西安交大 — 中国科大数论研讨会

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何伟鲲: Spectral gap in subgroups of $SL_d(\mathbf{Z}/n\mathbf{Z})$

Uniform spectral gap (of the discrete Laplace operators) of a family of finite groups is also known as expansion in groups or property (τ). In the case of $SL_2(\mathbf{Z}/n\mathbf{Z})$, a uniform spectral gap is present as a consequence of Selberg's 3/16 theorem and the Burger-Brooks correspondence. In the talk, I will explain how show uniform spectral gap in broader classes of groups, using combinatorics and dynamics. This talk is based on a joint work with Nicolas de Saxce, which is build on works of Bourgain–Gamburd and Bourgain–Varju among others.

黄治中: Quantitative strong approximation for quadrics

The classical Hasse–Minkowski theorem states that a non-singular integral quadratic form with at least three variables represents zero over $\mathbf{Z}$ if and only if it does so over $\mathbf{R}$ and every $\mathbf{Z}_p$. We shall report how Heath-Brown's delta circle method allows to obtain asymptotic formulas for counting integer zeros of bounded height on quadrics satisfying prescribed local conditions with optimal error terms. This is a quantitive and effective version of the Hasse–Minkowski theorem. It is also related to intrinsic Diophantine approximation on quadrics. This is based on joint work in progress with D. Schindler and A. Shute.

林永晓: Level of distribution of Hecke eigenvalues in arithmetic progressions

Let $(\lambda_F(n))_{n \geq 1}$ be Hecke eigenvalues attached to an automorphic form F on GL_d. Given q a modulus and a primitive class $a \pmod q$, it is an interesting question to know whether the sequence $(\lambda_F(n))_{n \leq X}$ is equidistributed when it is restricted to arithmetic progression $n \equiv a \pmod q$ with varying moduli q. Possibly subject to the Ramanujan-Petersson conjecture, the equidistribution holds when q varies with X in the range $q < X^{2/(d+1)}$. For general F and q it can be a difficult problem in passing this exponent. We will discuss examples where additional structural assumptions on either the Hecke eigenvalues $\lambda_F(n)$ or the moduli q allow improvement. I will explain how this is related to estimates for algebraically twisted sums involving $\lambda_F(n)$ which might be of independent interest. This is based on joint works with E. Kowalski, Ph. Michel, and W. Sawin.

齐治: Moments of central L-values for Maass forms over imaginary quadratic fields

I will talk about the twisted moments of central L-values for GL(2) Maass forms over imaginary quadratic fields. As a direct consequence, it can be shown that at least 33% of such central L-values do not vanish. This is joint work with Sheng-Chi Liu.

秦厚荣: TBA

石荣刚: Metric number theory and homogeneous dynamics

We review the connection between metric number theory and homogenous dynamics. We discuss some classical results and open problems. We report some recent progress on Dirichlet improvable vectors using homogeneous dynamics.

徐飞: Strong approximation for certain singular varieties via blowing up

We extend the concept of central strong approximation with Brauer–Manin obstruction for singular varieties by dropping the restriction of finiteness of Brauer groups of resolution of singularities up to constant. We prove that a variety defined by a polynomial represented by an isotropic binary quadratic form satisfies central strong approximation with Brauer–Manin obstruction by explicit blowing up. This is the last case of generalization Watson's result about the local-global principle for Diophantine equations reducible to quadratics. This is a joint work with Heng Song.

杨磊: An effective version of Ratner's equidistribution theorem for $\mathrm{SL}(3, \mathbf{R})$

In this talk I will prove an effective version of Ratner's equidistribution theorem for unipotent orbits in $\mathrm{SL}(3, \mathbf{R})/\mathrm{SL}(3, \mathbf{Z})$. The proof relies on controlling the dimension of the orbit along transversal directions. The key is to analyze a Kakeya type model related to the behavior of unipotent orbits.

杨鹏宇: Dirichlet improvable vectors on degenerate submanifolds

Dirichlet improvability is a notion in Diophantine approximation which was introduced by Davenport and Schmidt around 1969, and since then it has been extensively studied. In this talk I will present a recent result which states that if the affine span of a submanifold of $\mathbf{R}^n$ satisfies certain Diophantine/arithmetic conditions, then the set of Dirichlet improvable vectors on this submanifold is null with respect to the natural measure on the submanifold. Joint work with Nimish Shah.