the real applications. Also, the two-phase Erlang distribution has no memoryless property of exponential distribution, which is more suitable for actual deterioration modeling.

Table 2. Normality test for residual observations.

	Healthy oil data	Unhealthy oil data
Henze-Zirkler test (*p*-values)	0.1310	0.1710

Then the new state process $\{\Psi_t : t \in \mathbf{R}_+\}$ of lubricating oil can be modeled as a continuous semi-Markov process with the state space ψ. The instantaneous transition rates $Q = (q_{ij})_{i,j\in\psi}$ of the oil state are given by

$$Q = \begin{bmatrix} -\lambda_1 & \lambda_1 & 0 & 0 & 0 \\ 0 & -\lambda_1 & p_{12}\lambda_1 & 0 & p_{13}\lambda_1 \\ 0 & 0 & -\lambda_2 & \lambda_2 & 0 \\ 0 & 0 & 0 & -\lambda_2 & -\lambda_2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

The transition probability matrix of the state process Ψ_t for lubricating oil, $P(t) = \{P_{ij}(t)_{i,j\in\psi}\}$ can be given by

$$P(t) = \begin{bmatrix} P_{11}(t) & P_{12}(t) & P_{13}(t) & P_{14}(t) & P_{15}(t) \\ 0 & P_{22}(t) & P_{23}(t) & P_{24}(t) & P_{25}(t) \\ 0 & 0 & P_{33}(t) & P_{34}(t) & P_{35}(t) \\ 0 & 0 & 0 & P_{44}(t) & P_{45}(t) \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The transition probability matrix of the state process Ψ_t for lubricating oil, $P(t) = \{P_{ij}(t)_{i,j\in\psi}\}$ can be computed by solving the Kolmogorov backward differential equations showing as follows:

$P_{11}(t) = e^{-\lambda_1 t}$, $P_{12}(t) = \lambda_1 t \cdot e^{-\lambda_1 t}$,
$P_{13}(t) = \lambda_1^2 \cdot \left(\frac{e^{-\lambda_2 t} - e^{-\lambda_1 t}}{(\lambda_1-\lambda_2)^2} + t \cdot \frac{e^{-\lambda_1 t}}{\lambda_2-\lambda_1}\right)$,
$P_{14}(t) = \lambda_1^2\lambda_2 \cdot \left(2 \cdot \frac{e^{-\lambda_1 t} - e^{-\lambda_2 t}}{(\lambda_1-\lambda_2)^3} + t \cdot \frac{e^{-\lambda_1 t} + e^{-\lambda_2 t}}{(\lambda_1-\lambda_2)^2}\right)$,
$P_{22}(t) = e^{-\lambda_1 t}$, $P_{23}(t) = \lambda_1 \cdot \frac{e^{-\lambda_2 t} - e^{-\lambda_1 t}}{\lambda_1-\lambda_2}$,
$P_{24}(t) = \lambda_1\lambda_2 \cdot \left(\frac{e^{-\lambda_1 t} - e^{-\lambda_2 t}}{(\lambda_1-\lambda_2)^2} + t \cdot \frac{e^{-\lambda_2 t}}{\lambda_1-\lambda_2}\right)$,
$P_{33}(t) = e^{-\lambda_2 t}$, $P_{34}(t) = \lambda_2 t \cdot e^{-\lambda_2 t}$,
$P_{44}(t) = e^{-\lambda_2 t}$, $P_{i5}(t)_{i=1,\ldots,4} = 1 - \sum_{j=i}^{4} P_{ij}(t)$.

The residual process (Y_n) of lubricating oil histories is regarded as the observation process of the HSMM. The observations have been proved to follow a 2-dimensional normal distribution $N_2(\mu_i, \Sigma_i)$, $i \in \chi$, where $\mu_1, \mu_2 \in \mathbf{R}^2$ and $\Sigma_1, \Sigma_2 \in \mathbf{R}^{2\times 2}$. Given the state of lubricating oil $(X_{n\Delta} = i)$, the residual observations are conditionally independent, i.e., $Y_n | X_{n\Delta} = i \sim f_i(y_n) \sim^{iid} N_2(\mu_i, \Sigma_i)$. The oil's failure time is denoted as $\xi = \inf\{\, t \in \mathbf{R}_+ : X_t = 3\}$, $T\Delta < t \le (T+1)\Delta$, which is illustrated in Figure 6.

Let $O = \{F_1, F_2, \ldots, F_N, S_1, S_2, \ldots, S_M\}$ represent the observable data set with both the failure histories and suspension histories of lubricating oil, and $C = \{\bar{F}_1, \bar{F}_2, \ldots, \bar{F}_N, \bar{S}_1, \bar{S}_2, \ldots, \bar{S}_M\}$ denote the complete data set of oil histories, where $\{F_1, F_2, \ldots, F_N\}$ denote the $N = 11$ failure histories and $\{S_1, S_2, \ldots, S_M\}$ denotes the $M = 16$ suspension histories. $L(\lambda, \theta|O)$ is denoted as the associated likelihood fuction, and $\lambda = (\lambda_1, \lambda_2)$ and $\theta = (\mu_1, \Sigma_1, \mu_2, \Sigma_2)$ are the state parameters and observation parameters of the HSMM that need to be estimated. Parameter estimation can then be implemented based on the EM algorithm by iterative computations. There are two steps of EM algorithm: E-step and M-step. E-step is aimed at expectation computations of the so-called pseudo likelihood function. M-step is aimed at maximization computations to obtain updated HSMM parameters[31,25]. The flow chart of EM algorithm computation process is illustrated in Figure 7, and the specific steps are given as follows:

E-step: expectation computation with pseudo likelihood function

$$Q(\lambda, \theta|\hat{\lambda}, \hat{\theta}) := E_{\hat{\lambda},\hat{\theta}}(\ln L((\lambda, \theta|C)|O)) \tag{5}$$

M-step: maximization computation of updated parameters

$$\lambda^*, \theta^* \in \arg\max_{\lambda,\theta} Q(\lambda, \theta|\hat{\lambda}, \hat{\theta}) \tag{6}$$

where $\hat{\lambda}, \hat{\theta}$ are estimates of λ and θ in the M-step, and λ^*, θ^* are updated HSMM parameters obtained from the next M-step.

Estimation results are obtained and shown in Table 3 under the criterion $|(\lambda^*, \theta^*) - (\hat{\lambda}, \hat{\theta})| < 10^{-6}$, where the iteration is convergent. For more details on the computing process, see Khaleghei and Makis[21] and Jiang et al.[25]. In the next section, the obtained estimates λ and θ will be used to implement the oil RUL prediction.

Bayesian RUL prediction of lubricating oil

Lubricating oil RUL prediction model depends on the computation of the actual operational status, which can be obtained by the Bayesian posterior probability when it is in a state of unhealthy or warning. Let $\Pi_n(i)$, $i \in \{1, 2, 3, 4\}$ denote the posterior probability statistic when the oil is in the condition of state $i \in \psi$ at n^{th} sampling epoch. We assume that the oil is as healthy as new at the beginning, i.e., $\Pi_0(1) = 1$. By using the Bayes' rule, the posterior probability can be computed and updated by

$$\Pi_n(i) = \mathrm{P}\left(\Psi_{n\Delta} = i | \xi > n\Delta, y_1, y_2, \ldots, y_n, \vec{\Pi}_{n-1}\right) = \begin{cases} \dfrac{\sum_{i=1}^{k} P_{ik}(\Delta)\Pi_{n-1}(i)}{\sum_{j=1}^{2}\sum_{i=1}^{j} P_{ij}(\Delta)\Pi_{n-1}(i) + \dfrac{f(y_n|\mu_2, \sum_2)}{f(y_n|\mu_1, \sum_1)}\sum_{j=3}^{4}\sum_{i=1}^{j} P_{ij}(\Delta)\Pi_{n-1}(i)}, k \in \{1,2\} \\ \dfrac{\sum_{i=1}^{k} P_{ik}(\Delta)\Pi_{n-1}(i)}{\dfrac{f(y_n|\mu_1, \sum_1)}{f(y_n|\mu_2, \sum_2)}\sum_{j=1}^{2}\sum_{i=1}^{j} P_{ij}(\Delta)\Pi_{n-1}(i) + \sum_{j=3}^{4}\sum_{i=1}^{j} P_{ij}(\Delta)\Pi_{n-1}(i)}, k \in \{3,4\} \end{cases} \tag{7}$$

where

$$\frac{f(y_n|\mu_1, \sum_1)}{f(y_n|\mu_2, \sum_2)} = \sqrt{\frac{|\Sigma_2|}{|\Sigma_1|}} \cdot \frac{\exp\left(-\frac{1}{2}(y_n-\mu_1)'\Sigma_1^{-1}(y_n-\mu_1)\right)}{\exp\left(-\frac{1}{2}(y_n-\mu_2)'\Sigma_2^{-1}(y_n-\mu_2)\right)}$$

It is illustrated that the conditional reliability (CR) is calculated based on the posterior probability, which can also be adopted to evaluate and implement the oil RUL prediction. For $\xi > n\Delta$ and $t \geq 0$, CR function of lubricating oil representing that the oil is still operational at $n\Delta + t$ are given by

$$\begin{aligned} R(t|\vec{\Pi}_n) &= P(\xi > n\Delta + t|\xi > n\Delta,\ y_n, \vec{\Pi}_n) \\ &= \sum_{i=1}^{4}(1 - P_{i5}(t)) \cdot \Pi_n(i) \end{aligned} \tag{8}$$

where $\vec{\Pi}_n = (\Pi_n(1), \Pi_n(2), \Pi_n(3), \Pi_n(4))$. The MRL function at the nth sampling epoch can be obtained by integrating the reliability function, which is given by

$$\begin{aligned} \mu_{n\Delta} &= E(\xi - n\Delta|\xi > n\Delta, y_n, \vec{\Pi}_n) = \int_0^{\infty} R(t|\vec{\Pi}_n)\,\mathrm{d}t \\ &= \left(\frac{2}{\lambda_1} + \frac{2}{\lambda_2}\right)(\Pi_n(1) + \Pi_n(2)) + \frac{2}{\lambda_2}\Pi_n(3) \\ &\quad + \frac{1}{\lambda_2}\Pi_n(4) \end{aligned} \tag{9}$$

We apply the CR and MRL functions to one of the failure histories, whose residual observations are shown in Figure 4. Thus, the estimated MRL for this oil data history can be obtained and illustrated in Figure 8, where the comparison with the method of the HMM is also presented.

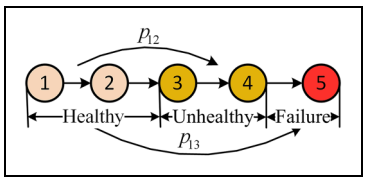

Figure 5. State transition of HSMM.

It can be seen that the MRL of lubricating oil goes down at the 169th sampling epoch. From the 1th to 169th sampling epochs, the oil services with good performances (in the healthy state), and after the 170th sampling epoch, the condition of lubricating oil changes to the unhealthy or warning state with unsatisfactory lubrication. This sharp drop of the MRL value is caused by the sudden wear accumulation of the four-ball tribo-pairs, since the MRL estimate is the sum of weighted posterior probability, which can vary with observation residuals. This drop phenomenon is quite common due to the discontinuity of the oil deterioration pattern. Additionally, the MRL obtained by the proposed HSMM presents a gradual decrease trend with the monitoring time. In comparison, the MRL obtained by the HMM remains almost stable before 169th sampling epochs, which violates the actual deterioration situation. On average, the proposed HSMM outperforms HMM when estimating the RUL of lubricating oil, which is more reliable. It also can be realized that HMM can indicate the state transition time of the lubricating oil, but cannot reflect the real deterioration process and MRL of the oil. Therefore, compared with the HMM, the HSMM gives more real and much better results on RUL prediction of lubricating oil.

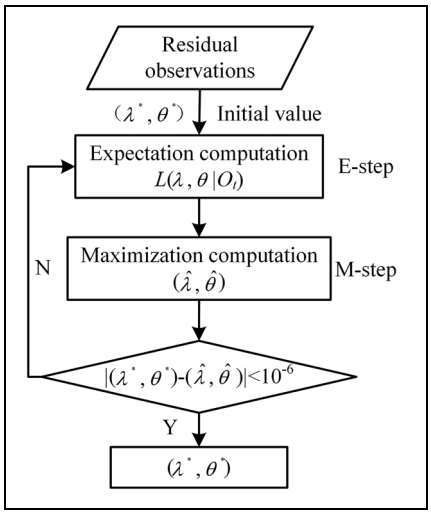

Figure 7. Flowchart of the EM algorithm computation process.

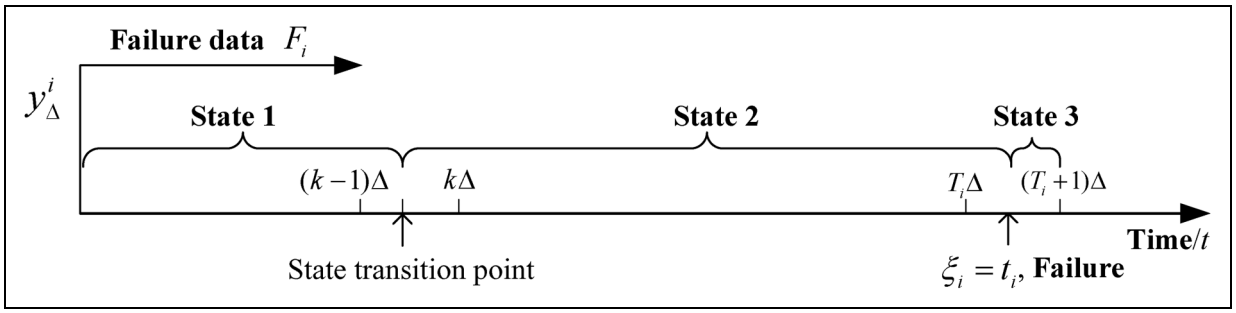

Figure 6. Time flow of the oil failure history.

Table 3. Parameter estimation results obtained by the EM algorithm.

Parameters	Initial values	First iteration	Second iteration	Final estimation
p_{12}	0.6	0.6646	0.8797	1
p_{13}	0.4	0.3354	0.1203	0
λ_1	0.15	0.0757	0.0825	0.6936
λ_2	0.5	0.6805	0.23992	0.0386
μ_1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.0004 \\ 0.0009 \end{pmatrix}$	$\begin{pmatrix} -0.0015 \\ 0.0002 \end{pmatrix}$	$\begin{pmatrix} -0.0051 \\ -0.0132 \end{pmatrix}$
μ_2	$\begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} -0.0095 \\ -0.0038 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0.0013 \end{pmatrix}$	$\begin{pmatrix} -0.0007 \\ 0.0018 \end{pmatrix}$
Σ_1	$\begin{pmatrix} 0.7 & 0.2 \\ 0.2 & 1 \end{pmatrix}$	$\begin{pmatrix} 0.0027 & 0.0001 \\ 0.0001 & 0.0018 \end{pmatrix}$	$\begin{pmatrix} 0.0030 & 0.0002 \\ 0.0002 & 0.0018 \end{pmatrix}$	$\begin{pmatrix} 0.0026 & -0.0002 \\ -0.0002 & 0.0015 \end{pmatrix}$
Σ_2	$\begin{pmatrix} 1.5 & 0.75 \\ 0.75 & 2 \end{pmatrix}$	$\begin{pmatrix} 0.0043 & 0.0010 \\ 0.0010 & 0.0019 \end{pmatrix}$	$\begin{pmatrix} 0.0042 & 0.0012 \\ 0.0012 & 0.0022 \end{pmatrix}$	$\begin{pmatrix} 0.0031 & 0.003 \\ 0.0003 & 0.0018 \end{pmatrix}$

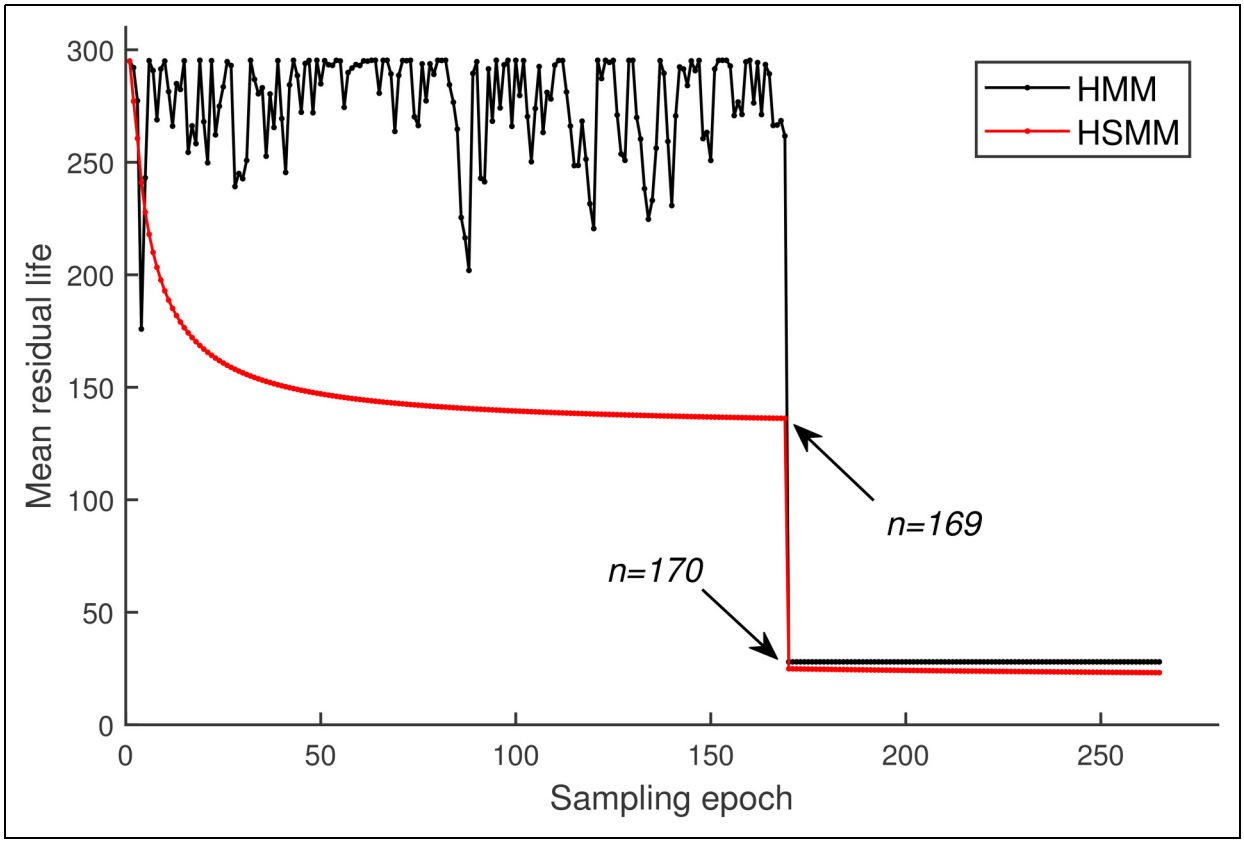

Figure 8. Estimated MRL for the failure oil data history.

Conclusions

In this paper, oil deterioration process is modeled as a stochastic process by a hidden semi-Markov chain with three macro-states. The real two-dimensional oil data are collected by on-line CM based on wear debris analysis from a four-ball tester. To predict the residual time of the deteriorating oil, following steps are presented. Firstly, a VAR model has been presented to calculate residual observations. Secondly, the HSMM is proposed for oil deterioration modeling with the obtained residuals as the observations, and sojourn times in each unobservable state are assumed to follow a two-phase Erlang distribution. Thirdly, an EM algorithm is employed, and the unknown parameters of the HSMM are obtained by maximizing the pseudo likelihood function. Ultimately, RUL prediction of lubricating oil is derived based on the Bayesian statistics for real lubricating oil data. Compared with the HMM-based method, HSMM is illustrated to be more efficient and realistic. The sojourn time that is assumed to follow Erlang distributions provides a better model indicator for the purpose of oil RUL prediction. In addition, the proposed approach shows the potential for future decision-makings of lubricating oil replacement strategy.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

This work was supported by the National Science Foundation of China (grant nos. 51905330, 62033001, and 51975455), and the International Collaborative Plan of Shaanxi Province (no. 2017kw-034). The author also gratefully acknowledges the support of K. C. Wang Education Foundation.

ORCID iDs

Chaoqun Duan https://orcid.org/0000-0002-8938-5474
Tonghai Wu https://orcid.org/0000-0003-1277-7848

References

1. Wen S and Huang P. *Principles of Tribology*. Beijing, China: Qinghua University Press, 2012.
2. Burkhart C, Johansson J, Ukonsaari J, et al. Performance of lubricating oils for wind turbine gear boxes and bearings. *Proc IMechE, Part J: J Engineering Tribology* 2018; 232: 62–72.
3. Zhuang X, Afshari SS, Yu T, et al. A hybrid model for wear prediction of a single revolute joint considering a time-varying lubrication condition. *Wear* 2020; 442-443: 203124.
4. Du Y, Wu T, Zhou S, et al. Remaining useful life prediction of lubricating oil with dynamic principal component analysis and proportional hazards model. *Proc IMechE, Part J: J Engineering Tribology* 2020; 234: 964–971.
5. De Voort FRV, Sedman J, Cocciardi RA, et al. FTIR condition monitoring of in-service lubricants: Ongoing developments and future perspectives. *Tribol Trans* 2006; 49: 410–418.
6. Ahmad R and Kamaruddin S. An overview of time-based and condition-based maintenance in industrial application. *Comput Ind Eng* 2012; 63: 135–149.
7. Yang Y, Wu Q, Wang Y, et al. Dynamic characteristics of cracked uncertain hollow-shaft. *Mech Syst Signal Process* 2019; 124: 36–48.
8. Cao W, Zhang H, Wang N, et al. The gearbox wears state monitoring and evaluation based on on-line wear debris features. *Wear* 2019; 426-427: 1719–1728.
9. Du Y, Wu T, Wang L, et al. Investigation on on-line monitoring method for lubricating oil deterioration. *Mech Ind* 2017; 18: 402.
10. Zhu J, Yoon JM, He D, et al. Online particle-contaminated lubrication oil condition monitoring and remaining useful life prediction for wind turbines. *Wind Energy* 2015; 18: 1131–1149.
11. Meng Z, Li J, Yin N, et al. Remaining useful life prediction of rolling bearing using fractal theory. *Measurement* 2020; 156: 107572.
12. Li Y, Wang S and Deng Z. Intelligent fault identification of rotary machinery using refined composite multi-scale lempel-ziv complexity. *J Manuf Syst* 2020. https://doi.org/10.1016/j.jmsy.2020.05.004
13. Li Y, Wang X, Si S, et al. Entropy based fault classification using the case western reserve university data: A benchmark study. *IEEE Trans Reliab* 2020; 69: 754–767.
14. Zhang D. An adaptive procedure for tool life prediction in face milling. *Proc IMechE, Part J: J Engineering Tribology* 2011; 225: 1130–1136.
15. Alaswad S and Xiang Y. A review on condition-based maintenance optimization models for stochastically deteriorating system. *Reliab Eng Syst Saf* 2017; 157: 54–63.
16. Lei Y, Li N, Guo L, et al. Machinery health prognostics. A systematic review from data acquisition to rul prediction. *Mech Syst Signal Process* 2018; 104: 799–834.
17. Makis V, Wu J and Gao Y. An application of DPCA to oil data for CBM modeling. *Eur J Oper Res* 2006; 174: 112–123.
18. Duan C and Deng C. Prognostics of health measures for machines with aging and dynamic cumulative damage. *IEEE ASME Trans Mechatron* 2020; 25: 2264–2275.
19. Du Y, Wu T and Makis V. Parameter estimation and remaining useful life prediction of lubricating oil with HMM. *Wear* 2017; 376-377: 1227–1233.
20. Tobonmejia DA, Medjaher K, Zerhouni N, et al. A data-driven failure prognostics method based on mixture of gaussians hidden markov models. *IEEE Trans Reliab* 2012; 61: 491–503.
21. Khaleghei A and Makis V. Parameter and residual life estimation for a hidden semi-markov model of a deteriorating system. *J Multidiscip Eng Sci Technol (JMEST)* 2015; 2: 2532–2541.
22. Kharoufeh JP, Solo CJ and Ulukus MY. Semi-markov models for degradation-based reliability. *LIE Trans* 2010; 42: 599–612.
23. Wang H, Zhou C, Hu B, et al. Tooth wear prediction of crowned helical gears in point contact. *Proc IMechE, Part J: J Engineering Tribology* 2020; 234: 947–963.
24. Duan C. Dynamic bayesian monitoring and detection for partially observable machines under multivariate observations. *Mech Syst Signal Process* 2021; 158: 107714.
25. Jiang R, Kim MJ and Makis V. Maximum likelihood estimation for a hidden semi-markov model with multivariate observations. *Qual Reliab Eng Int* 2012; 28: 783–791.
26. Dong M and He D. A segmental hidden semi-markov model (HSMM)-based diagnostics and prognostics framework and methodology. *Mech Syst Signal Process* 2007; 21: 2248–2266.
27. Duan C, Makis V and Deng C. Optimal Bayesian early fault detection for CNC equipment using hidden semi-Markov process. *Mech Syst Signal Process* 2019; 122: 290–306.
28. Makis V. Multivariate bayesian control chart. *Oper Res* 2008; 56: 487–496.
29. Kim MJ, Makis V and Jiang R. Parameter estimation for partially observable systems subject to random failure. *Appl Stoch Models Bus Ind* 2013; 29: 279–294.
30. Jiang R. *System availability maximization and residual life prediction under partial observations*, PhD Dissertation, Toronto, Canada: University of Toronto, 2011.
31. Mclachlan GJ and Krishnan T. *The EM algorithm and extensions*. Hoboken, New Jersey: John Wiley & Sons, 2007. https://doi.org/10.1002/9780470191613