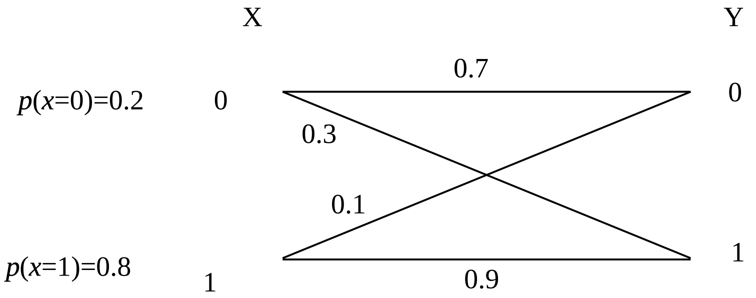
1. Please find $I(x=1;Y)$ of the followed binary channel



2. *Unused symbols.* Show that the capacity of the channel with probability transition matrix

$$P_{y|x}=\begin{pmatrix}\frac{2}{3} & \frac{1}{3} & 0\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ 0 & \frac{1}{3} & \frac{2}{3}\end{pmatrix}$$

is achieved by a distribution that places zero probability on one of input symbols. What is the capacity of this channel? Give an intuitive reason why that letter is not used.

3.*Channel capacity*. Calculate the capacity of the following channels with probability transition matrices:

(a) $X = Y = \{0, 1, 2\}$

$$P(y\,|\,x)=\begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{pmatrix}$$

(b) $X = Y = \{0, 1, 2\}$

$$P(y\,|\,x)=\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & 0\\ 0 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 0 & \frac{1}{2}\end{pmatrix}$$

4. *Noisy typewriter*. Consider the channel with $x, y \in \{0, 1, 2, 3\}$ and transition probabilities $p(y|x)$ given by the following matrix:

$$\begin{pmatrix}1/2 & 1/2 & 0 & 0\\ 0 & 1/2 & 1/2 & 0\\ 0 & 0 & 1/2 & 1/2\\ 1/2 & 0 & 0 & 1/2\end{pmatrix}$$

(a) Find the capacity of this channel.

(b) Define the random variable $z = g(y)$, where

$$g(y)=\begin{cases}A & if\ \ y\in\{0,1\}\\ B & if\ \ y\in\{2,3\}\end{cases}$$

For the following two PMFs(Probability mass function) for *x*, compute *I* (*X*; *Z*):

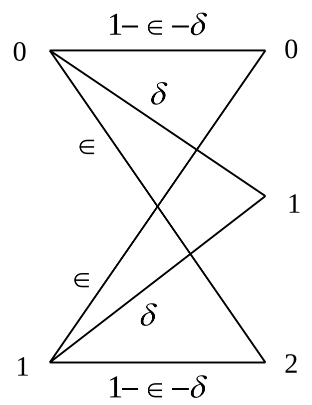
(i) $p(x)=\begin{cases}\frac{1}{2} & if\ \ x\in\{1,3\}\\ 0 & if\ \ x\in\{0,2\}\end{cases}$

(ii) $p(x)=\begin{cases}0 & if\ \ x\in\{1,3\}\\ \frac{1}{2} & if\ \ x\in\{0,2\}\end{cases}$

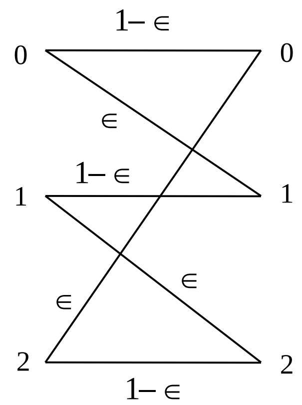
(c) Find the capacity of the channel between *x* and *z*, specifically where $x \in \{0, 1, 2, 3\}$, $z \in \{A,B\}$, and the transition probabilities *P*(*z*|*x*) are given by

$$p(Z=z\,|\,X=x)=\sum_{g(y_0)=z}P(Y=y_0\,|\,X=x)$$

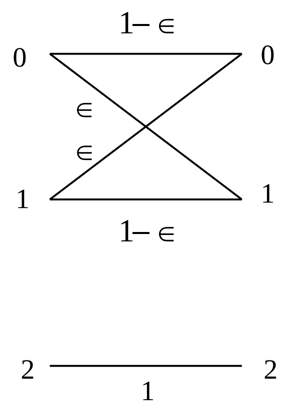
5. Judge the symmetry of the follwong channels and calculate their information channel capacity when the channel is symmetric



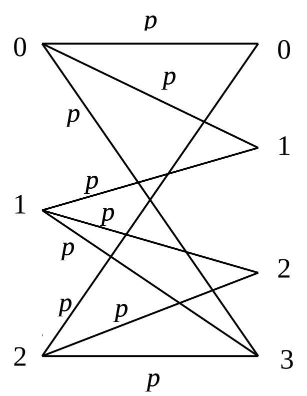
(a)



(b)

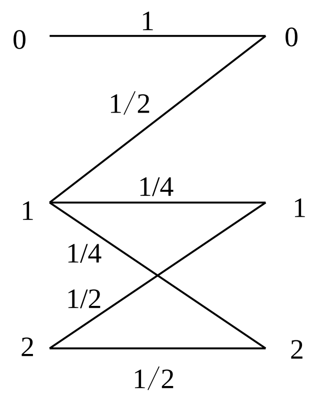


(c)

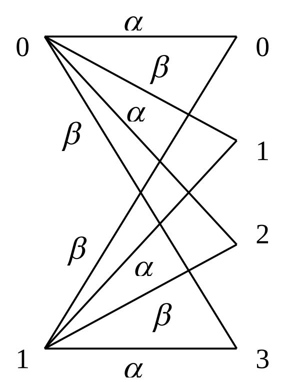


$p=\frac{1}{3}$

(d)



(e)



$\alpha=(1-p)/2$

$\beta=p/2$

(f)

6. *Z-channel*. The *Z*-channel has binary input and output alphabets and transition

probabilities $p(y|x)$ given by the following matrix:

$$\begin{pmatrix} 1 & 0 \\ 1/2 & 1/2 \end{pmatrix} \qquad x, y \in \{0,1\}$$

Find the capacity of the *Z*-channel and the maximizing input probability distribution.

7. *Codes of length 3 for a BSC.* For a binary symmetric channel with crossover probability $\varepsilon = 0.1$.

(a) Find the best code of length 3 with four codewords for this channel. What is the probability of error for this code? (Note that all possible received sequences should be mapped onto possible codewords.)

(b) What is the probability of error if we used all eight possible sequences of length 3 as codewords?