Problems of Ch3 Part 1

October 10, 2023

Relation between the units:

1 ft=0.3048m;1lb=0.454 kg; 1lb/ft^2=47.89N/m^2=47.89 Pa; 1^{o}R=5/9K
Due October 8^{th}, 2023

3.1 At a given point in the high-speed flow over an airplane wing, the local Mach number, pressure and temperature are 0.7, 0.9 atm and 250 K, respectively. Calculate the values of p_0,T_0,p^*,T^*, and a^* at this point.
Solution:
From isentropic relation

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2 \quad \frac{\rho_0}{\rho} = (1+\frac{\gamma-1}{2}M^2)^{\frac{1}{\gamma-1}} \quad and \quad \frac{p_0}{p} = (1+\frac{\gamma-1}{2}M^2)^{\gamma/(\gamma-1)}$$

Substitute $\gamma = 1.4$ and $M = 0.7$ into the above equations, we have

$$T_0 = 1.098T = 274.5K \quad p_0 = 1.387p = 1.2483atm$$

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The characteristic condition is given by setting $M = M^* = 1$ in the above equations. We have

$$T^* = T_0/1.2 = 228.75K \quad p* = p_0/1.893 = 0.659atm \quad and \quad a^* = \sqrt{\gamma RT^*} = 303.17m/s.$$

3.2 At a given point in a supersonic wind tunnel, the pressure and temperature are 5×10^4 N/m^2 and 200 K, respectively. The total pressure at this point is 1.5×10^6 N/m^2. Calculate the local Mach number and total temperature.
Solution:
$p_0/p = 1.5\times10^6/5\times10^4 = 30.$
Substitute $p_0/p = 30$ into $\frac{p_0}{p} = (1+\frac{\gamma-1}{2}M^2)^{\gamma/(\gamma-1)}$ with $\gamma = 1.4$, we have M=2.875.
Substitute M=2.875 into $\frac{T_0}{T} = 1+\frac{\gamma-1}{2}M^2$, we have $T_0/T = 2.653 \Rightarrow T_0 = 2.653T = 530.6$ K.

3.3 At a point in the flow over a high speed missile, the local velocity and temperature are 3000 ft/s and 500^oR, respectively. Calculate the Mach number and the characteristic Mach number M^* at this point.
Solution:
3000 ft/s=914.4 m/s and 500^{o}R=277.8K

$$M = \frac{V}{a} = \frac{V}{\sqrt{\gamma RT}} = \frac{914.4}{334} = 2.74.$$

$$T^* = \frac{T^*}{T_0}\frac{T_0}{T}T = \frac{1}{1.2}\times2.458\times277.8 = 569K$$

$$M^* = \frac{V}{\sqrt{\gamma RT^*}} = \frac{914.4}{\sqrt{1.4\times287\times569}} = 1.912$$

3.4 Consider a normal shock wave in air. The upstream conditions are given by M_1=3, p_1 = 1 atm, and ρ_1 =1.23kg/m^3. Calculate the downstream values of $p_2, T_2, \rho_2, M_2, u_2, p_{02}$, and T_{02}.

Solution:

At $M_1 = 3$, we have

$$\frac{p_2}{p_1} = 10.33, \quad \frac{T_2}{T_1} = 2.679, \quad \frac{\rho_2}{\rho_1} = 3.857, \quad ,\frac{p_{02}}{p_{01}} = 0.3283, \quad and \quad M_2 = 0.4752$$

Thus

$$p_2 = 10.33 p_1 = 10.33 atm$$
$$T_2 = 2.679 T_1 = 2.679(p_1/\rho_1 R) = 2.679 \times 1.01 \times 10^5/(1.23 \times 287) = 766.49K$$
$$\rho_2 = 3.857\rho_1 = 3.857 \times 1.23 = 4.744 kg/m^3$$
$$u_2 = M_2 \times a_2 = M_2 \times \sqrt{\gamma R T_2} = 0.4752 \times \sqrt{1.4 \times 287 \times 766.49} = 263.71 m/s$$
$$p_{02} = 0.3283 p_{01} = 0.3283 \times 36.73 \times p_1 = 12.06 atm$$
$$T_{02} = \frac{T_{02}}{T_2}\frac{T_2}{T_1}T_1 = 1.046 \times 2.679 \times 286.11 = 801.7K$$

3.5 Consider a Pitot static tube mounted on the nose of an experimental airplane. A Pitot tube measures the total pressure at the tip of the probe (hence sometimes called the Pitot pressure), and a Pitot static tube combines this with a simultaneous measurement of the free stream static pressure. The Pitot and free-stream static measurements are given below for three different flight conditions. Calculate the free stream Mach number at which the airplane is flying for each of the three different conditions:

a. Pitot pressure=$1.22 \times 10^5 N/m^2$, static pressure=1.01×10^5 N/m^2;

b. Pitot pressure=7222lb/ft^2, static pressure=2116 lb/ft^2;

c. Pitot pressure=13107lb/ft^2, static pressure=1020 lb/ft^2.

Solution:

For M=1, $p_{01}/p_1 = 1.893$. If the pressure ratio measured by the Pitot tube is larger than 1.893, then there will be a normal shock in front of the Pitot tube. The pressure measured by the Pitot tube is then the total pressure behind the normal shock.

(a) $\frac{p_{02}}{p_1} = 1.22 \times 10^5/(1.01 \times 10^5) = 1.21 < 1.893$. There is no normal shock. From isentropic relation, we know M=0.53.

(b) $\frac{p_{02}}{p_1} = 7222/2116 = 3.413 > 1.893$. A normal shock will form in front of the Pitot tube. From Rayleigh-Pitot relation, we know $M = 1.5$.

(c) $\frac{p_{02}}{p_1} = 13107/2116 = 12.762 > 1.893$. A normal shock will form in front of the Pitot tube. From Rayleigh-Pitot relation, we know $M = 3.1$

3.6 Consider compression of air by means of (a) a shock compression and (b) isentropic compression. Starting from the same initial condition of p_1 and v_1, plot to scale the pv diagram for both compression processes on the same graph. From the comparison, what can you say about the effectiveness of shock versus isentropic compression?

Solution:

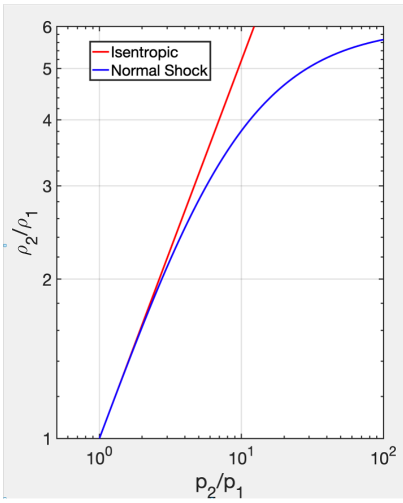
For isentropic conpression, we have

$$\frac{p_2}{p_1} = (\frac{\rho_2}{\rho_1})^1.4$$

For normal shock compression, we have the Hugoniot equation, i.e.,

$$\frac{p_1}{p_2} = \frac{(\frac{\gamma+1}{\gamma-1})\frac{\rho_2}{\rho_1} - 1}{(\frac{\gamma+1}{\gamma-1}) - \frac{\rho_2}{\rho_1}}$$

The two relations are plotted in the figure below. It is seen that for small $\frac{\rho_2}{\rho_1}$ the two methods are the same. For larger $\frac{\rho_2}{\rho_1}$, normal shock compression is less efficient than isentropic compression.



3.7 During the entry of the Apollo vehicle into the Earth's atmosphere, the Mach number at a given point on the trajectory was Ma=38 and the atmosphere temperature was 270K. Calculate the temperature at the stagnation point of the vehicle, assuming a calorically perfect gas with $\gamma = 1.4$. Do you think this is an accurate calculation? If not, why? If not, is your answer an overestimate or underestimate?

Solution:

The stagnation point temperature is the total temperature.

Assume isentropic process, we have $T_0 = (1 + \frac{\gamma-1}{2}M^2)T = (1 + 0.2 \times 38^2) \times 270 = 78246$ K. The calculation is overestimated. As the Mach number is much larger than 1, the flow is hypersonic flow. Due to the high temperature outside the vehicle, the air will be chemically reacting, which will reduce the heat capacity ratio γ. In addition, the presence of shock waves will further alter the temperature.