Solution to problems of Chapter 1

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Relation between the units:

1 ft=0.3048m
1lb=0.454 kg
1lb/ft^2=47.89N/m^2=47.89 Pa
1^{o}R=5/9K

1.1 At the nose of a missile in flight, the pressure and temperature are 5.6 atm and 850°R, respectively. Calculate the density and specific volume. (Note: 1 atm = 2116 lb/ft^2.)

Solution:

The temperature is $T = 850^o$R=$850 \times 5/9$K=472.2K
The pressure is p=5.6 atm=$5.6 \times 1.01 \times 10^5$ N/m^2=5.656×10^5N/m^2.
The density is $\rho = \frac{p}{RT} = \frac{5.656\times10^5}{287\times472.2} = 4.1735$ kg/m^3.
The specific volume is $v = \frac{1}{\rho}$ =0.2396m^3/kg

1.2 In the reservoir of a supersonic wind tunnel, the pressure and temperature of air are 10 atm and 320 K, respectively. Calculate the density, the number density, and the mole-mass ratio. (Note: 1 atm = 1.01×10^5 N/m^2.)

Solution:

The density is $\rho = \frac{p}{RT} = \frac{10\times1.01\times10^5}{287\times320} = 10.997$kg/m^3.
The number density is $n = \frac{p}{kT} = \frac{10\times1.01\times10^5}{1.38\times10^{-23}\times320} = 2.287 \times 10^{26}$/m^3.
The mole-mass ratio is $\eta = \frac{pv}{\mathscr{R}T} = \frac{p}{\rho\mathscr{R}T} = \frac{10\times1.01\times10^5}{10.997\times8314\times320} = 0.0345$kg·mol/kg.

1.3 For a calorically perfect gas, derive the relation $c_p - c_v = R$. Repeat the derivation for a thermally perfect gas.

Solution:

For calorically perfect gas $h = c_pT$ and c_vT.
From the definition of enthalpy, we have $h = e + pv \Rightarrow c_pT = c_vT + RT \Rightarrow c_p - c_v = R$.
For thermally perfect gas $dh = de + d(pv)$, $dh = c_pdT$ and $de = c_vdT$.
Thus we have $c_pdT = c_vdT + d(RT) \Rightarrow c_p = c_v + R \Rightarrow c_p - c_v = R$.

1.4 The pressure and temperature ratios across a given portion of a shock wave in air are $p_2/p_1 = 4.5$ and $T_2/T_1 = 1.687$, where 1 and 2 denote conditions ahead of and behind the shock wave, respectively. Calculate the change in entropy in units of (a) (ft · lb)/(slug·^{o}R) and (b) J/(kg· K).

Solution:

The change in entropy is given by

$$\Delta S = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} = \frac{\gamma R}{\gamma - 1}\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

. Substitute $\gamma = 1.4$ and R=287 J/(kg $\cdot$ K) into the above equation, we have $\Delta S = 93.6349$ J/(kg $\cdot$ K)

1.5 Assume that the flow of air through a given duct is isentropic. At one point in the duct, the pressure and temperature are $p_1 = 1800$ lb/ft^2 and $T_1 = 500$ ^{o}R, respectively. At a second point, the temperature is 400 ^{o}R. Calculate the pressure and density at this second point.

Solution:

For isentropic process, we have

$$\frac{p_2}{p_1} = (\frac{\rho_2}{\rho_1})^\gamma = (\frac{T_2}{T_1})^{\gamma/(\gamma-1)}$$

Thus

$$p_2 = p_1(\frac{T_2}{T_1})^{\gamma/(\gamma-1)} = 824.30lb/ft^2$$

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$$\rho_2 = \frac{p_2}{RT_2} = 0.6179kg/m^3$$

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1.6 Consider a room that is 20 ft long, 15 ft wide, and 8 ft high. For standard sea level conditions, calculate the mass of air in the room in slugs. Calculate the weight in pounds. (Note: If you do not know what standard sea level conditions are, consult any aerodynamics text, such as Refs. 1 and 104, for these values. Also, they can be obtained from any standard atmosphere table.)

Solution:

$V = 20 \times 15 \times 8 = 2400ft^3$

$\rho = 1.225kg/m^3 = 2.4 \times 10^{-3}slug/ft^3$ for standard sea level condition.

Thus $m = \rho V = 5.76slug = 5.76 \times 14.5935/0.454lb = 185.15lb$.

1.7 In the infinitesimal neighborhood surrounding a point in an inviscid flow, the small change in pressure, dp, that corresponds to a small change in velocity, dV, is given by the differential relation $dp = -\rho V dV$. (This equation is called Euler's Equation; it is derived in Chap. 6.) a. Using this relation, derive a differential relation for the fractional change in density, $d\rho/\rho$, as a function of the fractional change in velocity, dV/V, with the compressibility τ as a coefficient.

b. The velocity at a point in an isentropic flow of air is 10 m/s (a low speed flow), and the density and pressure are 1.23 kg/m^3 and 1.01×10^5 N/m^2, respectively (corresponding to standard sea level conditions). The fractional change in velocity at the point is 0.01. Calculate the fractional change in density.

c. Repeat part (b), except for a local velocity at the point of 1000 m/s (a high-speed flow). Compare this result with that from part (b), and comment on the differences.

Solution:

(a)

$$\tau = 1\frac{1}{V}\frac{dV}{dp} \tag{1}$$

and

$$V = \frac{1}{\rho} \Rightarrow dV = \frac{1}{\rho^2}d\rho \tag{2}$$

Substitute equation (2) into equation (1), we have

$$\tau = \frac{1}{\rho^2 V}\frac{d\rho}{dp} \tag{3}$$

. Substitute the Euler's equation into equation (3), we have

$$\tau = -\frac{1}{\rho}\frac{d\rho}{\rho V dV} \Rightarrow \frac{d\rho}{\rho} = -\tau\rho V^2\frac{dV}{V} \tag{4}$$

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(b). For isentropic flow, we have

$$\frac{p}{\rho^\gamma} = c \Rightarrow p = c\rho^\gamma \Rightarrow dp = \gamma c\rho^{\gamma-1}d\rho = \gamma c\frac{\rho^\gamma}{\rho} = \gamma\frac{p}{\rho}d\rho \Rightarrow \frac{dp}{d\rho} = \gamma\frac{p}{\rho} \tag{5}$$

$$\tau = \frac{1}{\rho}\frac{dp}{d\rho} = \frac{1}{\gamma p} \tag{6}$$

Substitute equation (6) into equation (4), we have

$$\frac{d\rho}{\rho} = -\frac{1}{\gamma p}\rho V^2\frac{dV}{V} \tag{7}$$

For V=10m/s, p=1.01×10^5N/m^2, $\rho = 1.23$kg/m^3, $\gamma = 1.4$ and $\frac{dV}{V} = 0.01$, we have $\frac{d\rho}{\rho} = -8.99\times10^{-6}$.
(c) For V=1000m/s, we have $\frac{d\rho}{\rho} = -8.99\times10^{-2}$
It can be seen that the larger the velocity, the larger the relative change in density.