Solution to problems of CH9

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9.2 The low speed lift coefficient for an NACA 2412 airfoil at an angle of attack of 4^o is 0.65. Using the Prandtl -Glauert rule, calculate the lift coefficient for $M_\infty = 0.7$.

Solution:

According to the Prandtl-Glauert rule for subsonic flow

$$C_l = \frac{C_{l0}}{\sqrt{1 - M_\infty^2}}$$

Thus,

$$C_l = \frac{0.65}{\sqrt{1 - 0.7^2}} = 0.91$$

9.3 In low speed flow, the pressure coefficient at a point on an airfoil is -0.9. Calculate the value of C_p at the same point for $M_\infty = 0.6$ by means of:

(a) The Prandtl-Glauert rule

(b) Laitone's correction

(c) The Karman-Tsien rule

Solution:

(a) According to the Prandtl-Glauert rule for subsonic flow

$$C_p = \frac{C_{p0}}{\sqrt{1 - M_\infty^2}}$$

Thus

$$C_p = \frac{-0.9}{\sqrt{1 - 0.6^2}} = -1.125$$

.

(b) According to the Laitone's rule for subsonic flow

$$C_p = \frac{C_{p0}}{\sqrt{1 - M_\infty^2} + [M_\infty^2(1 + \frac{\gamma - 1}{2} M_\infty^2)/(2\sqrt{1 - M_\infty^2})]C_{p0}}$$

Thus

$$C_p = \frac{-0.9}{\sqrt{1 - 0.6^2} + [0.6^2 \times (1 + \frac{1.4-1}{2} \times 0.6^2)/(2\sqrt{1 - 0.6^2})] \times (-0.9)} = -1.544$$

.

(c) According to the Karman-Tsien's rule for subsonic flow

$$C_p = \frac{C_{p0}}{\sqrt{1 - M_\infty^2} + \frac{M_\infty^2}{\sqrt{1+M_\infty^2}} \frac{C_{p0}}{2}}$$

Thus

$$C_p = \frac{-0.9}{\sqrt{1 - 0.6^2} + (0.6^2/\sqrt{1 + 0.6^2}) \times (-0.45)} = -1.2676$$

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9.4 Consider a flat plat with chord length c at an angle of attack α to a supersonic free stream of Mach number M_∞. Let L and D be the lift and drag per unit span, $S = c(1)$. Using linearised theory, derive the following expression for the lift and drag coefficients (where $C_L \equiv \frac{L}{\frac{1}{2}\rho_\infty V_\infty^2 S}$ and $C_D \equiv \frac{D}{\frac{1}{2}\rho_\infty V_\infty^2 S}$):

(a)$C_L = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$

(b)$C_D = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}$

Solution:

for supersonic flow.

The respective lift and drag force for a plate with angle of attack α is

$$L = (p_2 \cos\alpha - p_1 \cos\alpha)S$$

and

$$D = (p_2 \sin\alpha - p_1 \sin\alpha)S$$

Thus the lift and drag coefficient are, respectively,

$$C_L = \frac{(p_2 \cos\alpha - p_1 \cos\alpha)S}{\frac{1}{2}\rho_\infty V_\infty^2 S} = (C_{p_2} - C_{p_1})\cos\alpha$$

and

$$C_D = \frac{(p_2 \sin\alpha - p_1 \sin\alpha)S}{\frac{1}{2}\rho_\infty V_\infty^2 S} = (C_{p_2} - C_{p_1})\sin\alpha$$

Consider linearised flow, i.e., α is small, we have $\cos\alpha = 1$ and $\sin\alpha = \alpha$, we have

$$C_L = C_{p_2} - C_{p_1}$$

and

$$C_D = (C_{p_2} - C_{p_1})\alpha$$

According to the Prandtl Glauert rule, $C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$, $C_{p_1} = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}}$ and $C_{p_2} = \frac{2\alpha}{\sqrt{M_\infty^2 - 1}}$, we have

$$C_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \qquad and \qquad C_D = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}$$

9.5 For the flat plate in Problem 9.4, the quarter-chord point is located, by definition, at a distance equal to $c/4$ from the leading edge. Using linearised theory, derive the following expression for the moment coefficient about the quarter-chord point for supersonic flow:

$$C_{M_{c/4}} = \frac{-\alpha}{\sqrt{M_\infty^2 - 1}}$$

where $C_{M_{c/4}} \equiv M_{c/4}/\frac{1}{2}\rho_\infty V_\infty^2 Sc$, and as usual in aeronautical practice, a positive moment by convention is in the direction of increasing angle of attack.

Solution:

The moment at $\frac{c}{4}$ can be calculated as following:

$$M_{c/4} = (-p_1\frac{c}{4} \times 1 + p_2\frac{c}{4} \times 1)\frac{c}{8} + (p_1\frac{3c}{4} \times 1 - p_2\frac{3c}{4} \times 1)\frac{3c}{8} = (p_1 - p_2)\frac{c^2}{4}$$

Hence, the moment coefficient about the quard-chord point is

$$C_{M_{c/4}} = \frac{(p_1 - p_2)\frac{c^2}{4}}{\frac{1}{2}\rho_\infty V_\infty^2 c \times 1 \times c}$$

$$= (C_{p_1} - C_{p_2}) \times \frac{1}{4}$$

According to the Prandtl-Glauert rule, $C_{p_1} = -\frac{2\alpha}{\sqrt{M_\infty^2-1}}$ and $C_{p_2} = \frac{2\alpha}{\sqrt{M_\infty^2-1}}$, we have

$$C_{M_{c/4}} = \frac{-\alpha}{\sqrt{M_\infty^2 - 1}}$$

9.7 Consider a diamond-shaped airfoil such as that sketched in Fig 4.35. The half angle is ϵ, thickness is t, and chord length is c. For supersonic flow, use linearized theory to derive the following expression for C_D at $\alpha = 0$.

$$C_D = \frac{4}{\sqrt{M_\infty^2-1}}(\frac{t}{c})^2$$

Solution:

The drag on the diamond airfoil with unit span is

$$D = (p_1 \sin\epsilon + p_3 \sin\epsilon - p_4 \sin\epsilon - p_2 \sin\epsilon) \times l \times 1 = (p_1 + p_3 - p_4 - p_2)\frac{t}{2}$$

with l being the length of one side of the wing.

The drag coefficient is then

$$C_D = \frac{D}{\frac{1}{2} r\rho_\infty V_\infty^2 S} = \frac{(p_1 + p_3 - p_4 - p_2)\frac{t}{2}}{\frac{1}{2} r\rho_\infty V_\infty^2 c \times 1} = (C_{p_1} + C_{p_3} - C_{p_4} - C_{p_2})\frac{t}{2c}$$

Recall with linearised flow, we have

$$C_{p_1} = \frac{2\epsilon}{\sqrt{M_\infty^2 - 1}}, \quad C_{p_2} = -\frac{2\epsilon}{\sqrt{M_\infty^2 - 1}}, \quad C_{p_3} = \frac{2\epsilon}{\sqrt{M_\infty^2 - 1}} \quad and \quad C_{p_4} = -\frac{2\epsilon}{\sqrt{M_\infty^2 - 1}}$$

Thus,

$$C_D = \frac{4\epsilon}{\sqrt{M_\infty^2 - 1}}\frac{t}{c}$$

From the geometrical relation, we know that $\epsilon = \tan\epsilon = \frac{t}{c}$, hence we have

$$C_D = \frac{4}{\sqrt{M_\infty^2 - 1}}(\frac{t}{c})^2$$

9.11 At $\alpha = 0^o$, the minimum pressure coefficient for an NACA 0009 airfoil in low-speed flow is -0.25. Calculate the critical Mach number for this airfoil using

(a) The Prandtl-Glauert rule.

(b) The (more accurate) Karman-Tsien rule.

Solution: The critical pressure coefficient is given by

$$C_{p_{cr}} = \frac{1}{\gamma M_\infty^2}[(\frac{1 + \frac{\gamma-1}{2}M_\infty^2}{1 + \frac{\gamma-1}{2}})^{\frac{\gamma}{\gamma-1}} - 1] \tag{1}$$

(a) The Prandtl-Glauert correction for an airfoil is given by

$$C_p = \frac{C_{p0}}{\sqrt{1 - M_\infty^2}} \tag{2}$$

When the flow reaches the critical Mach number, we have $C_p = C_{p_{cr}}$. Thus,

$$\frac{C_{p0}}{\sqrt{1 - M_{cr}^2}} = \frac{1}{\gamma M_{cr}^2}[(\frac{1 + \frac{\gamma-1}{2}M_{cr}^2}{1 + \frac{\gamma-1}{2}})^{\frac{\gamma}{\gamma-1}} - 1] \tag{3}$$

(b) The Karman-Tsien correction states

$$C_p = \frac{C_{p0}}{\sqrt{1 - M_\infty^2} + \frac{M_\infty^2}{\sqrt{1+M_\infty^2}}\frac{C_{p0}}{2}} \tag{4}$$

Let equation (4) equals to equation (1), we have

$$\frac{1}{\gamma M_{cr}^2}[(\frac{1+\frac{\gamma-1}{2}M_{cr}^2}{1+\frac{\gamma-1}{2}})^{\frac{\gamma}{\gamma-1}}-1] = \frac{C_{p0}}{\sqrt{1-M_{cr}^2}+\frac{M_{cr}^2}{\sqrt{1+M_{cr}^2}}\frac{C_{p0}}{2}} \quad (5)$$

Substitute $C_{p0} = -0.25$ into equation (3) and (5), solve the resultant equation with M_{cr}, one can have $M_{cr} = 0.8035$ for Prandtl-Glauert rule and $M_{cr} = 0.795$ for the Karman-Tsien rule. Equations (3) and (5) are solved graphically. See the graph below.