西安交通大学电子与信息工程学院研究生课程

《等离子体电子学》

第五章 玻尔兹曼方程和带电粒子输运方程

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玻尔兹曼方程和带电粒子输运方程

- 玻尔兹曼方程

□ 相空间输运与玻尔兹曼方程推导

$$dn = g(v, r, t)\, dv\, dr.$$

↓

$$n = n(r, t).$$

↓

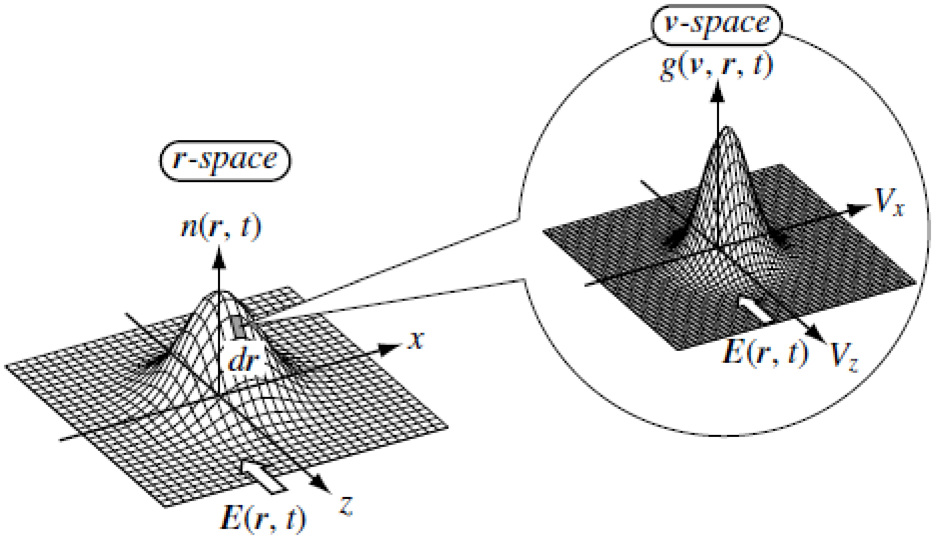
$$dn' = g(v', r', t + dt) dv' dr'.$$

↓

$$dv' dr' = \frac{\partial(r', v')}{\partial(r, v)} dv\, dr,$$

↓

$$dn' - dn = [g(v + \alpha dt, r + v dt, t + dt) - g(v, r, t)]\, dv\, dr$$
$$= \left[\frac{\partial}{\partial t} + v \cdot \frac{\partial}{\partial r} + \alpha \cdot \frac{\partial}{\partial v}\right] g(v, r, t)\, dv\, dr\, dt,$$

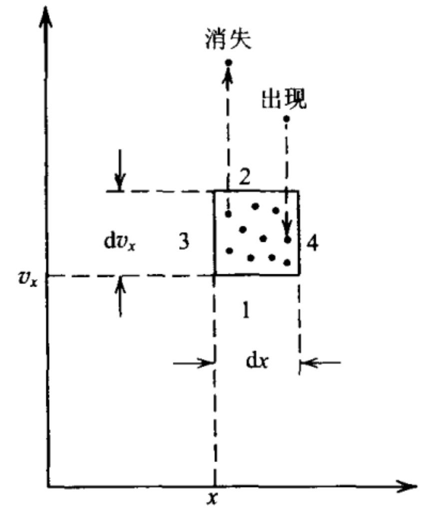


雅可比行列式

$$\frac{\partial(r', v')}{\partial(r, v)} = \begin{vmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} & \cdots & \frac{\partial x'}{\partial v_y} & \frac{\partial x'}{\partial v_z} \\ \frac{\partial y'}{\partial x} & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \frac{\partial v_y'}{\partial x} & \cdot & & \cdot & \cdot \\ \frac{\partial v_z'}{\partial x} & \cdot & & \cdot & \frac{\partial v_z'}{\partial v_z} \end{vmatrix} = 1.$$

- 玻尔兹曼方程(续)

  - 相空间输运与玻尔兹曼方程推导(续)



$$dn' - dn = [g(v + \alpha dt, r + v dt, t + dt) - g(v, r, t)]\, dv\, dr$$

关于v和r做泰勒级数展开，并保留一阶项

$$= \left[\frac{\partial}{\partial t} + v \cdot \frac{\partial}{\partial r} + \alpha \cdot \frac{\partial}{\partial v}\right] g(v, r, t)\, dv\, dr\, dt,$$

$$dn' - dn = J(g, F)\, dv\, dr\, dt.$$

$$\frac{\partial}{\partial t} g(v, r, t) + v \cdot \frac{\partial}{\partial r} g(v, r, t) + \alpha \cdot \frac{\partial}{\partial v} g(v, r, t) = J(g, F).$$

玻尔兹曼方程/Fokker-Planck方程

扩散　外场作用　碰撞项

$$F = m\alpha(r, t) = eE(r, t) + ev \times B(r, t),$$

$$\frac{\partial}{\partial t} g(v, r, t) + v \cdot \frac{\partial}{\partial r} g(v, r, t) + \alpha \cdot \frac{\partial}{\partial v} g(v, r, t) = 0.$$

无碰撞玻尔兹曼方程/弗拉索夫方程

玻尔兹曼方程和带电粒子输运方程

- 输运系数

  - 宏观量

    - 平均速度

平均量

$$n(\boldsymbol{r}, t) = \int g(\boldsymbol{v}, \boldsymbol{r}, t) d\boldsymbol{v}. \qquad \langle \mathbf{A}(\boldsymbol{r}, t) \rangle = \frac{\int \mathbf{A}(\boldsymbol{v}, \boldsymbol{r}, t) g(\boldsymbol{v}, \boldsymbol{r}, t) d\boldsymbol{v}}{\int g(\boldsymbol{v}, \boldsymbol{r}, t) d\boldsymbol{v}}.$$

$$\langle \boldsymbol{v}(\boldsymbol{r}, t) \rangle = \frac{1}{n} \int \boldsymbol{v} g(\boldsymbol{v}, \boldsymbol{r}, t) d\boldsymbol{v}$$

$$\boldsymbol{v}(\boldsymbol{r}, t) = \boldsymbol{v}_d(\boldsymbol{r}, t) + \boldsymbol{v}_r(\boldsymbol{v}, \boldsymbol{r}, t),$$

定向运动运动　　随机热运动

$$\langle \boldsymbol{v}_r \rangle = 0.$$

    - 平均动能

$$\langle \varepsilon(\boldsymbol{r}, t) \rangle = \frac{1}{n} \int \frac{1}{2} m v^2 g(\boldsymbol{v}, \boldsymbol{r}, t) d\boldsymbol{v} = \frac{1}{2} m \left(v_d^2 + \langle v_r^2 \rangle \right).$$

玻尔兹曼方程和带电粒子输运方程

- 输运系数（续）

  - 宏观量（续）

    - 流体速度

$$g(v, r, t) = g^0(v, t)n(r, t) + \mathbf{g}^1(v, t)\cdot\nabla_r n(r, t) + \mathbf{g}^2(v, t)\odot\nabla_r^2 n(r, t) + \cdots.$$

$$\int \mathbf{g}^k(v, t)dv = \begin{cases} 1; & k = 0 \\ 0; & k \neq 0 \end{cases}.$$

$$\langle v(r, t)\rangle = \frac{1}{n}\int v g^0(v, t)dv + \frac{1}{n}\int v \mathbf{g}^1(v, t)dv \cdot \nabla_r n(r, t).$$ 流体速度

    - 迁移速度 $v_d(t) = \frac{1}{n}\int v g^0(v, t)dv$

    - 扩散速度 $\mathbf{D}(t) = \frac{1}{n}\int v \mathbf{g}^1(v, t)dv.$

$$D_T(t) = \frac{1}{n}\int v_x g_x^1(v, t)dv = \frac{1}{n}\int v_y g_y^1(v, t)dv,$$ 横向扩散系数

$$D_L(t) = \frac{1}{n}\int v_z g_z^1(v, t)dv.$$ 纵向扩散系数

玻尔兹曼方程和带电粒子输运方程

- 输运系数（续）
  - 宏观量（续）
    - 压力张量

$$\begin{aligned}\mathbf{P}_T(\boldsymbol{r}, t) &= m\int \boldsymbol{v}\boldsymbol{v}g(\boldsymbol{v}, \boldsymbol{r}, t)d\boldsymbol{v}, \\ &= m\int \boldsymbol{v}_r\boldsymbol{v}_r g(\boldsymbol{v}, \boldsymbol{r}, t)d\boldsymbol{v} + mn\boldsymbol{v}_d\boldsymbol{v}_d, \\ &= mn(\boldsymbol{r}, t)\langle \boldsymbol{v}_r\boldsymbol{v}_r\rangle + mn(\boldsymbol{r}, t)\boldsymbol{v}_d\boldsymbol{v}_d.\end{aligned}$$

$$\mathbf{P} = \begin{vmatrix} P_x & 0 & 0 \\ 0 & P_y & 0 \\ 0 & 0 & P_z \end{vmatrix}, \quad p = p_x = p_y = p_z = \frac{1}{3}mn\langle v_r^2\rangle.$$

随机运动速度一般远大于定向运动速度，则近似各向同性

玻尔兹曼方程和带电粒子输运方程

- 输运系数（续）

  - 宏观量（续）

    - 能流矢量

$$Q(r,t)=\int \frac{1}{2}mv^2 v g(v,r,t)dv.$$

$$\begin{aligned}\langle v^2 v\rangle &= \langle[(v_d+v_r)\cdot(v_d+v_r)](v_d+v_r)\rangle \\ &= v_d^2 v_d + v_d^2\langle v_r\rangle + 2v_d v_d\cdot\langle v_r\rangle + 2v_d\cdot\langle v_r v_r\rangle + \langle v_r^2\rangle v_d + \langle v_r^2 v_r\rangle \\ &= v_d^2 v_d + 2v_d\cdot\langle v_r v_r\rangle + \langle v_r^2\rangle v_d + \langle v_r^2 v_r\rangle.\end{aligned}$$

$$\begin{aligned}Q(r,t) &= \frac{1}{2}mn\langle v^2 v\rangle \\ &= \frac{1}{2}mn\left(v_d^2+\langle v_r^2\rangle\right)v_d + mn\langle v_r v_r\rangle\cdot v_d + \frac{1}{2}mn\langle v_r^2 v_r\rangle \\ &= n(r,t)\langle\varepsilon(r,t)\rangle v_d + \mathbf{P}\cdot v_d + \frac{1}{2}mn(r,t)\langle v_r^2 v_r\rangle.\end{aligned}$$

$$q(r,t)=\frac{1}{2}mn(r,t)\langle v_r^2 v_r\rangle.$$

↑ 定向运动造成的能量输运

↑ 外场做功

↑ 随机运动产生的能量输运（热流）

玻尔兹曼方程和带电粒子输运方程

● 输运系数（续）

◻ 宏观量（续）

■ 扩散系数：可根据粒子的运动轨迹统计获得，与前面的计算公式物理意义一致

$$D_L(t) = \frac{1}{2}\frac{d}{dt}\left\langle (z(t) - \langle z(t)\rangle)^2 \right\rangle$$
$$= \langle z(t) v_z(t)\rangle - \langle z(t)\rangle\langle v_z(t)\rangle.$$
$$D_T(t) = \frac{1}{4}\frac{d}{dt}\langle x(t)^2 + y(t)^2\rangle$$
$$= \frac{1}{2}(\langle x(t) v_x(t)\rangle + \langle y(t) v_y(t)\rangle).$$

即空间二阶矩

$$\mathbf{M_k}(t) = <(\mathbf{r}(t) - <\mathbf{r}(t)>)^{\mathbf{k}}>. \quad (\mathbf{k} = \mathbf{3}, \mathbf{4}..)$$

$$\mathbf{D_3} = \frac{1}{3!}\frac{d}{dt}\mathbf{M_3}(t),$$

$$\mathbf{D_4} = \frac{1}{4!}\frac{d}{dt}\left(\mathbf{M_4}(t) - \mathbf{3}(\mathbf{M_2}(t))^{\mathbf{2}}\right).$$

空间高阶矩与3、4阶输运系数
空间三阶矩：斜度
空间四阶矩：峰度

玻尔兹曼方程和带电粒子输运方程

- 输运方程

  - 任意函数作用于玻尔兹曼方程

A(v,r,t): 任意函数

$$\frac{\partial}{\partial t}g(v,r,t)+v\cdot\frac{\partial}{\partial r}g(v,r,t)+\alpha\cdot\frac{\partial}{\partial v}g(v,r,t)=J(g,F).$$

$$\int \mathbf{A}(v,r,t)\frac{\partial g(v,r,t)}{\partial t}dv+\int \mathbf{A}(v,r,t)v\cdot\frac{\partial g(v,r,t)}{\partial r}dv + \int \mathbf{A}(v,r,t)\alpha\cdot\frac{\partial g(v,r,t)}{\partial v}dv=\int \mathbf{A}(v,r,t)J(g,F)dv.$$

$$\int \mathbf{A}\frac{\partial g}{\partial t}dv=\frac{\partial}{\partial t}\int \mathbf{A}g dv-\int\frac{\partial \mathbf{A}}{\partial t}g dv = \frac{\partial}{\partial t}(n(r,t)\langle\mathbf{A}\rangle)-n(r,t)\left\langle\frac{\partial \mathbf{A}}{\partial t}\right\rangle,$$

$$\int \mathbf{A}v\cdot\frac{\partial g}{\partial r}dv=\int\frac{\partial}{\partial r}\cdot(v\mathbf{A}g)dv-\int\left(\frac{\partial}{\partial r}\cdot v\mathbf{A}\right)g dv, = \frac{\partial}{\partial r}\cdot(n(r,t)\langle v\mathbf{A}\rangle)-n(r,t)\left\langle\frac{\partial}{\partial r}\cdot v\mathbf{A}\right\rangle.$$

$$\int \mathbf{A}\alpha\cdot\frac{\partial g}{\partial v}dv=\frac{\partial}{\partial v}\cdot\int\alpha\mathbf{A}g dv-\int\left(\frac{\partial}{\partial v}\cdot\alpha\mathbf{A}\right)g dv = \frac{\partial}{\partial v}\cdot(n(r,t)\langle\alpha\mathbf{A}\rangle)-n(r,t)\left\langle\frac{\partial}{\partial v}\cdot\alpha\mathbf{A}\right\rangle = -n(r,t)\left\langle\alpha\cdot\frac{\partial}{\partial v}\mathbf{A}\right\rangle.$$

玻尔兹曼方程和带电粒子输运方程

- 输运方程（续）

数密度守恒方程

$$\frac{\partial}{\partial t}(n(\mathbf{r},t)\langle\mathbf{A}(v,\mathbf{r},t)\rangle) - n(\mathbf{r},t)\left\langle\frac{\partial\mathbf{A}(v,\mathbf{r},t)}{\partial t}\right\rangle + \frac{\partial}{\partial\mathbf{r}}\cdot(n(\mathbf{r},t)\langle v\mathbf{A}(v,\mathbf{r},t)\rangle)$$
$$- n(\mathbf{r},t)\left\langle\frac{\partial}{\partial\mathbf{r}}\cdot v\mathbf{A}(v,\mathbf{r},t)\right\rangle - n(\mathbf{r},t)\left\langle\alpha\cdot\frac{\partial\mathbf{A}(v,\mathbf{r},t)}{\partial v}\right\rangle$$
$$= \int\mathbf{A}(v,\mathbf{r},t)J(g,F)dv. \quad (5.34)$$

$\mathbf{A} = 1$

$$\frac{\partial}{\partial t}n(\mathbf{r},t) + \frac{\partial}{\partial\mathbf{r}}\cdot(n(\mathbf{r},t)\langle v\rangle) = n_e(\mathbf{r},t)R_0(\mathbf{r},t).$$

$$R_0(\mathbf{r},t) = R_i(\mathbf{r},t) - R_a(\mathbf{r},t).$$

R_0: 粒子生成速率

玻尔兹曼方程和带电粒子输运方程

- 输运方程（续）

□ 动量守恒方程

v, r, t相互独立无关

$$\frac{\partial}{\partial t}(n(\boldsymbol{r},t)\langle \mathbf{A}(v,\boldsymbol{r},t)\rangle) - n(\boldsymbol{r},t)\left\langle \frac{\partial \mathbf{A}(v,\boldsymbol{r},t)}{\partial t}\right\rangle + \frac{\partial}{\partial \boldsymbol{r}}\cdot(n(\boldsymbol{r},t)\langle v\mathbf{A}(v,\boldsymbol{r},t)\rangle)$$
$$- n(\boldsymbol{r},t)\left\langle \frac{\partial}{\partial \boldsymbol{r}}\cdot v\mathbf{A}(v,\boldsymbol{r},t)\right\rangle - n(\boldsymbol{r},t)\left\langle \alpha\cdot\frac{\partial \mathbf{A}(v,\boldsymbol{r},t)}{\partial v}\right\rangle$$
$$= \int \mathbf{A}(v,\boldsymbol{r},t)J(g,F)dv. \quad (5.34)$$

$\mathbf{A} = \mathrm{m}v$

$$\frac{\partial}{\partial t}(mn(\boldsymbol{r},t)\langle v\rangle) + \frac{\partial}{\partial \boldsymbol{r}}\cdot(mn(\boldsymbol{r},t)\langle vv\rangle) - mn(\boldsymbol{r},t)\left\langle \alpha\cdot\frac{\partial}{\partial v}v\right\rangle = m\int vJ\,dv.$$

$\langle v\rangle = v_d$

代入数密度守恒方程 $\frac{\partial}{\partial t}n(\boldsymbol{r},t) + \frac{\partial}{\partial \boldsymbol{r}}\cdot(n(\boldsymbol{r},t)\langle v\rangle) = n_e(\boldsymbol{r},t)R_0(\boldsymbol{r},t).$

$$mn(\boldsymbol{r},t)\frac{\partial}{\partial t}v_d + mv_d\frac{\partial}{\partial t}n(\boldsymbol{r},t) = mn(\boldsymbol{r},t)\frac{\partial}{\partial t}v_d - mv_d\frac{\partial}{\partial \boldsymbol{r}}\cdot(n(\boldsymbol{r},t)v_d)$$
$$+ mn_e(\boldsymbol{r},t)v_dR_0,$$

玻尔兹曼方程和带电粒子输运方程

- 输运方程（续）

  - 动量守恒方程（续）

$$\frac{\partial}{\partial t}(mn(\boldsymbol{r},t)\langle \boldsymbol{v}\rangle) + \frac{\partial}{\partial \boldsymbol{r}}\cdot(mn(\boldsymbol{r},t)\langle \boldsymbol{v}\boldsymbol{v}\rangle) - mn(\boldsymbol{r},t)\left\langle \boldsymbol{\alpha}\cdot\frac{\partial}{\partial \boldsymbol{v}}\boldsymbol{v}\right\rangle = m\int \boldsymbol{v}J\,d\boldsymbol{v}.$$

$$\langle \boldsymbol{v}\rangle = \boldsymbol{v}_d$$

$$mn(\boldsymbol{r},t)\frac{\partial}{\partial t}\boldsymbol{v}_d + m\boldsymbol{v}_d\frac{\partial}{\partial t}n(\boldsymbol{r},t) = mn(\boldsymbol{r},t)\frac{\partial}{\partial t}\boldsymbol{v}_d - m\boldsymbol{v}_d\frac{\partial}{\partial \boldsymbol{r}}\cdot(n(\boldsymbol{r},t)\boldsymbol{v}_d) + mn_e(\boldsymbol{r},t)\boldsymbol{v}_d R_0,$$

$$\frac{\partial}{\partial \boldsymbol{r}}\cdot(mn(\boldsymbol{r},t)\boldsymbol{v}_d\boldsymbol{v}_d) + \frac{\partial}{\partial \boldsymbol{r}}\cdot(mn(\boldsymbol{r},t)\langle \boldsymbol{v}_r\boldsymbol{v}_r\rangle) = \boldsymbol{v}_d\frac{\partial}{\partial \boldsymbol{r}}\cdot(mn(\boldsymbol{r},t)\boldsymbol{v}_d) + mn(\boldsymbol{r},t)\left(\boldsymbol{v}_d\cdot\frac{\partial}{\partial \boldsymbol{r}}\right)\boldsymbol{v}_d + \frac{\partial}{\partial \boldsymbol{r}}\cdot(mn(\boldsymbol{r},t)\langle \boldsymbol{v}_r\boldsymbol{v}_r\rangle),$$

$$-mn(\boldsymbol{r},t)\left\langle \boldsymbol{\alpha}\cdot\frac{\partial}{\partial \boldsymbol{v}}\boldsymbol{v}\right\rangle = -n(\boldsymbol{r},t)e(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}).$$

$$mn(\boldsymbol{r},t)\frac{\partial}{\partial t}\boldsymbol{v}_d + mn(\boldsymbol{r},t)\boldsymbol{v}_d\left(\frac{\partial}{\partial \boldsymbol{r}}\cdot\boldsymbol{v}_d\right) + m\frac{\partial}{\partial \boldsymbol{r}}\cdot(n(\boldsymbol{r},t)\langle \boldsymbol{v}_r\boldsymbol{v}_r\rangle) = n(\boldsymbol{r},t)e(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}) - mn_e(\boldsymbol{r},t)\boldsymbol{v}_d R_0 + m\langle \boldsymbol{v}J\rangle.$$

玻尔兹曼方程和带电粒子输运方程

- 输运方程（续）

  - 动量守恒方程（续）

$$mn(\mathbf{r},t)\frac{\partial}{\partial t}\mathbf{v}_d + mn(\mathbf{r},t)\mathbf{v}_d\left(\frac{\partial}{\partial \mathbf{r}}\cdot\mathbf{v}_d\right) + m\frac{\partial}{\partial \mathbf{r}}\cdot\left(n(\mathbf{r},t)\langle\mathbf{v}_r\mathbf{v}_r\rangle\right) = n(\mathbf{r},t)e(\mathbf{E}+\mathbf{v}\times\mathbf{B}) - mn_e(\mathbf{r},t)\mathbf{v}_d R_0 + m\langle\mathbf{v}J\rangle.$$

$$m\langle\mathbf{v}J\rangle = -mn(\mathbf{r},t)\mathbf{v}_d R_m.$$

R_m: 动量转移碰撞速率

若E为常数，B=0，并假设定向运动速度v_d为不随时间和空间变化

$$R_m \gg R_0$$

$$mn\mathbf{v}_d R_m = en\mathbf{E} - m\frac{\partial}{\partial \mathbf{r}}\cdot\left(n\langle\mathbf{v}_r\mathbf{v}_r\rangle\right).$$

玻尔兹曼方程和带电粒子输运方程

- 输运方程（续）

  - 能量守恒方程

$$\frac{\partial}{\partial t}\left(n(\boldsymbol{r},t)\langle \mathbf{A}(v,\boldsymbol{r},t)\rangle\right) - n(\boldsymbol{r},t)\left\langle \frac{\partial \mathbf{A}(v,\boldsymbol{r},t)}{\partial t}\right\rangle + \frac{\partial}{\partial \boldsymbol{r}}\cdot\left(n(\boldsymbol{r},t)\langle v\mathbf{A}(v,\boldsymbol{r},t)\rangle\right)$$
$$- n(\boldsymbol{r},t)\left\langle \frac{\partial}{\partial \boldsymbol{r}}\cdot v\mathbf{A}(v,\boldsymbol{r},t)\right\rangle - n(\boldsymbol{r},t)\left\langle \alpha\cdot\frac{\partial \mathbf{A}(v,\boldsymbol{r},t)}{\partial v}\right\rangle$$
$$= \int \mathbf{A}(v,\boldsymbol{r},t)J(g,F)dv. \qquad (5.34)$$

replace **A** with $mv^2/2$

replace **A** with $mv^2/2$

$$-n(\boldsymbol{r},t)\left\langle \frac{1}{m}[eE(\boldsymbol{r},t)+ev\times B(\boldsymbol{r},t)]\cdot\frac{\partial}{\partial v}\frac{mv^2}{2}\right\rangle = -n(\boldsymbol{r},t)\langle eE\cdot v\rangle,$$

磁场不对带电粒子做功

$$\frac{\partial}{\partial t}\left(\frac{mn(\boldsymbol{r},t)}{2}\langle v^2\rangle\right) + \frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\frac{mn(\boldsymbol{r},t)}{2}\langle v^2 v\rangle\right) - n(\boldsymbol{r},t)eE\cdot v_d$$
$$= \int \frac{mv^2}{2}J\,dv.$$

玻尔兹曼方程和带电粒子输运方程

- 输运方程（续）

  - 能量守恒方程（续）

$$\frac{\partial}{\partial t}\left(\frac{mn(\boldsymbol{r},t)}{2}\langle v^2\rangle\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\frac{mn(\boldsymbol{r},t)}{2}\langle v^2\boldsymbol{v}\rangle\right)-n(\boldsymbol{r},t)e\boldsymbol{E}\cdot\boldsymbol{v}_d$$

$$=\int\frac{mv^2}{2}J\,d\boldsymbol{v}.$$

$$\begin{aligned}\boldsymbol{Q}(\boldsymbol{r},t)&=\frac{1}{2}mn\langle v^2\boldsymbol{v}\rangle\\&=\frac{1}{2}mn\left(v_d^2+\langle v_r^2\rangle\right)\boldsymbol{v}_d+mn\langle\boldsymbol{v}_r\boldsymbol{v}_r\rangle\cdot\boldsymbol{v}_d+\frac{1}{2}mn\left\langle v_r^2\boldsymbol{v}_r\right\rangle\\&=n(\boldsymbol{r},t)\langle\varepsilon(\boldsymbol{r},t)\rangle\boldsymbol{v}_d+\mathbf{P}\cdot\boldsymbol{v}_d+\frac{1}{2}mn(\boldsymbol{r},t)\left\langle v_r^2\boldsymbol{v}_r\right\rangle.\end{aligned}$$

代到第二项

$$\frac{\partial}{\partial t}(n(\boldsymbol{r},t)\langle\varepsilon(\boldsymbol{r},t)\rangle)+\frac{\partial}{\partial \boldsymbol{r}}\cdot(n(\boldsymbol{r},t)\langle\varepsilon(\boldsymbol{r},t)\rangle\boldsymbol{v}_d)+\frac{\partial}{\partial \boldsymbol{r}}\cdot(\mathbf{P}\cdot\boldsymbol{v}_d)$$

$$+\frac{\partial}{\partial \boldsymbol{r}}\cdot\boldsymbol{q}-n(\boldsymbol{r},t)e\boldsymbol{E}\cdot\boldsymbol{v}_d=\int\frac{mv^2}{2}J\,d\boldsymbol{v}.$$

玻尔兹曼方程和带电粒子输运方程

- 输运方程（续）

□ 方程汇总

$$\frac{\partial}{\partial t}n(\boldsymbol{r},t)+\frac{\partial}{\partial \boldsymbol{r}}\cdot(n(\boldsymbol{r},t)\langle \boldsymbol{v}\rangle)=n_e(\boldsymbol{r},t)R_0(\boldsymbol{r},t),$$

$$R_0(\boldsymbol{r},t)=R_i(\boldsymbol{r},t)-R_a(\boldsymbol{r},t).$$

$$mn(\boldsymbol{r},t)\frac{\partial}{\partial t}\boldsymbol{v}_d+mn(\boldsymbol{r},t)\boldsymbol{v}_d\left(\frac{\partial}{\partial \boldsymbol{r}}\cdot\boldsymbol{v}_d\right)+m\frac{\partial}{\partial \boldsymbol{r}}\cdot(n(\boldsymbol{r},t)\langle \boldsymbol{v}_r\boldsymbol{v}_r\rangle)$$
$$=n(\boldsymbol{r},t)e(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})-mn_e(\boldsymbol{r},t)\boldsymbol{v}_d R_0+m\langle \boldsymbol{v}J\rangle.$$

$$m\langle \boldsymbol{v}J\rangle=-mn(\boldsymbol{r},t)\boldsymbol{v}_d R_m.$$

$$\frac{\partial}{\partial t}(n(\boldsymbol{r},t)\langle\varepsilon(\boldsymbol{r},t)\rangle)+\frac{\partial}{\partial \boldsymbol{r}}\cdot(n(\boldsymbol{r},t)\langle\varepsilon(\boldsymbol{r},t)\rangle\boldsymbol{v}_d)+\frac{\partial}{\partial \boldsymbol{r}}\cdot(\mathbf{P}\cdot\boldsymbol{v}_d)$$
$$+\frac{\partial}{\partial \boldsymbol{r}}\cdot\boldsymbol{q}-n(\boldsymbol{r},t)e\boldsymbol{E}\cdot\boldsymbol{v}_d=\int\frac{mv^2}{2}J\,d\boldsymbol{v}.$$

$$\int\frac{mv^2}{2}J\,d\boldsymbol{v}=\frac{2m}{M}\langle\varepsilon(\boldsymbol{r},t)\rangle R_m n_e(\boldsymbol{r},t)$$
$$+\left(\Sigma\varepsilon_j R_j+\varepsilon_i R_i-\left\langle\varepsilon N Q_a(\varepsilon)\left(\frac{2\varepsilon}{m}\right)^{1/2}\right\rangle\right)n_e(\boldsymbol{r},t),$$

玻尔兹曼方程和带电粒子输运方程

- 玻尔兹曼方程的碰撞项

□ 积分碰撞

properties by the differential cross section $\sigma(\theta, \phi; \varepsilon)$ and the integral cross sections $Q(\varepsilon)$. The velocity distribution of neutral gas molecules is represented by $F(V', r, t)$, where gas molecules have a mass M and velocity V' before collision. The charged particles of mass m and velocity v' before collision are described by the corresponding velocity distribution function $g(v', r, t)$. When we consider a small element of the phase space $dv\, dr$, then the number of the charged particles that enter this element during a short time dt is equal to

进入相空间单元dvdr中的带电粒子数(碰撞前)

$$J_{in} = \int_{\Omega} \int_{V'} F(V', r, t) dV' g(v', r, t) dv' v'_{\gamma} \sigma(\theta', \phi'; v'_{\gamma}) d\Omega' dr\, dt.$$

离开相空间单元dvdr的带电粒子数（碰撞后）

$$J_{out} = \int_{\Omega} \int_{V} F(V, r, t) dV g(v, r, t) dv v_{\gamma} \sigma(\theta, \phi; v_{\gamma}) d\Omega dr\, dt,$$

↓

dt时间内在dvdr单元内粒子数的改变量

$$J\, dv\, dr\, dt = \{J_{in} - J_{out}\}.$$

玻尔兹曼方程和带电粒子输运方程

● 玻尔兹曼方程的碰撞项（续）

- 电子-气体分子碰撞积分
  - 定向运动速度远小于热运动速度，故电子速度分布接近于球对称
  - 可在速度空间用用球谐函数展开

$$g(v) = \sum_{mn} g_{mn}(v) Y^e_{mn}(\theta, \varphi),$$

$$Y^e_{mn}(\theta, \varphi) = P^m_n(\cos\theta) \cos m\varphi$$

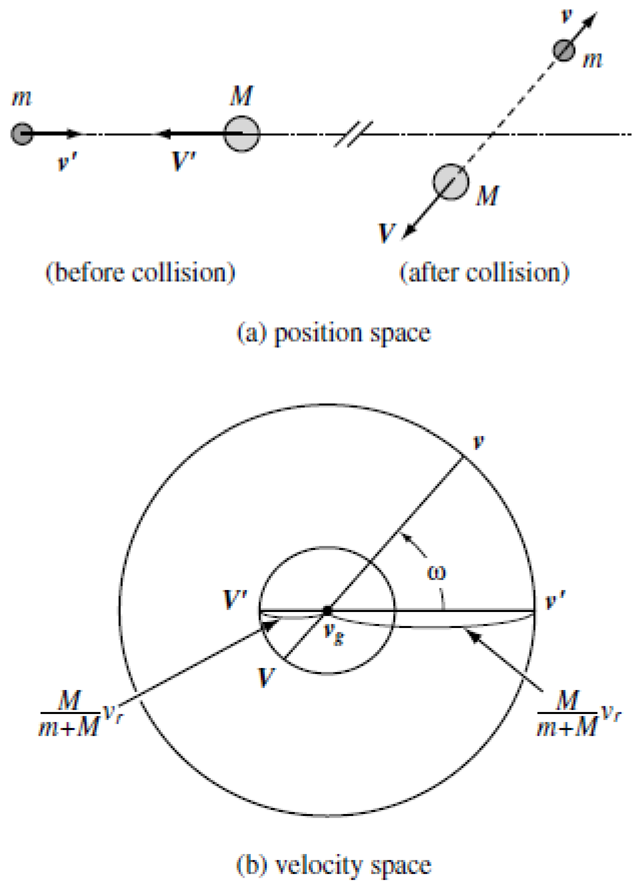


FIGURE 5.2
The definition of a collision in (a) a laboratory frame of reference (real space) and (b) a center-of-mass frame of reference (velocity space).

玻尔兹曼方程和带电粒子输运方程

- 玻尔兹曼方程的碰撞项（续）

  - 弹性碰撞项J_{elas}

    - 简化条件

i. The relative velocity between the electron and the gas molecule is not changed after the collision; 碰撞前后相对速度不变

ii. The velocity of gas molecules is much less than the velocity of electrons, $|V| \ll |v|$, and we represent the velocity distribution of gas molecules with the number density N by $F(V) = N\delta(V)$, where δ is Dirac's delta function; and 简化的中性粒子速度分布函数

iii. From the momentum and energy conservation equations before and after collisions, v' and v, we have the relation

$$\frac{v'^2 - v^2}{v^2} = \frac{2m}{M+m}(1 - \cos\omega), \quad (5.46)$$

能量转移系数

玻尔兹曼方程和带电粒子输运方程

- 玻尔兹曼方程的碰撞项（续）

  - 弹性碰撞项J_{elas}（续）

$$\frac{v'^2 - v^2}{v^2} = \frac{2m}{M+m}(1-\cos\omega),$$

$$dv'/dv = (v'/v)^3,$$

将dv'变换为dv

$$\begin{aligned} J_{elas}dv =& \int_{\Omega'}\int_{V'} N\delta(V')g(v')v_\gamma\sigma(v_\gamma,\omega)d\Omega' dv' dV' \\ &- \int_{\Omega}\int_{V} N\delta(V)g(v)v_\gamma\sigma(v_\gamma,\omega)d\Omega dv\, dV \\ =& N\left(\int_{\Omega'}\frac{v'^3}{v^3}g(v')v'\sigma(v',\omega)d\Omega' - \int_{\Omega}g(v)v\sigma(v,\omega)d\Omega\right)dv. \end{aligned}$$

$$g(v) = \sum_{mn} g_{mn}(v)Y^e_{mn}(\theta,\varphi),$$

$$\sigma(v,\omega) = \sum_{n'}\sigma_{n'}(v)P_{n'}(\cos\omega).$$

$$\begin{aligned} &N\frac{v'^4}{v^3}\sum_{mn}\sum_{n'} g_{mn}(v')\sigma_{n'}(v')\left[\int Y^e_{mn}(\theta',\varphi')P_{n'}(\cos\theta')P_n(\cos\theta)d\Omega'\right. \\ &+2\sum_{m=1}^{n'}\frac{(n-m)!}{(n+m)!}\int_{\Omega} Y^e_{mn}(\theta',\varphi')\left\{Y^e_{mn}(\theta',\varphi')Y^e_{mn}(\theta,\varphi)\right. \\ &\left.\left.+\ Y^0_{mn}(\theta',\varphi')Y^0_{mn}(\theta,\varphi)\right\}d\Omega'\right] = N\frac{v'^4}{v^3}\left[\sum_n g_{0n}(v')\sigma_n(v')\frac{4\pi}{2n+1}P_n(\cos\theta)\right. \\ &\left.+\sum_{m=1}^{n}\sum_n g_{mn}(v')\sigma(v')\frac{4\pi}{2n+1}Y^e_{mn}(\theta,\varphi)\right]. \end{aligned}$$

利用球谐函数性质，将θ'和φ'变换为θ和φ

玻尔兹曼方程和带电粒子输运方程

- 玻尔兹曼方程的碰撞项（续）

□ 弹性碰撞项J_{elas}（续）

$$\int_0^\pi P_n(\cos\theta)P_{n'}(\cos\theta)\sin\theta d\theta = \begin{cases} \dfrac{2}{2n+1}, & n = n' \\ 0, & n \neq n' \end{cases}.$$

$$\begin{aligned} J_{elas} &= N\sum_{mn}\int_\Omega Y^e_{mn}(\theta,\varphi)\frac{1}{v^3}\left[v'^4 g_{mn}(v')\sigma(v',\omega)P_n(\cos\omega) - v^4 g_{mn}(v)\sigma(v,\omega)\right]d\Omega \\ &= N\sum_{mn}\int_\Omega Y^e_{mn}(\theta,\varphi)\frac{1}{v^3}\left[v'^4 g_{mn}(v')\sigma(v',\omega) - v^4 g_{mn}(v)\sigma(v,\omega)\right]P_n(\cos\omega)d\Omega \\ &\quad -N\sum_{mn}\int_\Omega Y^e_{mn}(\theta,\varphi)g_{mn}(v)v\sigma(v,\omega)\left\{1 - P_n(\cos\omega)\right\}d\Omega. \qquad (5.50) \end{aligned}$$

$$\frac{v'^2 - v^2}{v^2} = \frac{2m}{M+m}(1-\cos\omega),$$

$$\Delta v^2 = v'^2 - v^2.$$

将v′变换为v

$$\begin{aligned} J_{elas} \cong\ & N\sum_{mn}\int_\Omega Y^e_{mn}(\theta,\varphi)\frac{1}{v}\frac{2m}{M+m}(1-\cos\omega)\frac{\partial[v^4 g_{mn}(v)\sigma(v,\omega)]}{\partial(v^2)}P_n(\cos\omega)d\Omega \\ & - N\sum_{mn}Y^e_{mn}(\theta,\varphi)g_{mn}(v)v\sigma(v,\omega)\left\{1 - P_n(\cos\omega)\right\}d\Omega. \qquad (5.51) \end{aligned}$$

玻尔兹曼方程和带电粒子输运方程

- 玻尔兹曼方程的碰撞项（续）

  - 弹性碰撞项J_{elas}（续）

$$J_{elas} \cong N\sum_{mn}\int_{\Omega} Y^{e}_{mn}(\theta,\varphi)\frac{1}{v}\frac{2m}{M+m}(1-\cos\omega)\frac{\partial[v^4 g_{mn}(v)\sigma(v,\omega)]}{\partial(v^2)}P_n(\cos\omega)d\Omega$$
$$-N\sum_{mn} Y^{e}_{mn}(\theta,\varphi)g_{mn}(v)v\sigma(v,\omega)\{1-P_n(\cos\omega)\}d\Omega. \quad (5.51)$$

$(m=0, n=0),$ 　 $P_0(\cos\omega)=1,$ 　 取各向同性特例情况

$$J^{00}_{elas} = N\frac{2m}{M+m}\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^4 g_{00}(v)Q_m(v)\right\}.$$

$$Q_m(v) = \int_{\Omega}(1-\cos\omega)\sigma(v,\omega)d\Omega.$$

考虑带电粒子与气体分子碰撞对速度造成的影响

$v = v_0 + O(V).$

$$O(V^2) = -g_{00}(v_0^2)\Big/\frac{\partial g_{00}(v_0^2)}{\partial(v^2)} = \frac{2kT_g}{m}$$

$$J^{00}_{elas} = N\frac{m}{M+m}\frac{1}{v^2}\frac{\partial}{\partial v}\left[v^4 Q_m(v)\left(g_{00}(v)+\frac{kT_g}{mv}\frac{\partial}{\partial v}g_{00}(v)\right)\right]$$

玻尔兹曼方程和带电粒子输运方程

- 玻尔兹曼方程的碰撞项（续）

□ 激发碰撞项J_{ex}

i. The change of kinetic energy in jth inelastic collisions ε_j usually satisfies the relation $\varepsilon_j \gg kT_g$ with gas molecule $F(V) = N\delta(V)$; and

ii. From the energy conservation we have

$$\frac{1}{2}mv'^2 = \frac{1}{2}mv^2 + \varepsilon_j. \tag{5.55}$$

$$v'dv' = vdv,$$

将dv′变换为dv

$$J_{exj}dv = N\int_{\Omega'}\int_V g(v')v'\sigma_j(v',\omega)\delta(V')d\Omega'dv'dV' - N\int_{\Omega}\delta(V)g(v)v\sigma_j(v,\omega)d\Omega dv\,dV$$
$$= N\frac{1}{v}\left[\int g(v')v'^2\sigma_j(v',\omega)d\Omega' - g(v)v^2\int\sigma_j(v,\omega)d\Omega\right]dv,$$

$$Q_j(v) = \int_{\Omega}\sigma_j(v,\omega)d\Omega.$$

利用球谐函数展开，将θ′和φ′变换为θ和φ

$$J_{exj} = N\frac{1}{v}\sum Y_{mn}(\theta,\varphi)\left[g_{mn}(v')v'^2\int_{\Omega}\sigma_j(v',\omega)P_n(\cos\omega)d\Omega - g_{mn}(v)v^2Q_j(v)\right], \tag{5.57}$$

玻尔兹曼方程和带电粒子输运方程

玻尔兹曼方程的碰撞项（续）

电离碰撞项J_{ion}

i. The change of kinetic energy in ionization ε_i usually satisfies the relation $\varepsilon_i \gg kT_g$ with gas molecule $F(V) = N\delta(V)$.

ii. In the principle of indistinguishability, the incoming and newly produced electrons at ionization cannot be distinguished after collision, but experimentally we know that there exist a pair of electrons with high and low energy. We assume that the rest of the kinetic energy is shared by two electrons according to the ratio $(1-\Delta):\Delta$. Then, the velocity element can be expressed as

两个电子分享动能的比例

$$dv' = \frac{1}{\Delta}\frac{v'}{v}dv_2 \quad \text{or} \quad dv' = \frac{1}{(1-\Delta)}\frac{v'}{v}dv_1. \tag{5.58}$$

iii. From the energy conservation, we have

$$\frac{1}{2}mv'^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \varepsilon_i. \tag{5.59}$$

玻尔兹曼方程和带电粒子输运方程

- 玻尔兹曼方程的碰撞项（续）

  - 电离碰撞项J_{ion}(续)

$$
\begin{aligned}
J_{ion} &= N\frac{1}{(1-\Delta)v}\int_{\Omega} g(v')v'^2\sigma_i(v',\omega)d\Omega + N\frac{1}{\Delta v}\int_{\Omega} g(v'')v''^2\sigma_i(v'',\omega)d\Omega \\
&\quad -N\frac{1}{v}g(v)v^2\int_{\Omega}\sigma_i(v,\omega)d\Omega \\
&= N\sum_{mn} Y_{mn}(\theta,\varphi)\frac{1}{v}\left\{\frac{1}{(1-\Delta)}\int_{\Omega} g_{mn}(v')v'^2\sigma_i(v',\omega)P_n(\cos\omega)d\Omega\right. \\
&\quad \left. +\frac{1}{\Delta}\int_{\Omega} g_{mn}(v'')v''^2\sigma_i(v'',\omega)P_n(\cos\omega)d\Omega - Ng_{mn}v^2Q_i(v)\right\}, \qquad (5.60)
\end{aligned}
$$

$$
Q_i(v) = \int \sigma_i(v,\omega)d\Omega.
$$

玻尔兹曼方程和带电粒子输运方程

- 玻尔兹曼方程的碰撞项（续）

  - 附着碰撞项J_{att}

Electron attachment is a very specific and nonconservative process wherein an electron with energy ε is lost in collision with threshold energy ε_a. The collision operator is simplified under $J_{in} = 0$ as

不新产生电子

$$J_{atta} = -Ng(v)v\int_{\Omega}\sigma_a(v,\omega)d\Omega$$
$$= -N\sum_{mn} Y^e_{mn}(\theta,\varphi)g_{mn}(v)vQ_a(v),$$

where the integrated attachment cross section is given by

$$Q_a(v) = \int_{\Omega}\sigma_a(v,\omega)d\Omega.$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程

  - 球谐函数及其性质

    - 分布函数的密度梯度展开

- 在非热平衡低温等离子体中，密度梯度是小量
- 准热平衡：电子从外场获得的能量与它和分子发生非弹性碰撞消耗的能量相等
- 分布函数g(v,r,t)可以通过球谐函数展开为g(v,t)和n(r,t)的乘积

$$g(v, r, t) = \sum_k g^k(v, t) \otimes (\nabla_r)^k n(r, t)$$

$$= \sum_k \sum_{mn} g^k_{mn}(v, t) Y^e_{mn}(\theta, \varphi) \otimes (\nabla_r)^k n(r, t),$$

$$Y^m_n(\theta, \varphi) = \Theta(\theta)\Psi_m(\varphi), \qquad \Psi_m(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi}$$

$$\Theta(\theta) = (-1)^m \left(\frac{2n+1}{2} \frac{(n-m)!}{(n+m)!} \right)^{1/2} P^m_n(\cos\theta), \qquad -n \le m \le n.$$

$$Y^m_n(\theta, \varphi) = (-1)^m \left(\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!} \right)^{1/2} P^m_n(\cos\theta) e^{im\varphi}. \qquad (5.64)$$

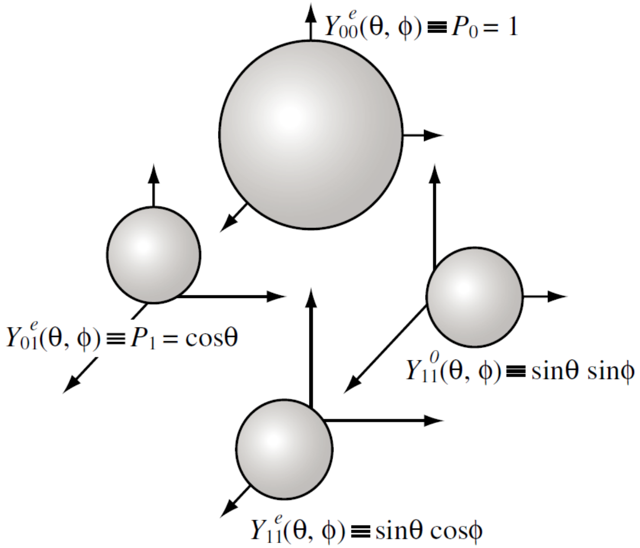
Their orthogonality is given by

$$\int_\varphi \int_\theta Y^{m_1}_{n_1}(\theta, \varphi) Y^{m_2}_{n_2}(\theta, \varphi) d\Omega = \delta_{n_1, n_2} \delta_{m_1, m_2}. \qquad (5.65)$$

- 电子的玻尔兹曼方程（续）

  - 球谐函数及其性质（续）

    - 球谐函数的一种简洁的形式



$$Y^e_{mn}(\theta,\varphi) = P^m_n(\cos\theta)\cos m\varphi$$

$$Y^0_{mn}(\theta,\varphi) = P^m_n(\cos\theta)\sin m\varphi.$$

归一化

$$\int_0^{2\pi}\int_0^{\pi}\left[Y^e_{mn}(\theta,\varphi)\quad\text{or}\quad Y^0_{mn}(\theta,\varphi)\right]^2 d\Omega = \begin{cases}\dfrac{4\pi}{2(2n+1)}\dfrac{(n+m)!}{(n-m)!}, & n=1,2,3,\ldots\\ 4\pi, & n=0.\end{cases}$$

$$(2n+1)\cos\theta\, Y^e_{mn}(\theta,\varphi) = (n+m)Y^e_{m(n-1)}(\theta,\varphi) + (n-m+1)Y^e_{m(n+1)}(\theta,\varphi)$$

递推关系

$$(2n+1)\cos^2\theta\frac{\partial}{\partial\cos\theta}Y^e_{mn}(\theta,\varphi) = (n+1)(n+m)Y^e_{m(n-1)}(\theta,\varphi) - (n-m+1)Y^e_{m(n+1)}(\theta,\varphi),$$

求导关系

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 球谐函数及其性质（续）

$$
\begin{aligned}
P_n(\cos\omega) &= P_n(\cos\theta_1)P_n(\cos\theta_2) \\
&\quad + 2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_n^m(\cos\theta_1)P_n^m(\cos\theta_2)\cos m(\varphi_1-\varphi_2) \\
&= P_n(\cos\theta_1)P_n(\cos\theta_2) \\
&\quad + 2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}\left[Y_{mn}^e(\theta_1,\varphi_1)Y_{mn}^e(\theta_2,\varphi_2)+Y_{mn}^0(\theta_1,\varphi_1)Y_{mn}^0(\theta_2,\varphi_2)\right].
\end{aligned}
$$

加法定理

TABLE 5.1

Legendre Polynomials

P_n	
$P_0(\cos\theta)$	1
$P_1(\cos\theta)$	$\cos\theta$
$P_2(\cos\theta)$	$(3\cos^2\theta-1)/2$
$P_3(\cos\theta)$	$(5\cos^3\theta-3\cos\theta)/2$
$P_4(\cos\theta)$	$(35\cos^4\theta-30\cos^2\theta+3)/8$

常用勒让德多项式

TABLE 5.2

Associated Legendre Polynomials

P_n^m	
$P_1^1(\cos\theta)$	$\sin\theta$
$P_2^1(\cos\theta)$	$3\cos\theta\sin\theta$
$P_2^2(\cos\theta)$	$3\sin^2\theta$
$P_3^1(\cos\theta)$	$3(5\cos^2\theta-1)\sin\theta/2$
$P_3^2(\cos\theta)$	$15\cos\theta\sin^2\theta$
$P_3^3(\cos\theta)$	$15\sin^3\theta$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 球谐函数及其性质（续）

$$\int_0^\pi P_n(\cos\theta)P_{n'}(\cos\theta)\sin\theta d\theta = \begin{cases} \dfrac{2}{2n+1}, & n = n' \\ 0, & n \neq n' \end{cases}.$$

正交性及任意函数的球谐函数展开

$$f(\cos\theta) = \sum_{n=0}^{\infty} a_n P_n(\cos\theta),$$

任意theta函数展开

$$a_n = \frac{2n+1}{2}\int_0^\pi f(\cos\theta)P_n(\cos\theta)\sin\theta d\theta,$$

$$f(\cos\theta) = \sum_{n=0}^{\infty}\frac{2n+1}{2}\left(\int_0^\pi f(\cos\theta')P_n(\cos\theta')\sin\theta' d\theta'\right)P_n(\cos\theta).$$

$$f(\theta,\varphi) = \sum a_{mn} Y_{mn}^e(\theta,\varphi),$$

$$a_{mn} = \frac{2(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}\int_0^{2\pi}\int_0^\pi f(\theta,\varphi)Y_{mn}^e(\theta,\varphi)d\Omega.$$

任意函数展开

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 电子的速度分布函数

    - 空间密度梯度展开后的最低三阶项及其满足的玻尔兹曼方程

$g^0(v,t)$, $g^1(v,t)$, and $g^2(v,t)$, are defined by the equations

↓ 根据玻尔兹曼方程

$$\frac{\partial}{\partial t}g^0(v,t) + \alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t) + R_0(t)g^0(v,t) = J(g^0, F),$$

$$\frac{\partial}{\partial t}g^1(v,t) + \alpha(t)\cdot\frac{\partial}{\partial v}g^1(v,t) + R_0(t)g^1(v,t) = J(g^1, F) + vg^0(v,t) - v_d(t)g^0(v,t),$$

$$\frac{\partial}{\partial t}\mathrm{g}^2(v,t) + \alpha(t)\cdot\frac{\partial}{\partial v}\mathrm{g}^2(v,t) + R_0(t)\mathrm{g}^2(v,t) = J(g^2, F) + vg^1(v,t) - v_d(t)g^1(v,t) + \mathrm{D}(t)g^0(v,t),$$

- g_0独立，与其它分布函数间无耦合，因此可以先求出g_0，然后再求g_1, g_2
- 二项展开近似方法中，只需求解g_0和g_1，忽略g_2

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 均匀数密度条件下的电子速度分布函数g_0

$$m = 0,$$

$$g^0(v,t) = \sum_{n=0} g^0_{0n}(v,t) Y^e_{0n}(\theta,\varphi) = \sum_n g^0_n(v,t) P_n(\theta).$$

球谐函数展开

$$\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t)$$

$$= \alpha_z(t)\left(\cos\theta\frac{\partial g^0}{\partial v} + \frac{\sin^2\theta}{v}\frac{\partial g^0}{\partial\cos\theta}\right)$$

$$= \sum_n \alpha_z(t)\cos\theta P_n(\theta)\frac{\partial g^0_n(v)}{\partial v} + \sum_n \alpha_z(t)\frac{\sin^2\theta}{v} g^0_n(v)\frac{\partial P_n(\theta)}{\partial\cos\theta}.$$

根据球谐函数的求导关系和递推关系

$$\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t)$$

$$= \alpha_z(t)\sum_n\left(\frac{n+1}{2n+1}P_{n+1}(\theta) + \frac{n}{2n+1}P_{n-1}(\theta)\right)\frac{\partial g^0_n(v)}{\partial v}$$

$$+ \alpha_z(t)\sum_n\frac{g^0_n(v)}{v}\left(-\frac{n(n+1)}{2n+1}P_{n+1}(\theta) + \frac{n(n+1)}{2n+1}P_{n-1}(\theta)\right).$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 均匀数密度条件下的电子速度分布函数g_0（续）

$$
\begin{aligned}
\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t)
&= \alpha_z(t)\sum_n\left(\frac{n+1}{2n+1}P_{n+1}(\theta)+\frac{n}{2n+1}P_{n-1}(\theta)\right)\frac{\partial g_n^0(v)}{\partial v}\\
&+\alpha_z(t)\sum_n\frac{g_n^0(v)}{v}\left(-\frac{n(n+1)}{2n+1}P_{n+1}(\theta)+\frac{n(n+1)}{2n+1}P_{n-1}(\theta)\right).
\end{aligned}
$$

重组并调整求和符号的n

$$
\begin{aligned}
\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t)
&= \alpha_z(t)\sum_n\left(\frac{n}{2n-1}\frac{\partial g_{n-1}^0(v)}{\partial v}+\frac{n+1}{2n+3}\frac{\partial g_{n+1}^0(v)}{\partial v}\right.\\
&\left.-\frac{(n-1)n}{2n-1}\frac{g_{n-1}^0(v)}{v}+\frac{(n+1)(n+2)}{2n+3}\frac{g_{n+1}^0(v)}{v}\right)P_n(\theta)\\
&=\sum_n\left\{\alpha_z(t)\frac{n}{2n-1}\left(\frac{\partial}{\partial v}-\frac{n-1}{v}\right)g_{n-1}^0(v,t)\right.\\
&\left.+\alpha_z(t)\frac{n+1}{2n+3}\left(\frac{\partial}{\partial v}+\frac{n+2}{v}\right)g_{n+1}^0(v)\right\}P_n(\theta).
\end{aligned}
$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 均匀数密度条件下的电子速度分布函数g_0（续）

$$\frac{\partial}{\partial t}g^0(v,t)+\alpha(t)\cdot\frac{\partial}{\partial v}g^0(v,t)+R_0(t)g^0(v,t)=J(g^0,F),$$

↓

$$\begin{aligned}&\frac{\partial}{\partial t}g_n^0(v,t)+\alpha_z(t)\frac{n}{2n-1}\left(\frac{\partial}{\partial v}-\frac{n-1}{v}\right)g_{n-1}^0(v,t)\\&\quad+\alpha_z(t)\frac{n+1}{2n+3}\left(\frac{\partial}{\partial v}+\frac{n+2}{v}\right)g_{n+1}^0(v,t)\\&\quad+R_0(t)g_n^0(v,t)-J(g^0,F)=0.\end{aligned}$$

可求解的0维玻尔兹曼方程

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 均匀数密度条件下的电子速度分布函数g_0（续）

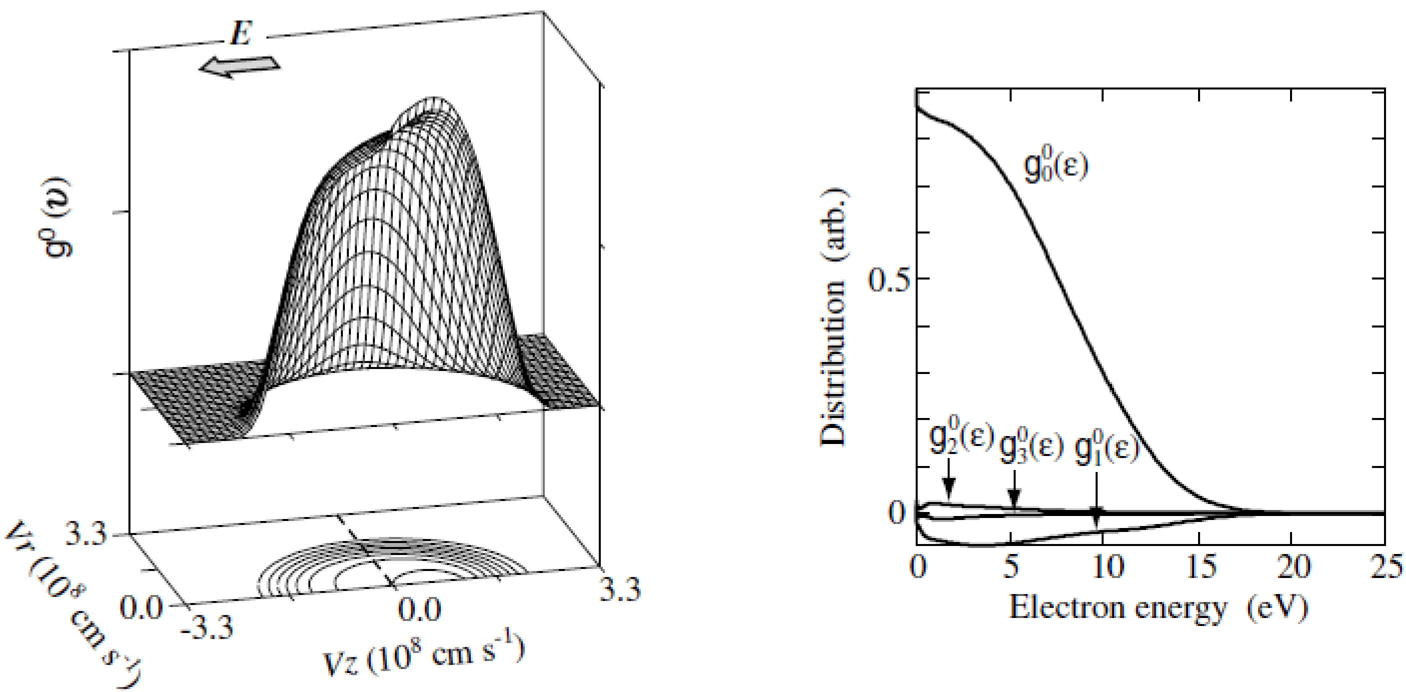


FIGURE 5.4

Lowest-order solution for the velocity distribution function for electrons at 100 Td in Ar: (a) $g^0(v)$ and (b) $g_n^0(v)$ $(n = 0, 1, 2, \ldots)$.

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1

$$
\frac{\partial}{\partial t}g^1(v,t)+\alpha(t)\cdot\frac{\partial}{\partial v}g^1(v,t)+R_0(t)g^1(v,t)
$$
$$
=J(g^1,F)+vg^0(v,t)-v_d(t)g^0(v,t),
$$

$$
g^1(v,t)=g_x^1(v,t)i+g_y^1(v,t)j+g_z^1(v,t)k.
$$

$$
\begin{vmatrix}\frac{\partial}{\partial t}g_x^1(v,t)\\ \frac{\partial}{\partial t}g_y^1(v,t)\\ \frac{\partial}{\partial t}g_z^1(v,t)\end{vmatrix}+\begin{vmatrix}\alpha_z(t)\frac{\partial}{\partial v_z}g_x^1(v,t)\\ \alpha_z(t)\frac{\partial}{\partial v_z}g_y^1(v,t)\\ \alpha_z(t)\frac{\partial}{\partial v_z}g_z^1(v,t)\end{vmatrix}+R_0\begin{vmatrix}g_x^1\\ g_y^1\\ g_z^1\end{vmatrix}+\begin{vmatrix}v_xg^0\\ v_yg^0\\ (v_z-v_d)g^0\end{vmatrix}=\begin{vmatrix}J(g_x^1,F)\\ J(g_y^1,F)\\ J(g_z^1,F)\end{vmatrix}.
$$

下面对g_z^1和g_x^1分别求解，g_y^1的求解与g_x^1相同

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

求解g_z^1

$$g_z^1(v,t)=\sum_n g_{z_{0n}}^1(v,t)Y_{0n}^e(\theta,\varphi)=\sum_n g_{z_n}^1(v,t)P_n(\theta).$$

$$\begin{aligned}(v_z-v_d)g^0(v,t)&=\sum_n (v\cos\theta-v_d)g_n^0P_n(\theta)\\&=\sum_n\left(v\frac{n}{2n+1}P_{n-1}(\theta)g_n^0+v\frac{n+1}{2n+1}P_{n+1}(\theta)g_n^0-v_dg_n^0P_n(\theta)\right)\\&=\sum_n\left(\frac{n+1}{2n+3}vg_{n+1}^0(v,t)+\frac{n}{2n-1}vg_{n-1}^0(v,t)-v_dg_n^0(v,t)\right)P_n(\theta),\end{aligned}$$

$$\begin{aligned}&\frac{\partial}{\partial t}g_{z_n}^1(v,t)+\alpha_z(t)\frac{n}{2n-1}\left(\frac{\partial}{\partial v}-\frac{n-1}{v}\right)g_{z_{n-1}}^1(v,t)\\&\qquad+\alpha_z(t)\frac{n+1}{2n+3}\left(\frac{\partial}{\partial v}+\frac{n+2}{v}\right)g_{z_{n+1}}^1(v,t)+R_0(t)g_{z_n}^1(v,t)-J\left(g_z^1,F\right)\\&\qquad=-\frac{n+1}{2n+3}vg_{n+1}^0(v,t)-\frac{n}{2n-1}vg_{n-1}^0(v,t)+v_dg_n^0(v,t).\qquad(5.90)\end{aligned}$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

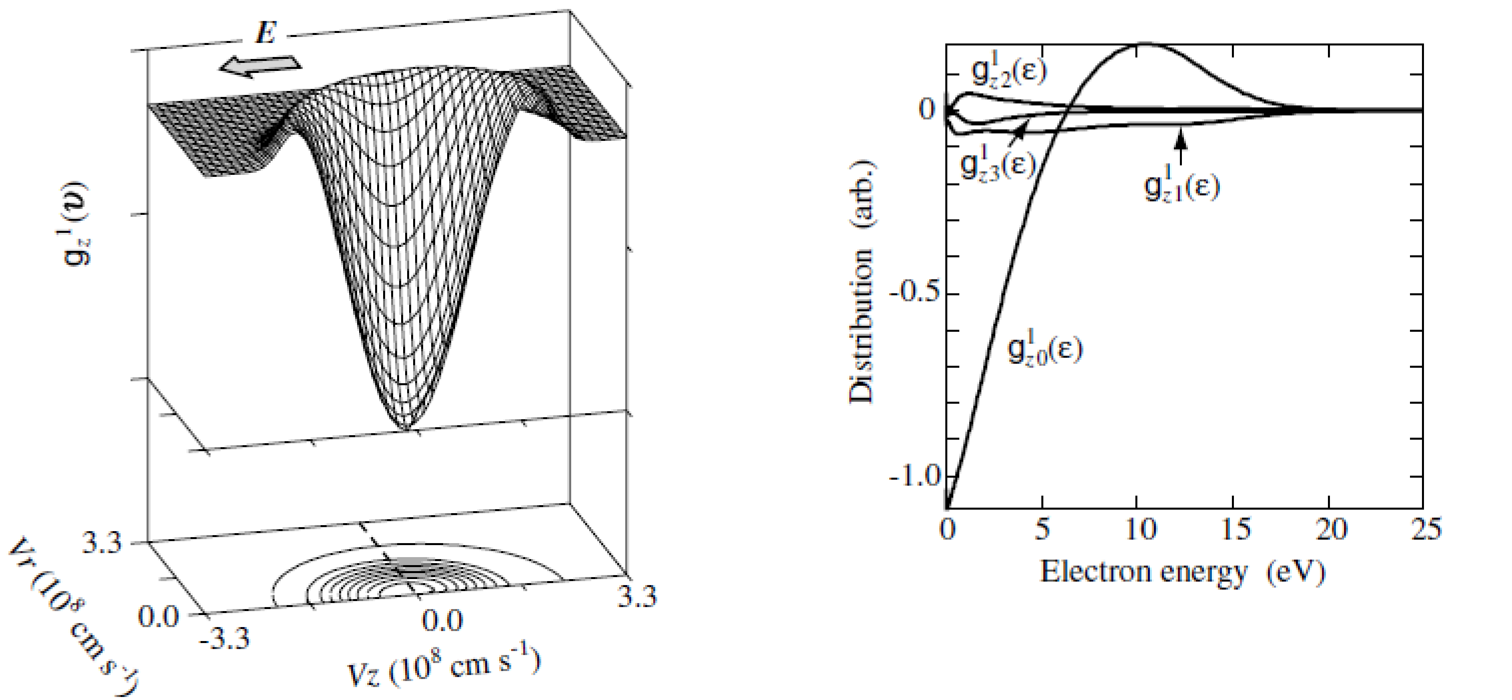


FIGURE 5.5
The longitudinal component (z) of the first-order solution for the velocity distribution function for electrons at 100 Td in Ar: (a) $g_z^1(v)$ and (b) $g_{zn}^1(\varepsilon)$ ($n = 0, 1, 2, \ldots$).

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

求解g_x^1

$$g_x^1(v,t)=\sum g_{x_{1n}}^1(v,t)Y_{1n}^e(\theta,\varphi)=\sum g_{x_n}^1(v,t)P_n^1(\theta)\cos\varphi.$$

$$\alpha_z(t)\frac{\partial}{\partial v_z}g_x^1(v,t)=\alpha_z(t)\left(\cos\theta\frac{\partial g_x^1}{\partial v}+\frac{\sin^2\theta}{v}\frac{\partial g_x^1}{\partial\cos\theta}+\frac{\partial\varphi}{\partial v_z}\frac{\partial g_x^1}{\partial\varphi}\right)$$

$$=\sum_n\alpha_z(t)\cos\theta\,P_n^1(\theta)\cos\varphi\frac{\partial g_{x_n}^1}{\partial v}+\sum_n\alpha_z(t)\frac{g_{x_n}^1}{v}\cos\varphi\sin^2\theta\frac{\partial P_n^1(\theta)}{\partial\cos\theta}.$$

When we use Equations 5.71 and 5.72 to replace $\cos\theta\,P_n^1(\theta)$ and $\sin^2\theta\,\partial P_n^1(\theta)/\partial\cos\theta$, we obtain

$$=\alpha_z(t)\cos\varphi\sum_{n=1}\left(\frac{n}{2n+1}P_{n+1}^1(\theta)+\frac{n+1}{2n+1}P_{n-1}^1(\theta)\right)\frac{\partial g_{x_n}^1(v,t)}{\partial v}$$

$$+\alpha_z(t)\cos\varphi\sum_{n=1}\left(-\frac{n^2}{2n+1}P_{n+1}^1(\theta)+\frac{(n+1)^2}{2n+1}P_{n-1}^1(\theta)\right)\frac{g_{x_n}^1(v,t)}{v},$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

$$\alpha_z(t)\frac{\partial}{\partial v_z}g_x^1(v,t) = \alpha_z(t)\sum_{n=1}\left(\frac{n-1}{2n-1}\frac{\partial g^1_{x_{n-1}}(v,t)}{\partial v} + \frac{n+2}{2n+3}\frac{\partial g^1_{x_{n+1}}(v,t)}{\partial v} - \frac{(n-1)^2}{2n-1}\frac{g^1_{x_{n-1}}(v,t)}{v} + \frac{(n+2)^2}{2n+3}\frac{g^1_{x_{n+1}}(v,t)}{v}\right)P_n^1(\theta)\cos\varphi$$

$$= \sum_{n=1}\left\{\alpha_z(t)\frac{n-1}{2n-1}\left(\frac{\partial}{\partial v} - \frac{n-1}{v}\right)g^1_{x_{n-1}}(v,t) + \alpha_z(t)\frac{n+2}{2n+3}\left(\frac{\partial}{\partial v} + \frac{n+2}{v}\right)g^1_{x_{n+1}}(v,t)\right\}P_n^1(\theta)\cos\varphi.$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

$$
\begin{aligned}
v_x g^0(v,t) &= \sum_{n=0} v \sin\theta \cos\varphi P_n(\theta) g_n^0(v,t) \\
&= \sum_{n=0} v g_n^0(v,t) \cos\varphi \left(\frac{1}{2n+1} P_{n+1}^1(\theta) - \frac{1}{2n+1} P_{n-1}^1(\theta) \right) \\
&= \sum \left(v \frac{1}{2n-1} g_{n-1}^0(v,t) - v \frac{1}{2n+3} g_{n+1}^0(v,t) \right) P(\theta) \cos\varphi .
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial t} g_{x_n}^1(v,t) + \alpha_z(t) \frac{n-1}{2n-1} \left(\frac{\partial}{\partial v} - \frac{n-1}{v} \right) g_{x_{n-1}}^1(v,t) \\
&\qquad + \alpha_z(t) \frac{n+2}{2n+3} \left(\frac{\partial}{\partial v} + \frac{n+2}{v} \right) g_{x_{n+1}}^1(v,t) + R_0(t) g_{x_n}^1(v,t) - J(g_x^1, F) \\
&\qquad = -\frac{1}{2n-1} v g_{n-1}^0(v,t) + \frac{1}{2n+3} v g_{n+1}^0(v,t). \qquad (5.94)
\end{aligned}
$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 正比于密度梯度的电子速度分布函数g_1（续）

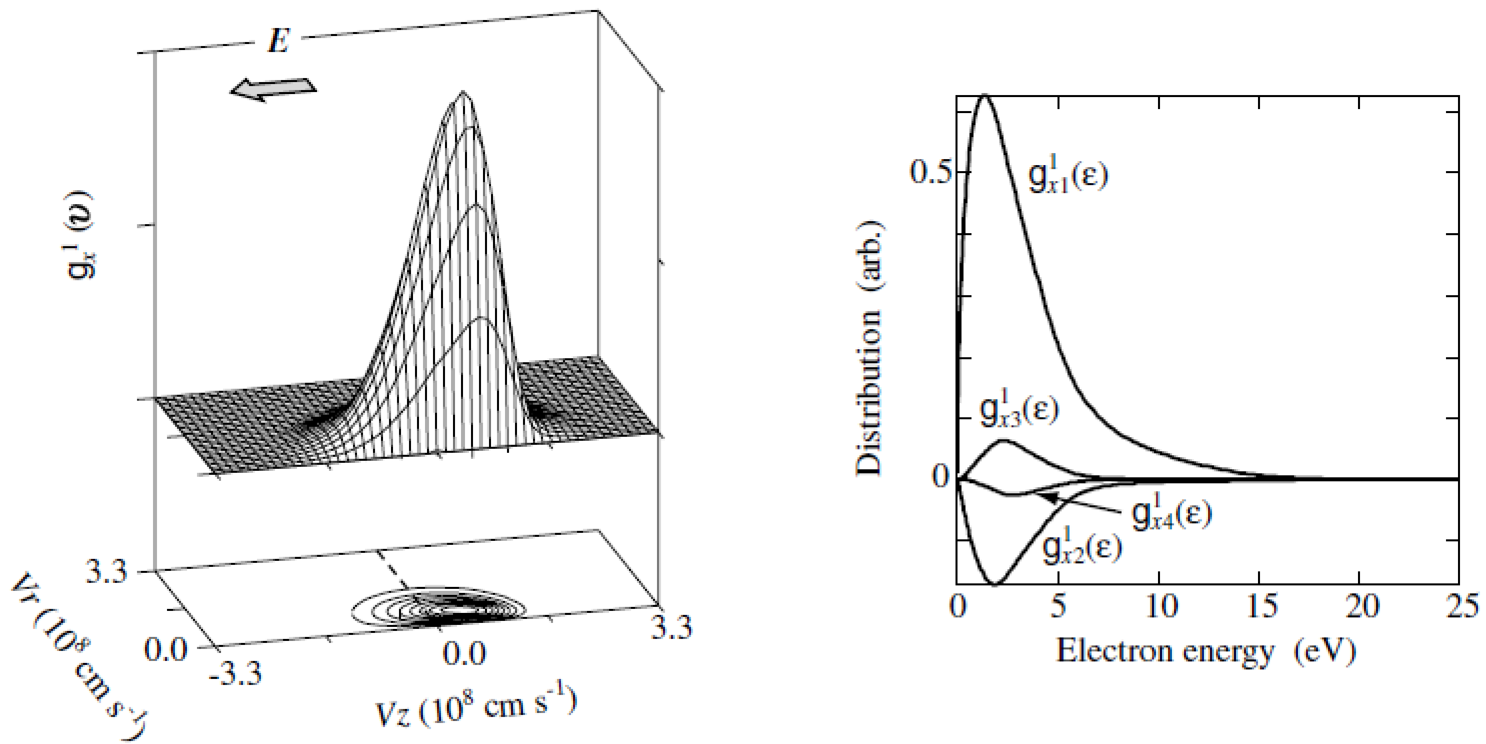


FIGURE 5.5
The longitudinal component (z) of the first-order solution for the velocity distribution function for electrons at 100 Td in Ar: (a) $g_z^1(v)$ and (b) $g_{zn}^1(\varepsilon)$ ($n = 0, 1, 2, \ldots$).

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 所有碰撞项的梯度展开和球谐展开小结

TABLE 5.3

Each of the Collision Terms Appearing in $J(g, F)$

Collision Type	Collision Integral	Expanded Collision Term
Elastic	$J_m\left(g_0^0\right)$	$\frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left\{NQ_m(v)v^4\left(g_0^0+\frac{kT_g}{mv}\frac{\partial g_0^0}{\partial v}\right)\right\}$
		$=\frac{m}{4\pi}\frac{2m}{M}\left[\left(\varepsilon-\frac{1}{2}kT_g\right)\frac{\partial}{\partial\varepsilon}NQ_m(\varepsilon)+\left(\frac{3}{2}-\frac{kT_g}{4\varepsilon}\right)NQ_m\right]f_0^0$
		$+\frac{m}{4\pi}\frac{2m}{M}\left[(\varepsilon+kT_g)NQ_m(\varepsilon)+kT_g\varepsilon\frac{\partial}{\partial\varepsilon}NQ_m(\varepsilon)\right]\frac{\partial}{\partial\varepsilon}f_0^0$
		$+\frac{m}{4\pi}\frac{2m}{M}kT_g\varepsilon NQ_m(\varepsilon)\frac{\partial}{\partial\varepsilon}f_0^0$
	$J_m\left(g_1^0\right)$	$-NQ_m(v)vg_1^0+\frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left\{\left(NQ_v(v)-NQ_m(v)\right)v^4g_1^0\right\}$
		$=\frac{m}{4\pi}NQ_m(\varepsilon)f_1^0$
		$+\frac{m}{4\pi}\frac{2m}{M}\left[\frac{3}{2}\left(NQ_v(\varepsilon)-NQ_m(\varepsilon)\right)\right.$
		$\left.+\varepsilon\frac{\partial}{\partial\varepsilon}\left[NQ_v(\varepsilon)-NQ_m(\varepsilon)\right]\right]f_1^0$
		$+\frac{2m}{M}\varepsilon\left(NQ_v(\varepsilon)-NQ_m(\varepsilon)\right)\frac{\partial}{\partial\varepsilon}f_1^0$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 所有碰撞项的梯度展开和球谐展开小结（续）

	$J_m\left(g_2^0\right)$	$-\frac{3}{2}NQ_v(v)vg_2^0=\frac{m}{4\pi}\left(-\frac{3}{2}\right)NQ_v(\varepsilon)$
	$J_m\left(g_{x11}^0\right)$	$-NQ_m(v)vg_{x11}^1=-\frac{m}{4\pi}NQ_m(\varepsilon)f_{x11}^1$
	$J_m\left(g_{x12}^0\right)$	$-\frac{3}{2}NQ_v(v)vg_{x12}^1=\frac{m}{4\pi}\left(-\frac{3}{2}\right)NQ_v(\varepsilon)f_{x12}^1$
Excitation	$J_j\left(g_0^0\right)$	$\frac{1}{v}\left\{v'^2NQ_j(v')g_0^0(v')-v^2NQ_j(v)g_0^0(v)\right\}$ $=\frac{m}{4\pi}\frac{1}{\sqrt{\varepsilon}}\frac{\partial}{\partial\varepsilon}\int_{\varepsilon}^{\varepsilon+\varepsilon_j}\sqrt{\varepsilon}NQ_j(\varepsilon)f_0^0(\varepsilon)d\varepsilon$
	$J_j\left(g_1^0 or g_2^0\right)$	$-NQ_j(v)vg_1^0\left(or g_2^0\right)=-\frac{m}{4\pi}NQ_j(\varepsilon)f_1^0\left(or f_2^0\right)$
	$J_j\left(g_{x11}^0 or g_{x12}^0\right)$	$-NQ_j(v)vg_{x11}^1\left(or g_{x12}^1\right)=-\frac{m}{4\pi}NQ_j(\varepsilon)f_{x11}^1\left(or f_{x12}^1\right)$
Ionization	$J_i\left(g_0^0\right)$	$\frac{1}{v}\left\{\frac{1+k}{k}v'^2NQ_i(v')g_0^0+(1+k)v'^2NQ_i(v')g_0^0-v^2NQ_i(v)g_0^0\right\}$ $=\frac{m}{4\pi}\frac{1}{\sqrt{\varepsilon}}\left(\frac{\partial}{\partial\varepsilon}\int_{\varepsilon}^{(1+k)\varepsilon+\varepsilon_i}\sqrt{\varepsilon}NQ_i(\varepsilon)f_0^0(\varepsilon)d\varepsilon+\frac{\partial}{\partial\varepsilon}\int_0^{\frac{1+k}{k}\varepsilon+\varepsilon_i}\sqrt{\varepsilon}NQ_i(\varepsilon)f_0^0(\varepsilon)d\varepsilon\right)$
	$J_i\left(g_1^0 or g_2^0\right)$	$-NQ_i(v)vg_1^0\left(or g_2^0\right)=-\frac{m}{4\pi}NQ_i(\varepsilon)f_1^0\left(or f_2^0\right)$
	$J_i\left(g_{x11}^0 or g_{x12}^0\right)$	$-NQ_i(v)vg_{x11}^1\left(or g_{x12}^1\right)=-\frac{m}{4\pi}NQ_i(\varepsilon)f_{x11}^1\left(or f_{x12}^1\right)$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 所有碰撞项的梯度展开和球谐展开小结（续）

Attachment	$J_a\left(g_0^0\right)$	$-NQ_a(v)vg_0^0=-\frac{m}{4\pi}NQ_a(\varepsilon)f_0^0$
	$J_a\left(g_1^0 or g_2^0\right)$	$-NQ_a(v)vg_1^0\left(or g_2^0\right)=-\frac{m}{4\pi}NQ_a(\varepsilon)f_1^0\left(or f_2^0\right)$
	$J_a\left(g_{x_{11}}^0 or g_{x_{12}}^0\right)$	$-NQ_a(v)vg_{x_{11}}^1\left(or g_{x_{12}}^1\right)=-\frac{m}{4\pi}NQ_a(\varepsilon)f_{x_{11}}^1\left(or f_{x_{12}}^1\right)$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 电子输运参数

$$
\begin{aligned}
g(v, r, t) &= \sum_k \mathrm{g}^k(v, t) \otimes (\nabla_r)^k n(r, t) \\
&= \sum_k \sum_{mn} g^k_{mn}(v, t) Y^e_{mn}(\theta, \varphi) \otimes (\nabla_r)^k n(r, t) \\
&= \left(\sum_{n=0} g^0_n(v, t) P_n(\theta) \right) n(r, t) \\
&\quad + \left(\sum_{n=1} g^1_{x_{1n}}(v, t) Y^e_{1n}(\theta, \varphi) i + \sum_{n=1} g^1_{y_{1n}}(v, t) Y^e_{1n}(\theta, \varphi) j \right. \\
&\quad \left. + \sum_{n=1} g^1_{z_n}(v, t) P_n(\theta) k \right) \cdot \frac{\partial}{\partial r} n(r, t) + O\left(\nabla_r^2 n\right) \\
&= \left(g^0_0(v, t) + g^0_1(v, t) \cos\theta + g^0_2(v, t) \frac{3\cos^2\theta - 1}{2} + \cdots \right) n(r, t) \\
&+ \left(g^1_{x_1}(v, t) \sin\theta \cos\varphi + g^1_{x_2}(v, t) 3 \cos\theta \sin\theta \cos\varphi + \cdots \right) i \cdot \frac{\partial}{\partial r} n(r, t) \\
&+ \left(g^1_{y_1}(v, t) \sin\theta \cos\varphi + g^1_{y_2}(v, t) 3 \cos\theta \sin\theta \cos\varphi + \cdots \right) j \cdot \frac{\partial}{\partial r} n(r, t) \\
&+ \left(g^1_{z_0}(v, t) + g^1_{z_1}(v, t) \cos\theta + g^1_{z_2}(v, t) \frac{3\cos^2\theta - 1}{2} + \cdots \right) k \cdot \frac{\partial}{\partial r} n(r, t) \\
&+ O\left(\nabla_r^2 n\right). \qquad (5.95)
\end{aligned}
$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 电子输运参数（续）

$$
\begin{aligned}
g(v, r, t) = & \left(g_0^0(v, t) + g_1^0(v, t)\cos\theta + g_2^0(v, t)\frac{3\cos^2\theta - 1}{2} + \cdots\right) n(r, t) \\
& + \left(g_{x_1}^1(v, t)\sin\theta\cos\varphi + g_{x_2}^1(v, t) 3\cos\theta\sin\theta\cos\varphi + \cdots\right) i \cdot \frac{\partial}{\partial r} n(r, t) \\
& + \left(g_{y_1}^1(v, t)\sin\theta\cos\varphi + g_{y_2}^1(v, t) 3\cos\theta\sin\theta\cos\varphi + \cdots\right) j \cdot \frac{\partial}{\partial r} n(r, t) \\
& + \left(g_{z_0}^1(v, t) + g_{z_1}^1(v, t)\cos\theta + g_{z_2}^1(v, t)\frac{3\cos^2\theta - 1}{2} + \cdots\right) k \cdot \frac{\partial}{\partial r} n(r, t) \\
& + O(\nabla_r^2 n). \qquad (5.95)
\end{aligned}
$$

两项展开近似

$$
g(v) = [g_0^0(v) + g_1^0(v)\cos\theta + O(g_2^0)] n_e,
$$

迁移速度

$$
\begin{aligned}
v_d(t) &= \int v g^0(v, t) dv \\
&= \sum_n \int_0^\pi v\cos\theta g_n^0(v, t) P_n(\theta) v^2 dv d\Omega \\
&= \frac{4\pi}{3} \int v^3 g_1^0(v, t) dv.
\end{aligned}
$$

(只保留前两项)

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 电子输运参数（续）

    - 扩散系数

两项展开近似条件

$$D_T(t) = \int v_x g_x^1(v,t)dv = \int v_y g_y^1(v,t)dv$$
$$= \sum_n \int v \sin\theta \cos\varphi g^1_{x_{1n}}(v,t) Y^e_{1n}(\theta,\varphi) v^2 dv d\Omega$$
$$= \frac{4\pi}{3} \int v^3 g^1_{x_1}(v,t)dv,$$

$$D_L(t) = \int v_z g_z^1(v,t)dv$$
$$= \sum_n \int v \cos\theta g^1_{z_n}(v,t) P_n(\theta) v^2 dv d\Omega$$
$$= \frac{4\pi}{3} \int v^3 g^1_{z_1}(v,t)dv.$$

玻尔兹曼方程和带电粒子输运方程

- 电子的玻尔兹曼方程（续）

  - 电子输运参数（续）

    - 有效电子产生系数

$$
\begin{aligned}
R_0(t) &= R_i(t) - R_a(t) \\
&= N\int [Q_i(v) - Q_a(v)]\, v g(v, t) dv \\
&= 4\pi N \int [Q_i(v) - Q_a(v)]\, v^3 g_0^0(v, t) dv.
\end{aligned}
$$

    - 平均动能

$$
\begin{aligned}
\langle \varepsilon(r, t)\rangle &= \int \frac{1}{2} m v^2 g(v, r, t) dv \\
&= \sum_n \int \frac{1}{2} m v^2 g_n^0(v, t) P_n(\theta) v^2 dv d\Omega \\
&\quad - \nabla_z n k \sum_n \int \frac{1}{2} m v^2 g_{z_n}^1(v, t) P_n(\theta) v^2 dv d\Omega \\
&\quad - \nabla_x n i \sum_n \int \frac{1}{2} m v^2 g_{x_{1n}}^1(v, t) Y_{1n}^e(\theta, \varphi) v^2 dv d\Omega \\
&= 2\pi m \int v^4 g_0^0(v, t) dv - \left(2\pi m \int v^4 g_{z_0}^1(v, t) dv\right) \nabla_z n k.
\end{aligned}
$$

《等离子体电子学》

第五章 玻尔兹曼方程和带电粒子输运方程

本章待续

下一章：第六章 气体中带电粒子输运的一般性质

（在"幻灯片放映"模式中时单击该箭头）