# Problems of Ch3 Part 1

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Relation between the units:

1 ft=0.3048m;1lb=0.454 kg; 1lb/ft$^2$=47.89N/m$^2$=47.89 Pa; 1$^o$R=5/9K

3.8 Consider air entering a heated duct at $p_1 = 1$ atm and $T_1 = 288$ K. Ignore the effect of friction. Calculate the amount of heat per mass (in joules per kilogram) necessary to chock the flow at the exit of the duct, as well as the pressure and temperature at the duct exit, for an inlet Mach number of (a) $M_1 = 2.0$ and (b) $M_1 = 0.2$.

Solution:

(a) When $M_1 = 2.0$, we have

$$\frac{T_1}{T^*} = 0.5289, \quad \frac{p_1}{p^*} = 0.3636, \quad \frac{T_{01}}{T_1} = 1.8 \quad and \quad \frac{p_{01}}{p_1} = 7.824$$

When chocked, the flow at the exit reaches sonic condition.

Hence

$$\frac{T_2}{T^*} = 1, \quad \frac{p_2}{p^*} = 1, \quad \frac{T_{02}}{T_2} = 1.2 \quad and \quad \frac{p_{02}}{p_2} = 1.893$$

Thus the temperature at the exit is

$$T_2 = \frac{T_2}{T^*}\frac{T^*}{T_1}T_1 = 1 \times \frac{1}{0.5289} \times 288 = 544.53K,$$

The pressure at the exit is

$$p_2 = \frac{p_2}{p^*}\frac{p^*}{p_1}p_1 = 1 \times \frac{1}{0.3636} \times 1atm = 2.75atm$$

$$q = c_p(T_{02} - T_{01}) = 1004 \times (1.2 \times T_2 - 1.8 \times T_1) = 1.36 \times 10^5 \quad J/kg.$$

(b) When $M_1 = 0.2$, we have

$$\frac{T_1}{T^*} = 0.2066, \quad \frac{p_1}{p^*} = 2.273, \quad \frac{T_{01}}{T_1} = 1.008 \quad and \quad \frac{p_{01}}{p_1} = 1.028$$

When chocked, the flow at the exit reaches sonic condition.

Hence

$$\frac{T_2}{T^*} = 1, \quad \frac{p_2}{p^*} = 1, \quad \frac{T_{02}}{T_2} = 1.2 \quad and \quad \frac{p_{02}}{p_2} = 1.893$$

Thus the temperature at the exit is

$$T_2 = \frac{T_2}{T^*}\frac{T^*}{T_1}T_1 = 1 \times \frac{1}{0.2066} \times 288 = 1394K,$$

The pressure at the exit is

$$p_2 = \frac{p_2}{p^*}\frac{p^*}{p_1}p_1 = 1 \times \frac{1}{2.273} \times 1atm = 0.44 \quad atm$$

$$q = c_p(T_{02} - T_{01}) = 1004 \times (1.2 \times T_2 - 1.008 \times T_1) = 1.39 \times 10^6 \quad J/kg.$$

3.9 Air enters the combustor of a jet engine at $p_1 = 10atm$ and $T_1 = 1000^oR$, and $M_1 = 0.2$. Fuel is inject and burned, with a fuel-air ratio (by mass) of 0.06. The heat released during the combustion is $4.5 \times 10^8$ ft-lb per slug of fuel. Assuming one-dimensional frictionless flow with $\gamma = 1.4$ for the fuel-air mixture, calculate $M_2, p_2$ and $T_2$ at the exit of the combustor.

Solution:

$$T_1 = 1000^oR = 1000 \times 5/9 = 555.5K$$

$$q = 4.5 \times 10^8 ft - lb \quad per \quad slug = 4.5 \times 10^8 \times \frac{0.3048 \times 0.4536}{14.5936} = 4.26 \times 10^6 J/kg$$

When $M_1 = 0.2$, we have $T_{01}/T_1 = 1.008$ and $T_{01}/T^* = 0.1736$.
Thus

$$T_{01} = 1.008T_1 = 560K$$

and

$$T^* = T_{01}/0.1736 = 3225.8K$$

The heat released per unit mass is $q^{'} = 0.06 \times q/1.06 = 2.41 \times 10^5$ J/kg.
Thus

$$T_{02} = q^{'}/c_p + T_{01} = 2.41 \times 10^5/1005 + 560 = 800K.$$

and $T_{02}/T^* = 800/3225.8 = 0.2480$.
From appendix A3, we know $M_2 = 0.25$. For $M = 0.25, T_2/T^* = 0.2643, p_2/p^* = 0.2234$; For $M = 0.2, T_1/T^* = 0.2066, p_2/p^* = 2.235$.
Thus

$$T_2 = \frac{T_2}{T^*}\frac{T^*}{T_1}T_1 = 0.2643 \times \frac{1}{0.2066} \times 555.5 = 710.64K$$

$$p_2 = \frac{p_2}{p^*}\frac{p^*}{p_1}p_1 = 2.235 \times \frac{1}{2.273} \times 10 = 9.83atm.$$

3.10 For the inlet condition of Prob 3.9, calculate the maximum fuel-air ratio beyond which the flow will be chocked at the exit.

Solution:

When chocked, $M_2 = 1$. $q^{'} = c_p(T_{02} - T_{01})$.
From problem 3.9, we know $T_{01} = 560$ K and $T_{02} = T_{02}^* = T_{01}^* = 3225.8$K
Thus $q^{'} = 2.679 \times 10^6$ J/kg.
Let $x$ be the gas fuel ratio when the flow is chocked. Thus we have

$$q^{'} = \frac{xq}{1+x} \Rightarrow x = 0.0684$$

The maximum fuel to air ratio is 0.0684 (when the flow is chocked).

3.11 At the inlet to the combustor of a supersonic combustion ramjet (SCRAMjet, 冲压发动机，无需涡轮涡扇结构), the flow Mach number is supersonic. For a fuel-air ratio (by mass) of 0.03 and a combustor exit temperature of $4800^o R$, calculate the inlet Mach number above which the flow will be unchoked. Assuming one dimensional frictionless flow with $\gamma = 1.4$ with the heat release per slug of fuel equal to $4.5 \times 10^8 ft - lb$ (assuming one dimensional flow).

Solution:

The heat released per unit mass of fule is

$$q = \frac{0.03 \times 4.5 \times 10^8 \times 0.093 J/kg}{1.03} = 1.22 \times 10^6 J/kg$$

When the flow is chocked, we have the Mach number at the exit is M=1. Thus

$$T_{02} = 1.2 T_2 = 1.2 \times 4800 \times 5/9 = 3200K.$$

$$T_{02} = T_{02}^* = T_{01}^* = 3200K$$

$$T_{01} = T_{02} - q/c_p = 3200 - 1.22 \times 10^6/1004 = 1986K.$$

Thus $T_{01}/T_{01}^* = 0.62$. From appendix A3, we know $M_1$=0.3 or 3.4.
As the flow is supersonic, $M_1$ must be larger than 1. Thus $M_1 = 3.4$

3.12 Air is flowing through a pipe of 0.02-m inside diameter and 40-m length. The conditions at the exit of the pipe are $M_2 = 0.5$, $p_2 = 1atm$, and $T_2 = 270K$. Assuming adiabatic, one-dimensional, with a local friction coefficient of 0.005, calculate $M_1, p_1$ and,$T_1$.

Solution:

From appendix A4, for $M_2 = 0.5$,

$$\frac{4\bar{f}L_2^*}{D} = 1.069, \quad \frac{p_2}{p^*} = 2.138 \quad and \quad \frac{T_2}{T^*} = 1.143$$

Thus

$$\frac{4fL_1^*}{D} = \frac{4fL}{D} + \frac{4fL_2^*}{D} = \frac{4 \times 0.005 \times 40}{0.02} + 1.069 = 41.069$$

From appendix A4, we know for $\frac{4fL_1^*}{D} = 41.069$,

$$M_1 = 0.127, \frac{p_1}{p^*} = 9.116 \quad and \quad \frac{T_1}{T^*} = 1.197$$

Thus

$$p_1 = \frac{p_1}{p^*}\frac{p^*}{p_2}p_2 = 9.116 \times \frac{1}{2.138} \times 1 = 4.26atm$$

$$T_1 = \frac{T_1}{T^*}\frac{T^*}{T_2}T_2 = 1.179 \times \frac{1}{1.143} \times 270 = 282.8K$$

3.13 Consider the adiabatic flow of air through a pipe of 0.2-ft inside diameter and 3-ft length. The inlet flow conditions are $M_1 = 2.5$ and $p_1 = 0.5$ atm, and $T_1 = 520^oR$. Assuming the local friction coefficient equals a constant of 0.005, calculate the following flow conditions at the exit: $M_2, p_2, T_2$, and $p_{02}$.

Solution:

$T_1 = T_1 = 520^oR = 288.89K$

From appendix A4, for $M_1 = 2.5$,

$$\frac{4fL_1^*}{D} = 0.4320 \quad \frac{p_1}{p^*} = 0.2921 \quad and \quad \frac{T_1}{T^*} = 0.5333$$

Thus

$$\frac{4fL_2^*}{D} = \frac{4fL_1^*}{D} - \frac{4fL}{D} = 0.4320 - \frac{4 \times 0.005 \times 3}{0.2} = 0.132$$

From appendix A4, for $\frac{4fL_2^*}{D} = 0.132$, we have

$$M_2 = 1.5, \frac{p_2}{p^*} = 0.6065 \quad and \quad \frac{T_2}{T^*} = 0.8276$$

Thus

$$p_2 = \frac{p_2}{p^*}\frac{p^*}{p_1}p_1 = 0.6065 \times \frac{1}{0.2921} \times 0.5 = 1.04atm$$

$$T_2 = \frac{T_2}{T^*}\frac{T^*}{T_1}T_1 = 0.8276 \times \frac{1}{0.5333} \times 288.89 = 448.31K$$

From appendix A1, for $M = 1.5$, $p_{02}/p_2 = 3.671 \Rightarrow p_{02} = 3.671 \times p_2 = 3.82$ atm.

3.14 The stagnation chamber of a wind tunnel is connected to a high-pressure air bottle farm which is outside the laboratory building. The two are connected by a long pipe of 4-m inside diameter. If the static pressure ratio between the bottle farm and the stagnation chamber is 10, and the bottle farm static pressure is 100 atm, how long can the pipe be without choking? Assume adiabatic, subsonic, one-dimensional flow with a friction coefficient of 0.005.

Solution:

The static pressure of the stagnation chamber is $p_2 = 100/10 = 10$ atm.
When the pipe is chocked, the condition at the exit is $M_2 = 1$,

$$\frac{4fL_2^*}{D} = 0 \quad T_2 = T_2^* \quad p_2 = p_2^*$$

Let $L$ be the length of the pipe without choking. We have

$$\frac{4fL}{D} = \frac{4fL_1^*}{D} - \frac{4fL_2^*}{D}$$

The pressure ratio at the inlet is

$$\frac{p_1}{p_1^*} = \frac{p_1}{p_2^*} = 10$$

Thus the inlet Mach number is $M_1 = 0.1(p/p^* = 10.94)$ and $\frac{4fL_1^*}{D} = 66.92$. Thus we have

$$\frac{4fL}{D} = 66.92$$

with D=4 and f=0.005, we have $L$=13384 m.

3.16 Consider a Mach 2.5 flow of air entering a constant area duct. Heat is added to this flow in the duct; the amount of heat added is equal to 30 percent of the total enthalpy at the entrance to the duct. Calculate the Mach number at the exit of the duct. Comment on the fluid dynamic significance of this problem, where the exit Mach number does not depends on a number for the actual heat added, but rather only on the dimensionless ratio of heat added to the total enthalpy in the inflowing gas.

Solution:

Since

$$q = 0.3h_{01} = h_{02} - h_{01}$$

,

$$T_{02} = 1.3T_{01}.$$

For $M = 2.5$m, $\frac{T_{01}}{T_{01}^*} = 0.7101$, thus $\frac{T_{02}}{T_{02}^*} = \frac{T_{02}}{T_{01}}\frac{T_{01}}{T_{01}^*} = 1.3 \times 0.7101 = 0.9231$.
From appendix A3, we have $M_2$=1.44.
The results suggest that it is the ratio of the heat added per total enthalpy that determines the Mach number of the exit flow.