西安交通大学电子与信息工程学院研究生课程

《等离子体电子学》

第五章 玻尔兹曼方程和带电粒子输运方程(2)

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玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程

  - 球谐函数及其性质

    - 分布函数的密度梯度展开

- 在非热平衡低温等离子体中，密度梯度是小量
- 准热平衡：电子从外场获得的能量与它和分子发生非弹性碰撞消耗的能量相等
- 分布函数g(v,r,t)可以通过球谐函数展开为g(v,t)和n(r,t)的乘积

$$g(v, r, t) = \sum_k g^k(v, t) \otimes (\nabla_r)^k n(r, t)$$

$$= \sum_k \sum_{mn} g_{mn}^k(v, t) Y_{mn}^e(\theta, \varphi) \otimes (\nabla_r)^k n(r, t),$$

$$Y_n^m(\theta, \varphi) = \Theta(\theta)\Psi_m(\varphi), \qquad \Psi_m(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi}$$

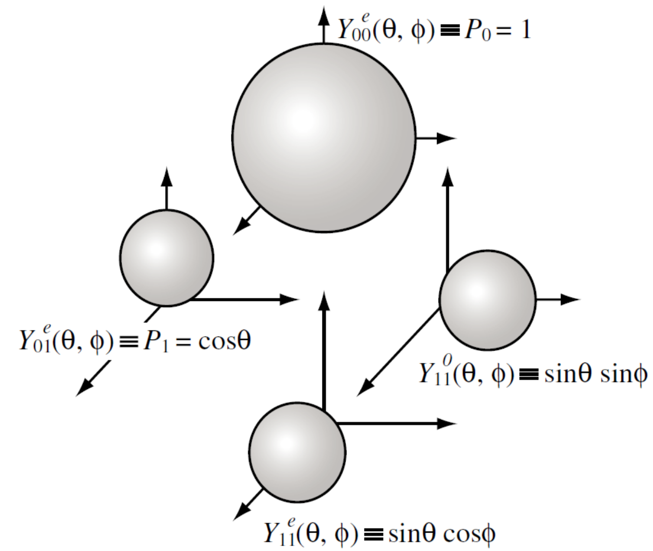
$$\Theta(\theta) = (-1)^m \left(\frac{2n+1}{2} \frac{(n-m)!}{(n+m)!} \right)^{1/2} P_n^m(\cos\theta), \qquad -n \le m \le n.$$

$$Y_n^m(\theta, \varphi) = (-1)^m \left(\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!} \right)^{1/2} P_n^m(\cos\theta) e^{im\varphi}. \qquad (5.64)$$

Their orthogonality is given by

$$\int_\varphi \int_\theta Y_{n_1}^{m_1}(\theta, \varphi) Y_{n_2}^{m_2}(\theta, \varphi) d\Omega = \delta_{n_1, n_2} \delta_{m_1, m_2}. \qquad (5.65)$$

- 电子的玻尔兹曼方程（续）
  - 球谐函数及其性质（续）
    - 球谐函数的一种简洁的形式



$$Y^e_{mn}(\theta,\varphi)=P^m_n(\cos\theta)\cos m\varphi$$

$$Y^0_{mn}(\theta,\varphi)=P^m_n(\cos\theta)\sin m\varphi.$$

归一化

$$\int_0^{2\pi}\int_0^{\pi}\left[Y^e_{mn}(\theta,\varphi)\quad\text{or}\quad Y^0_{mn}(\theta,\varphi)\right]^2 d\Omega
=\begin{cases}\dfrac{4\pi}{2(2n+1)}\dfrac{(n+m)!}{(n-m)!}, & n=1,2,3,\ldots\\ 4\pi, & n=0.\end{cases}$$

$$(2n+1)\cos\theta\,Y^e_{mn}(\theta,\varphi)=(n+m)Y^e_{m(n-1)}(\theta,\varphi)+(n-m+1)Y^e_{m(n+1)}(\theta,\varphi)$$

递推关系

$$(2n+1)\cos^2\theta\frac{\partial}{\partial\cos\theta}Y^e_{mn}(\theta,\varphi)$$
$$=(n+1)(n+m)Y^e_{m(n-1)}(\theta,\varphi)-(n-m+1)Y^e_{m(n+1)}(\theta,\varphi),$$

求导关系

玻尔兹曼方程和带电粒子输运方程

● 电子的玻尔兹曼方程（续）

□ 球谐函数及其性质（续）

加法定理

$$
\begin{aligned}
P_n(\cos\omega) = {} & P_n(\cos\theta_1)P_n(\cos\theta_2) \\
& + 2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_n^m(\cos\theta_1)P_n^m(\cos\theta_2)\cos m(\varphi_1-\varphi_2) \\
= {} & P_n(\cos\theta_1)P_n(\cos\theta_2) \\
& + 2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}\left[Y_{mn}^{e}(\theta_1,\varphi_1)Y_{mn}^{e}(\theta_2,\varphi_2)+Y_{mn}^{0}(\theta_1,\varphi_1)Y_{mn}^{0}(\theta_2,\varphi_2)\right].
\end{aligned}
$$

常用勒让德多项式

TABLE 5.1

Legendre Polynomials

P_n	
$P_0(\cos\theta)$	1
$P_1(\cos\theta)$	$\cos\theta$
$P_2(\cos\theta)$	$(3\cos^2\theta-1)/2$
$P_3(\cos\theta)$	$(5\cos^3\theta-3\cos\theta)/2$
$P_4(\cos\theta)$	$(35\cos^4\theta-30\cos^2\theta+3)/8$

TABLE 5.2

Associated Legendre Polynomials

P_n^m	
$P_1^1(\cos\theta)$	$\sin\theta$
$P_2^1(\cos\theta)$	$3\cos\theta\sin\theta$
$P_2^2(\cos\theta)$	$3\sin^2\theta$
$P_3^1(\cos\theta)$	$3(5\cos^2\theta-1)\sin\theta/2$
$P_3^2(\cos\theta)$	$15\cos\theta\sin^2\theta$
$P_3^3(\cos\theta)$	$15\sin^3\theta$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 球谐函数及其性质（续）

$$\int_0^\pi P_n(\cos\theta) P_{n'}(\cos\theta)\sin\theta d\theta = \begin{cases} \dfrac{2}{2n+1}, & n = n' \\ 0, & n \neq n' . \end{cases}$$

正交性及任意函数的球谐函数展开

$$f(\cos\theta) = \sum_{n=0}^{\infty} a_n P_n(\cos\theta),$$

任意theta函数展开

$$a_n = \frac{2n+1}{2}\int_0^\pi f(\cos\theta) P_n(\cos\theta)\sin\theta d\theta,$$

$$f(\cos\theta) = \sum_{n=0}^{\infty} \frac{2n+1}{2}\left(\int_0^\pi f(\cos\theta') P_n(\cos\theta')\sin\theta' d\theta'\right) P_n(\cos\theta).$$

$$f(\theta,\varphi) = \sum a_{mn} Y^e_{mn}(\theta,\varphi),$$

$$a_{mn} = \frac{2(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}\int_0^{2\pi}\int_0^\pi f(\theta,\varphi) Y^e_{mn}(\theta,\varphi) d\Omega.$$

任意函数展开

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 电子的速度分布函数

    - 空间密度梯度展开后的最低三阶项及其满足的玻尔兹曼方程

$g^0(v,t)$, $g^1(v,t)$, and $g^2(v,t)$, are defined by the equations

↓ 根据玻尔兹曼方程

$$\frac{\partial}{\partial t}g^0(v,t)+\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t)+R_0(t)g^0(v,t)=J(g^0,F),$$

$$\frac{\partial}{\partial t}g^1(v,t)+\alpha(t)\cdot\frac{\partial}{\partial v}g^1(v,t)+R_0(t)g^1(v,t)$$
$$=J(g^1,F)+vg^0(v,t)-v_d(t)g^0(v,t),$$

$$\frac{\partial}{\partial t}\mathbf{g}^2(v,t)+\alpha(t)\cdot\frac{\partial}{\partial v}\mathbf{g}^2(v,t)+R_0(t)\mathbf{g}^2(v,t)$$
$$=J(g^2,F)+vg^1(v,t)-v_d(t)g^1(v,t)+\mathbf{D}(t)g^0(v,t),$$

- g_0独立，与其它分布函数间无耦合，因此可以先求出g_0，然后再求g_1, g_2
- 二项展开近似方法中，只需求解g_0和g_1，忽略g_2

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 均匀数密度条件下的电子速度分布函数g_0

$$m=0,\quad g^0(v,t)=\sum_{n=0} g^0_{0n}(v,t)Y^e_{0n}(\theta,\varphi)=\sum_n g^0_n(v,t)P_n(\theta).$$

$$\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t)$$

$$=\alpha_z(t)\left(\cos\theta\frac{\partial g^0}{\partial v}+\frac{\sin^2\theta}{v}\frac{\partial g^0}{\partial\cos\theta}\right)$$

$$=\sum_n \alpha_z(t)\cos\theta P_n(\theta)\frac{\partial g^0_n(v)}{\partial v}+\sum_n \alpha_z(t)\frac{\sin^2\theta}{v}g^0_n(v)\frac{\partial P_n(\theta)}{\partial\cos\theta}.$$

球谐函数展开

根据球谐函数的求导关系和递推关系

$$\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t)$$

$$=\alpha_z(t)\sum_n\left(\frac{n+1}{2n+1}P_{n+1}(\theta)+\frac{n}{2n+1}P_{n-1}(\theta)\right)\frac{\partial g^0_n(v)}{\partial v}$$

$$+\alpha_z(t)\sum_n\frac{g^0_n(v)}{v}\left(-\frac{n(n+1)}{2n+1}P_{n+1}(\theta)+\frac{n(n+1)}{2n+1}P_{n-1}(\theta)\right).$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 均匀数密度条件下的电子速度分布函数g_0（续）

$$
\begin{aligned}
\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t) \\
&= \alpha_z(t)\sum_n\left(\frac{n+1}{2n+1}P_{n+1}(\theta)+\frac{n}{2n+1}P_{n-1}(\theta)\right)\frac{\partial g_n^0(v)}{\partial v} \\
&+ \alpha_z(t)\sum_n\frac{g_n^0(v)}{v}\left(-\frac{n(n+1)}{2n+1}P_{n+1}(\theta)+\frac{n(n+1)}{2n+1}P_{n-1}(\theta)\right).
\end{aligned}
$$

重组并调整求和符号的n

$$
\begin{aligned}
\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t) \\
&= \alpha_z(t)\sum_n\left(\frac{n}{2n-1}\frac{\partial g_{n-1}^0(v)}{\partial v}+\frac{n+1}{2n+3}\frac{\partial g_{n+1}^0(v)}{\partial v}\right. \\
&\left.-\frac{(n-1)n}{2n-1}\frac{g_{n-1}^0(v)}{v}+\frac{(n+1)(n+2)}{2n+3}\frac{g_{n+1}^0(v)}{v}\right)P_n(\theta) \\
&= \sum_n\left\{\alpha_z(t)\frac{n}{2n-1}\left(\frac{\partial}{\partial v}-\frac{n-1}{v}\right)g_{n-1}^0(v,t)\right. \\
&\left.+\alpha_z(t)\frac{n+1}{2n+3}\left(\frac{\partial}{\partial v}+\frac{n+2}{v}\right)g_{n+1}^0(v)\right\}P_n(\theta).
\end{aligned}
$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 均匀数密度条件下的电子速度分布函数g_0（续）

$$\frac{\partial}{\partial t}g^0(v,t)+\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t)+R_0(t)g^0(v,t)=J(g^0,F),$$

$$\begin{aligned}
\frac{\partial}{\partial t}g_n^0(v,t)&+\alpha_z(t)\frac{n}{2n-1}\left(\frac{\partial}{\partial v}-\frac{n-1}{v}\right)g_{n-1}^0(v,t)\\
&+\alpha_z(t)\frac{n+1}{2n+3}\left(\frac{\partial}{\partial v}+\frac{n+2}{v}\right)g_{n+1}^0(v,t)\\
&+R_0(t)g_n^0(v,t)-J(g^0,F)=0.
\end{aligned}$$

可求解的0维玻尔兹曼方程

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 均匀数密度条件下的电子速度分布函数g_0（续）

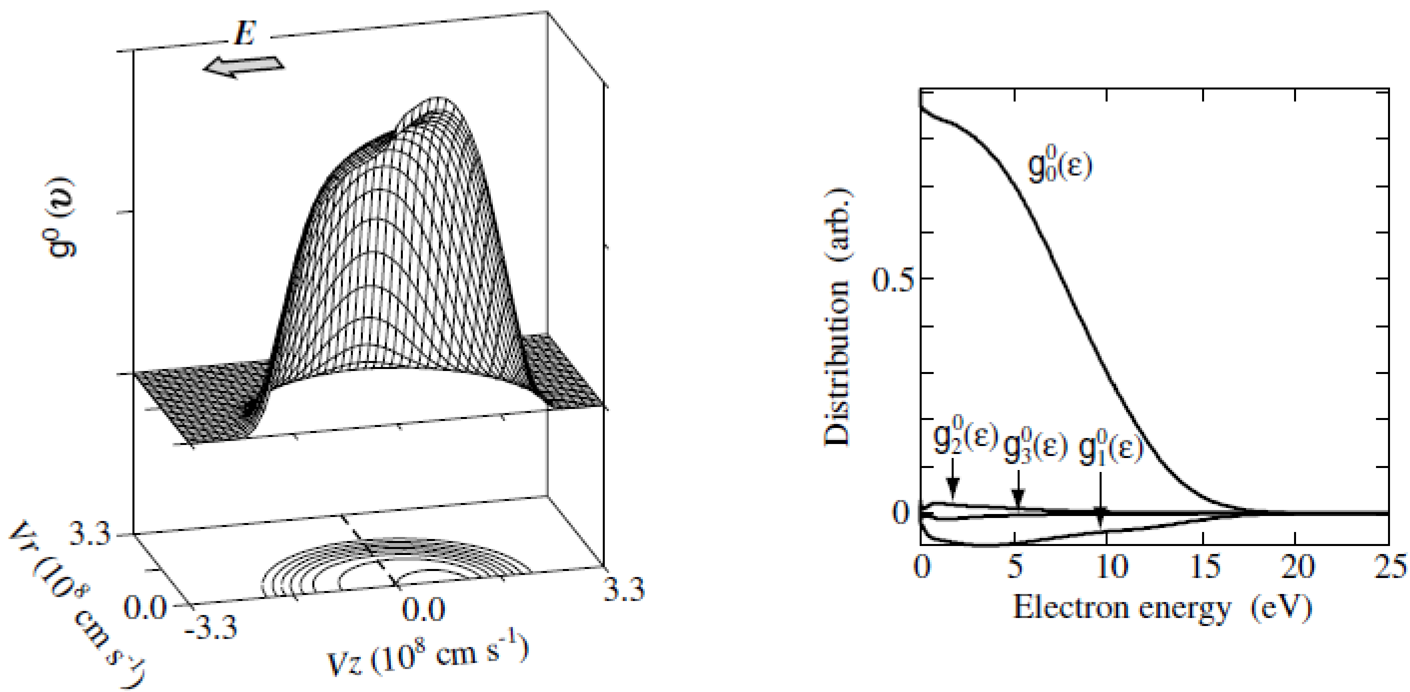


FIGURE 5.4
Lowest-order solution for the velocity distribution function for electrons at 100 Td in Ar: (a) $g^0(v)$ and (b) $g_n^0(v)$ ($n = 0, 1, 2, \ldots$).

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1

$$
\frac{\partial}{\partial t} g^1(v,t) + \alpha(t)\cdot\frac{\partial}{\partial v} g^1(v,t) + R_0(t) g^1(v,t) = J(g^1, F) + v g^0(v,t) - v_d(t) g^0(v,t),
$$

$$
g^1(v,t) = g_x^1(v,t)i + g_y^1(v,t)j + g_z^1(v,t)k.
$$

$$
\begin{vmatrix} \frac{\partial}{\partial t} g_x^1(v,t) \\ \frac{\partial}{\partial t} g_y^1(v,t) \\ \frac{\partial}{\partial t} g_z^1(v,t) \end{vmatrix} + \begin{vmatrix} \alpha_z(t)\frac{\partial}{\partial v_z} g_x^1(v,t) \\ \alpha_z(t)\frac{\partial}{\partial v_z} g_y^1(v,t) \\ \alpha_z(t)\frac{\partial}{\partial v_z} g_z^1(v,t) \end{vmatrix} + R_0 \begin{vmatrix} g_x^1 \\ g_y^1 \\ g_z^1 \end{vmatrix} + \begin{vmatrix} v_x g^0 \\ v_y g^0 \\ (v_z - v_d) g^0 \end{vmatrix} = \begin{vmatrix} J(g_x^1, F) \\ J(g_y^1, F) \\ J(g_z^1, F) \end{vmatrix}.
$$

下面对g_z^1和g_x^1分别求解，g_y^1的求解与g_x^1相同

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

求解g_z^1

$$g_z^1(v,t)=\sum_n g_{z0n}^1(v,t)Y_{0n}^e(\theta,\varphi)=\sum_n g_{z_n}^1(v,t)P_n(\theta).$$

$$\begin{aligned}
&(v_z-v_d)g^0(v,t)\\
&\quad=\sum_n(v\cos\theta-v_d)g_n^0P_n(\theta)\\
&\quad=\sum_n\left(v\frac{n}{2n+1}P_{n-1}(\theta)g_n^0+v\frac{n+1}{2n+1}P_{n+1}(\theta)g_n^0-v_dg_n^0P_n(\theta)\right)\\
&\quad=\sum_n\left(\frac{n+1}{2n+3}vg_{n+1}^0(v,t)+\frac{n}{2n-1}vg_{n-1}^0(v,t)-v_dg_n^0(v,t)\right)P_n(\theta),
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial}{\partial t}g_{z_n}^1(v,t)+\alpha_z(t)\frac{n}{2n-1}\left(\frac{\partial}{\partial v}-\frac{n-1}{v}\right)g_{z_{n-1}}^1(v,t)\\
&\qquad+\alpha_z(t)\frac{n+1}{2n+3}\left(\frac{\partial}{\partial v}+\frac{n+2}{v}\right)g_{z_{n+1}}^1(v,t)+R_0(t)g_{z_n}^1(v,t)-J\left(g_z^1,F\right)\\
&\qquad=-\frac{n+1}{2n+3}vg_{n+1}^0(v,t)-\frac{n}{2n-1}vg_{n-1}^0(v,t)+v_dg_n^0(v,t). \qquad (5.90)
\end{aligned}$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

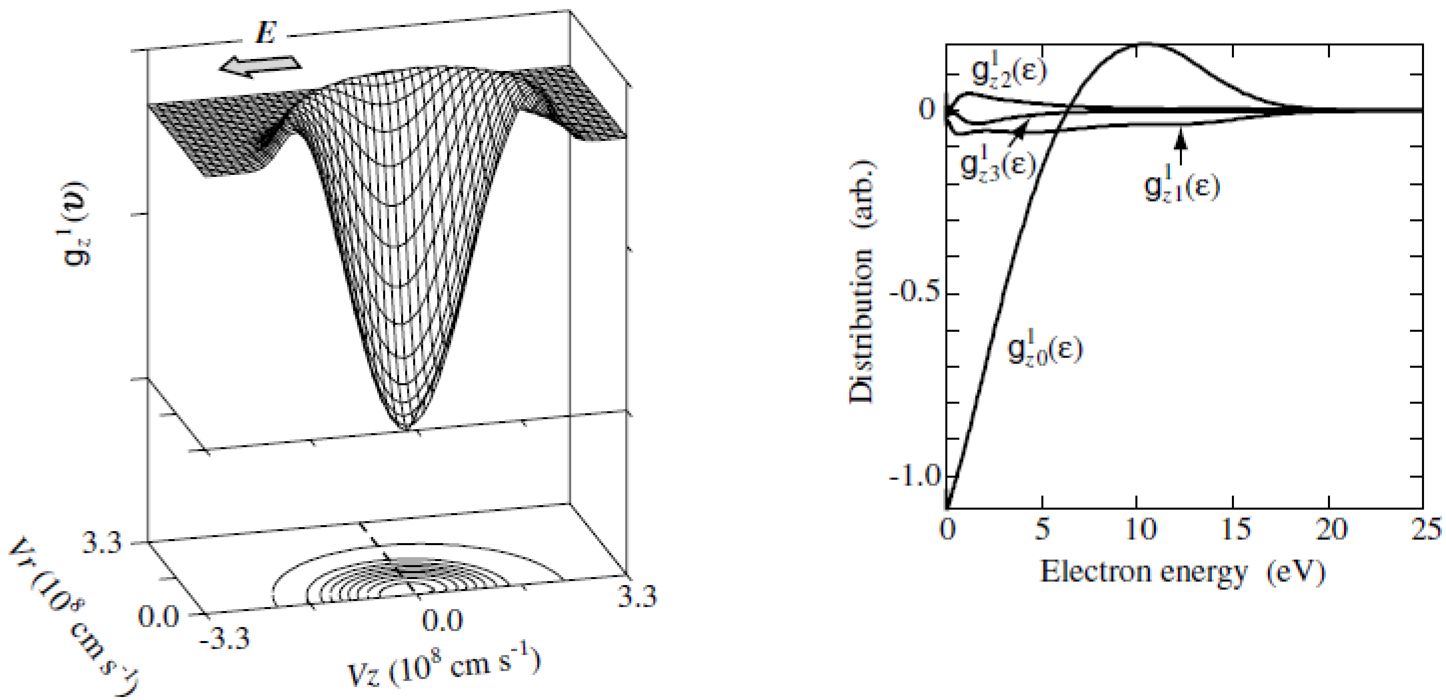


FIGURE 5.5
The longitudinal component (z) of the first-order solution for the velocity distribution function for electrons at 100 Td in Ar: (a) $g_z^1(v)$ and (b) $g_{zn}^1(\varepsilon)$ ($n = 0, 1, 2, \ldots$).

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

$$g_x^1(v,t) = \sum g_{x1n}^1(v,t) Y_{1n}^e(\theta,\varphi) = \sum g_{x_n}^1(v,t) P_n^1(\theta)\cos\varphi.$$

求解g_x^1

$$\alpha_z(t)\frac{\partial}{\partial v_z} g_x^1(v,t) = \alpha_z(t)\left(\cos\theta\frac{\partial g_x^1}{\partial v} + \frac{\sin^2\theta}{v}\frac{\partial g_x^1}{\partial\cos\theta} + \frac{\partial\varphi}{\partial v_z}\frac{\partial g_x^1}{\partial\varphi}\right)$$

$$= \sum_n \alpha_z(t)\cos\theta\, P_n^1(\theta)\cos\varphi\frac{\partial g_{x_n}^1}{\partial v} + \sum_n \alpha_z(t)\frac{g_{x_n}^1}{v}\cos\varphi\sin^2\theta\frac{\partial P_n^1(\theta)}{\partial\cos\theta}.$$

When we use Equations 5.71 and 5.72 to replace $\cos\theta\, P_n^1(\theta)$ and $\sin^2\theta\,\partial P_n^1(\theta)/\partial\cos\theta$, we obtain

$$= \alpha_z(t)\cos\varphi\sum_{n=1}\left(\frac{n}{2n+1}P_{n+1}^1(\theta) + \frac{n+1}{2n+1}P_{n-1}^1(\theta)\right)\frac{\partial g_{x_n}^1(v,t)}{\partial v}$$

$$+\alpha_z(t)\cos\varphi\sum_{n=1}\left(-\frac{n^2}{2n+1}P_{n+1}^1(\theta) + \frac{(n+1)^2}{2n+1}P_{n-1}^1(\theta)\right)\frac{g_{x_n}^1(v,t)}{v},$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

$$
\begin{aligned}
\alpha_z(t)\frac{\partial}{\partial v_z}g_x^1(v,t) &= \alpha_z(t)\sum_{n=1}\left(\frac{n-1}{2n-1}\frac{\partial g_{x_{n-1}}^1(v,t)}{\partial v}+\frac{n+2}{2n+3}\frac{\partial g_{x_{n+1}}^1(v,t)}{\partial v}\right.\\
&\quad\left.-\frac{(n-1)^2}{2n-1}\frac{g_{x_{n-1}}^1(v,t)}{v}+\frac{(n+2)^2}{2n+3}\frac{g_{x_{n+1}}^1(v,t)}{v}\right)P_n^1(\theta)\cos\varphi\\
&= \sum_{n=1}\left\{\alpha_z(t)\frac{n-1}{2n-1}\left(\frac{\partial}{\partial v}-\frac{n-1}{v}\right)g_{x_{n-1}}^1(v,t)\right.\\
&\quad\left.+\alpha_z(t)\frac{n+2}{2n+3}\left(\frac{\partial}{\partial v}+\frac{n+2}{v}\right)g_{x_{n+1}}^1(v,t)\right\}P_n^1(\theta)\cos\varphi.
\end{aligned}
$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

$$
\begin{aligned}
v_x g^0(v,t) &= \sum_{n=0} v \sin\theta \cos\varphi P_n(\theta) g_n^0(v,t) \\
&= \sum_{n=0} v g_n^0(v,t) \cos\varphi \left(\frac{1}{2n+1} P_{n+1}^1(\theta) - \frac{1}{2n+1} P_{n-1}^1(\theta) \right) \\
&= \sum \left(v \frac{1}{2n-1} g_{n-1}^0(v,t) - v \frac{1}{2n+3} g_{n+1}^0(v,t) \right) P(\theta) \cos\varphi .
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial t} g_{x_n}^1(v,t) + \alpha_z(t) \frac{n-1}{2n-1} \left(\frac{\partial}{\partial v} - \frac{n-1}{v} \right) g_{x_{n-1}}^1(v,t) \\
&\qquad + \alpha_z(t) \frac{n+2}{2n+3} \left(\frac{\partial}{\partial v} + \frac{n+2}{v} \right) g_{x_{n+1}}^1(v,t) + R_0(t) g_{x_n}^1(v,t) - J(g_x^1, F) \\
&\qquad = -\frac{1}{2n-1} v g_{n-1}^0(v,t) + \frac{1}{2n+3} v g_{n+1}^0(v,t). \qquad (5.94)
\end{aligned}
$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

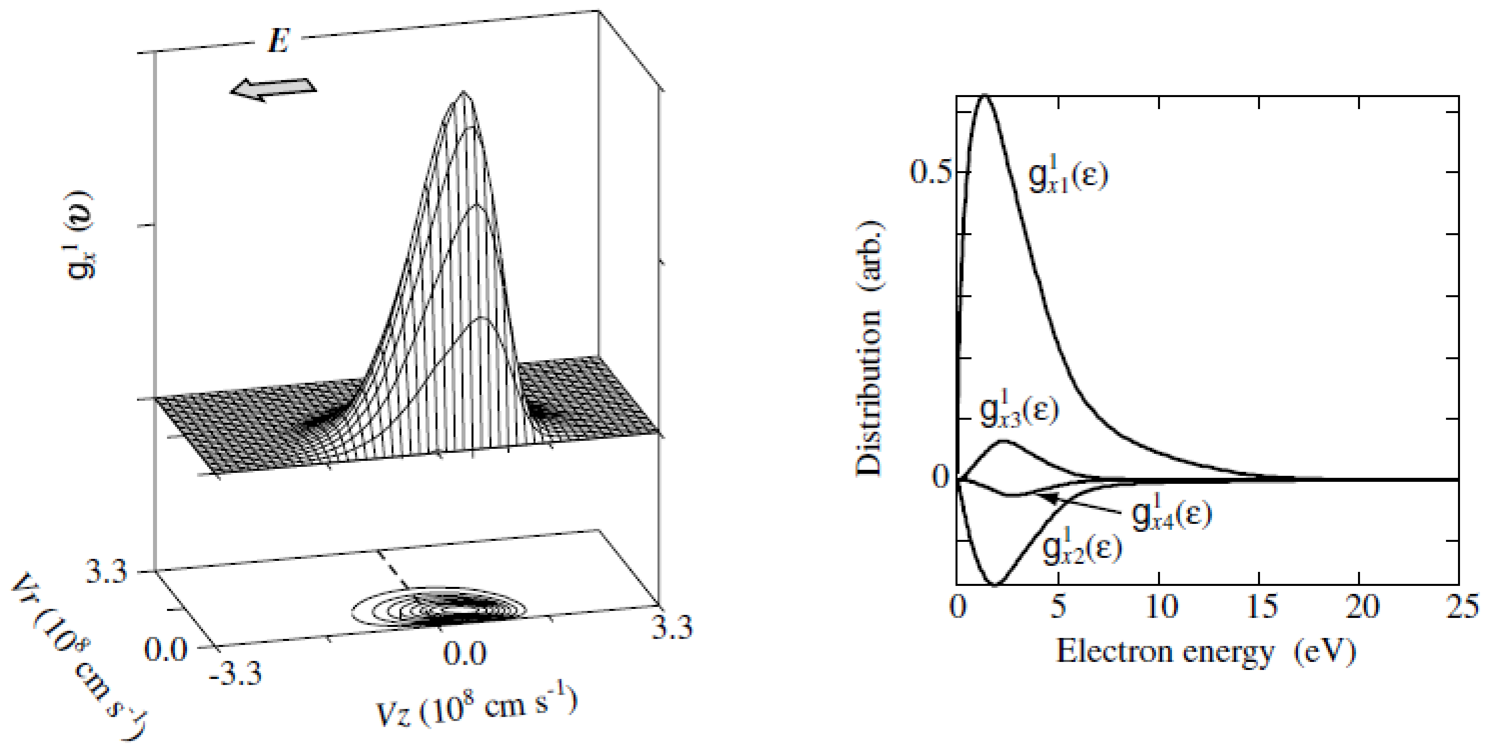


FIGURE 5.5

The longitudinal component (z) of the first-order solution for the velocity distribution function for electrons at 100 Td in Ar: (a) $g_z^1(v)$ and (b) $g_{zn}^1(\varepsilon)$ ($n = 0, 1, 2, \ldots$).

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）
  - 所有碰撞项的梯度展开和球谐展开小结

TABLE 5.3

Each of the Collision Terms Appearing in $J(g, F)$

Collision Type	Collision Integral	Expanded Collision Term
Elastic	$J_m\left(g_0^0\right)$	$\frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left\{NQ_m(v)v^4\left(g_0^0+\frac{kT_g}{mv}\frac{\partial g_0^0}{\partial v}\right)\right\}$
		$=\frac{m}{4\pi}\frac{2m}{M}\left[\left(\varepsilon-\frac{1}{2}kT_g\right)\frac{\partial}{\partial\varepsilon}NQ_m(\varepsilon)+\left(\frac{3}{2}-\frac{kT_g}{4\varepsilon}\right)NQ_m\right]f_0^0$
		$+\frac{m}{4\pi}\frac{2m}{M}\left[(\varepsilon+kT_g)NQ_m(\varepsilon)+kT_g\varepsilon\frac{\partial}{\partial\varepsilon}NQ_m(\varepsilon)\right]\frac{\partial}{\partial\varepsilon}f_0^0$
		$+\frac{m}{4\pi}\frac{2m}{M}kT_g\varepsilon NQ_m(\varepsilon)\frac{\partial}{\partial\varepsilon}f_0^0$
	$J_m\left(g_1^0\right)$	$-NQ_m(v)vg_1^0+\frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left\{\left(NQ_v(v)-NQ_m(v)\right)v^4g_1^0\right\}$
		$=\frac{m}{4\pi}NQ_m(\varepsilon)f_1^0$
		$+\frac{m}{4\pi}\frac{2m}{M}\left[\frac{3}{2}\left(NQ_v(\varepsilon)-NQ_m(\varepsilon)\right)\right.$
		$\left.+\varepsilon\frac{\partial}{\partial\varepsilon}\left[NQ_v(\varepsilon)-NQ_m(\varepsilon)\right]\right]f_1^0$
		$+\frac{2m}{M}\varepsilon\left(NQ_v(\varepsilon)-NQ_m(\varepsilon)\right)\frac{\partial}{\partial\varepsilon}f_1^0$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 所有碰撞项的梯度展开和球谐展开小结（续）

	$J_m\left(g_2^0\right)$	$-\frac{3}{2}NQ_v(v)vg_2^0=\frac{m}{4\pi}\left(-\frac{3}{2}\right)NQ_v(\varepsilon)$
	$J_m\left(g_{x11}^0\right)$	$-NQ_m(v)vg_{x11}^1=-\frac{m}{4\pi}NQ_m(\varepsilon)f_{x11}^1$
	$J_m\left(g_{x12}^0\right)$	$-\frac{3}{2}NQ_v(v)vg_{x12}^1=\frac{m}{4\pi}\left(-\frac{3}{2}\right)NQ_v(\varepsilon)f_{x12}^1$
Excitation	$J_j\left(g_0^0\right)$	$\frac{1}{v}\left\{v'^2NQ_j(v')g_0^0(v')-v^2NQ_j(v)g_0^0(v)\right\}$ $=\frac{m}{4\pi}\frac{1}{\sqrt{\varepsilon}}\frac{\partial}{\partial\varepsilon}\int_{\varepsilon}^{\varepsilon+\varepsilon_j}\sqrt{\varepsilon}NQ_j(\varepsilon)f_0^0(\varepsilon)d\varepsilon$
	$J_j\left(g_1^0 or g_2^0\right)$	$-NQ_j(v)vg_1^0\left(or g_2^0\right)=-\frac{m}{4\pi}NQ_j(\varepsilon)f_1^0\left(or f_2^0\right)$
	$J_j\left(g_{x11}^0 or g_{x12}^0\right)$	$-NQ_j(v)vg_{x11}^1\left(or g_{x12}^1\right)=-\frac{m}{4\pi}NQ_j(\varepsilon)f_{x11}^1\left(or f_{x12}^1\right)$
Ionization	$J_i\left(g_0^0\right)$	$\frac{1}{v}\left\{\frac{1+k}{k}v'^2NQ_i(v')g_0^0+(1+k)v'^2NQ_i(v')g_0^0-v^2NQ_i(v)g_0^0\right\}$ $=\frac{m}{4\pi}\frac{1}{\sqrt{\varepsilon}}\left(\frac{\partial}{\partial\varepsilon}\int_{\varepsilon}^{(1+k)\varepsilon+\varepsilon_i}\sqrt{\varepsilon}NQ_i(\varepsilon)f_0^0(\varepsilon)d\varepsilon+\frac{\partial}{\partial\varepsilon}\int_0^{\frac{1+k}{k}\varepsilon+\varepsilon_i}\sqrt{\varepsilon}NQ_i(\varepsilon)f_0^0(\varepsilon)d\varepsilon\right)$
	$J_i\left(g_1^0 or g_2^0\right)$	$-NQ_i(v)vg_1^0\left(or g_2^0\right)=-\frac{m}{4\pi}NQ_i(\varepsilon)f_1^0\left(or f_2^0\right)$
	$J_i\left(g_{x11}^0 or g_{x12}^0\right)$	$-NQ_i(v)vg_{x11}^1\left(or g_{x12}^1\right)=-\frac{m}{4\pi}NQ_i(\varepsilon)f_{x11}^1\left(or f_{x12}^1\right)$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 所有碰撞项的梯度展开和球谐展开小结（续）

Attachment	$J_a\left(g_0^0\right)$	$-NQ_a(v)vg_0^0 = -\frac{m}{4\pi}NQ_a(\varepsilon)f_0^0$
	$J_a\left(g_1^0 or g_2^0\right)$	$-NQ_a(v)vg_1^0\left(or g_2^0\right) = -\frac{m}{4\pi}NQ_a(\varepsilon)f_1^0\left(or f_2^0\right)$
	$J_a\left(g_{x_{11}}^0 or g_{x_{12}}^0\right)$	$-NQ_a(v)vg_{x_{11}}^1\left(or g_{x_{12}}^1\right) = -\frac{m}{4\pi}NQ_a(\varepsilon)f_{x_{11}}^1\left(or f_{x_{12}}^1\right)$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 电子输运参数

$$
\begin{aligned}
g(v, r, t) &= \sum_k \mathrm{g}^k(v, t) \otimes (\boldsymbol{\nabla}_r)^k\, n(r, t) \\
&= \sum_k \sum_{mn} g^k_{mn}(v, t) Y^e_{mn}(\theta, \varphi) \otimes (\boldsymbol{\nabla}_r)^k\, n(r, t) \\
&= \left(\sum_{n=0} g^0_n(v, t) P_n(\theta) \right) n(r, t) \\
&\quad + \left(\sum_{n=1} g^1_{x_{1n}}(v, t) Y^e_{1n}(\theta, \varphi) i + \sum_{n=1} g^1_{y_{1n}}(v, t) Y^e_{1n}(\theta, \varphi) j \right. \\
&\quad \left. + \sum_{n=1} g^1_{z_n}(v, t) P_n(\theta) k \right) \cdot \frac{\partial}{\partial r} n(r, t) + O\left(\nabla_r^2 n\right) \\
&= \left(g^0_0(v, t) + g^0_1(v, t) \cos\theta + g^0_2(v, t) \frac{3\cos^2\theta - 1}{2} + \cdots \right) n(r, t) \\
&+ \left(g^1_{x_1}(v, t) \sin\theta \cos\varphi + g^1_{x_2}(v, t) 3\cos\theta \sin\theta \cos\varphi + \cdots \right) i \cdot \frac{\partial}{\partial r} n(r, t) \\
&+ \left(g^1_{y_1}(v, t) \sin\theta \cos\varphi + g^1_{y_2}(v, t) 3\cos\theta \sin\theta \cos\varphi + \cdots \right) j \cdot \frac{\partial}{\partial r} n(r, t) \\
&+ \left(g^1_{z_0}(v, t) + g^1_{z_1}(v, t) \cos\theta + g^1_{z_2}(v, t) \frac{3\cos^2\theta - 1}{2} + \cdots \right) k \cdot \frac{\partial}{\partial r} n(r, t) \\
&+ O\left(\nabla_r^2 n\right). \qquad (5.95)
\end{aligned}
$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 电子输运参数（续）

$$\begin{aligned} g(v,r,t) \;=\; & \left(g_0^0(v,t) + g_1^0(v,t)\cos\theta + g_2^0(v,t)\frac{3\cos^2\theta - 1}{2} + \cdots\right) n(r,t) \\ & + \left(g_{x_1}^1(v,t)\sin\theta\cos\varphi + g_{x_2}^1(v,t)3\cos\theta\sin\theta\cos\varphi + \cdots\right) i\cdot\frac{\partial}{\partial r}n(r,t) \\ & + \left(g_{y_1}^1(v,t)\sin\theta\cos\varphi + g_{y_2}^1(v,t)3\cos\theta\sin\theta\cos\varphi + \cdots\right) j\cdot\frac{\partial}{\partial r}n(r,t) \\ & + \left(g_{z_0}^1(v,t) + g_{z_1}^1(v,t)\cos\theta + g_{z_2}^1(v,t)\frac{3\cos^2\theta - 1}{2} + \cdots\right) k\cdot\frac{\partial}{\partial r}n(r,t) \\ & + O\left(\nabla_r^2 n\right). \end{aligned} \tag{5.95}$$

两项展开近似

$$g(v) = \left[g_0^0(v) + g_1^0(v)\cos\theta + O\left(g_2^0\right)\right]n_e,$$

迁移速度

$$\begin{aligned} v_d(t) &= \int v g^0(v,t)dv \\ &= \sum_n \int_0^\pi v\cos\theta g_n^0(v,t)P_n(\theta)v^2 dv d\Omega \\ &= \frac{4\pi}{3}\int v^3 g_1^0(v,t)dv. \end{aligned}$$

(只保留前两项)

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 电子输运参数（续）

    - 扩散系数

两项展开近似条件

$$
\begin{aligned}
D_T(t) &= \int v_x g_x^1(v,t)\,dv = \int v_y g_y^1(v,t)\,dv \\
&= \sum_n \int v \sin\theta \cos\varphi\, g_{x_{1n}}^1(v,t)\, Y_{1n}^e(\theta,\varphi)\, v^2\, dv\, d\Omega \\
&= \frac{4\pi}{3} \int v^3 g_{x_1}^1(v,t)\,dv,
\end{aligned}
$$

$$
\begin{aligned}
D_L(t) &= \int v_z g_z^1(v,t)\,dv \\
&= \sum_n \int v \cos\theta\, g_{z_n}^1(v,t)\, P_n(\theta)\, v^2\, dv\, d\Omega \\
&= \frac{4\pi}{3} \int v^3 g_{z_1}^1(v,t)\,dv.
\end{aligned}
$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 电子输运参数（续）

    - 有效电子产生系数

$$
\begin{aligned}
R_0(t) &= R_i(t) - R_a(t) \\
&= N\int [Q_i(v) - Q_a(v)]\, v g(v,t) dv \\
&= 4\pi N\int [Q_i(v) - Q_a(v)]\, v^3 g_0^0(v,t) dv.
\end{aligned}
$$

    - 平均动能

$$
\begin{aligned}
\langle \varepsilon(r,t)\rangle &= \int \frac{1}{2} m v^2 g(v,r,t) dv \\
&= \sum_n \int \frac{1}{2} m v^2 g_n^0(v,t) P_n(\theta) v^2 dv d\Omega \\
&\quad -\nabla_z nk \sum_n \int \frac{1}{2} m v^2 g_{z_n}^1(v,t) P_n(\theta) v^2 dv d\Omega \\
&\quad -\nabla_x ni \sum_n \int \frac{1}{2} m v^2 g_{x_{1n}}^1(v,t) Y_{1n}^e(\theta,\varphi) v^2 dv d\Omega \\
&= 2\pi m \int v^4 g_0^0(v,t) dv - \left(2\pi m \int v^4 g_{z_0}^1(v,t) dv\right) \nabla_z nk.
\end{aligned}
$$

● 期中测试与期末大作业安排

- 期中测试（占总成绩30%）
  - 形式：开卷
  - 测试内容：第1至第6章中的主要物理概念、原理和方法
  - 测试时间：第14周周一（5月23日）上课时间
- 期末大作业（占总成绩60%）
  - 利用OOPIC Pro软件，对第9、10、11章中的单频电容耦合放电、双频电容耦合放电、感应耦合放电和磁增强放电情况进行数值模拟，最后一节课PPT介绍建模情况和模拟结果，并提交数值试验报告
  - 形式： 提交一份报告、最后一节课做PPT报告

《等离子体电子学》

第五章 玻尔兹曼方程和带电粒子输运方程

本章结束

下一章：第六章 气体中带电粒子输运的一般性质

课件下载：ftp://202.117.18.164/incoming/PE_2015/

（在“幻灯片放映”模式中时单击该箭头）