第四章 单端口网络与多端口网络

- 4.1 基本定义
- 4.2 互联网络
- 4.3 网络特性及其应用
- 4.4 散射参量 （S参数）

4 单端口和多端口网络

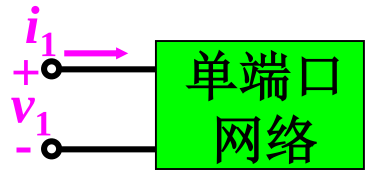
网络模型的作用

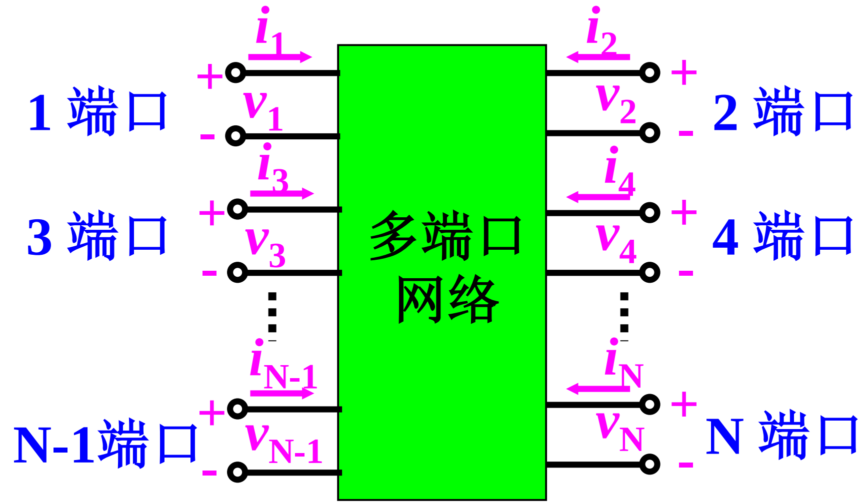
- ✓ 网络模型可以大量减少无源和有源器件数目；
- ✓ 避开电路的复杂性和非线性效应；
- ✓ 简化网络输入和输出特性的关系；
- ✓ 可以通过实验确定网络输入和输出参数，而不必了解系统内部的结构。

典型的网络

单口网络	负载，振荡器…
双口网络	滤波器、放大器、衰减器、隔离器…
多口网络	混频器、功分器、环行器、合成器…

4.1 基本定义





$$v_1 = Z_{11}i_1 + Z_{12}i_2 + \cdots + Z_{1N}i_N$$

$$v_2 = Z_{21}i_1 + Z_{22}i_2 + \cdots + Z_{2N}i_N$$

$$\vdots$$

$$v_N = Z_{N1}i_1 + Z_{N2}i_2 + \cdots + Z_{NN}i_N$$

$$\begin{Bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{Bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{Bmatrix}$$

$$\begin{Bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{Bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{Bmatrix}$$

$$Z_{nm} = \frac{v_n}{i_m}\bigg|_{i_k=0(\,for\,k\neq m)}$$

端口参量：[V]、[I]

网络参量：[Z]、[Y]

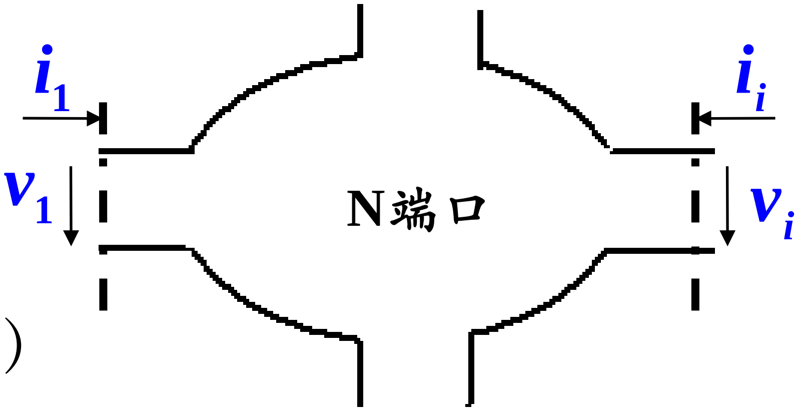
$$Y_{nm} = \frac{i_n}{v_m}\bigg|_{v_k=0(k\neq m)}$$

4.1 基本定义

$$Z_{nm}=\left.\frac{v_n}{i_m}\right|_{i_k=0\,(for\ k\neq m)}$$

其它端口开路

m端口到n端口的转移阻抗（互阻抗）



$$Z_{ii}=\left.\frac{v_i}{i_i}\right|_{i_k=0,\,k\neq i}$$

其它端口开路，

i端口的输入阻抗（自阻抗）

$$[Z]=[Y]^{-1}$$

$$Y_{nm}=\left.\frac{i_n}{v_m}\right|_{v_k=0,\,k\neq m}$$

其它端口短路，

m端口到n端口的转移导纳（互导纳）

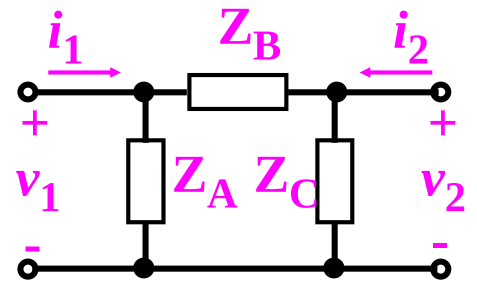
$$Y_{ii}=\left.\frac{i_i}{v_i}\right|_{v_k=0,\,k\neq i}$$

其它端口短路，

i端口的输入导纳（自导纳）

4.1 基本定义

例4.1 求π形网络的阻抗矩阵和导纳矩阵。



解：

$$Z_{11}=\frac{v_1}{i_1}\Big|_{i_2=0}=\frac{Z_A(Z_B+Z_C)}{Z_A+Z_B+Z_C}\qquad Z_{12}=\frac{v_1}{i_2}\Big|_{i_1=0}=\frac{Z_AZ_C}{Z_A+Z_B+Z_C}$$

$$Z_{21}=\frac{v_2}{i_1}\Big|_{i_2=0}=\frac{Z_AZ_C}{Z_A+Z_B+Z_C}\qquad Z_{22}=\frac{v_2}{i_2}\Big|_{i_1=0}=\frac{Z_C(Z_A+Z_B)}{Z_A+Z_B+Z_C}$$

$$Y_{11}=\frac{i_1}{v_1}\Big|_{v_2=0}=\frac{1}{Z_A}+\frac{1}{Z_B}\qquad Y_{12}=\frac{i_1}{v_2}\Big|_{v_1=0}=-\frac{1}{Z_B}$$

$$Y_{21}=\frac{i_2}{v_1}\Big|_{v_2=0}=-\frac{1}{Z_B}\qquad Y_{22}=\frac{i_2}{v_2}\Big|_{v_1=0}=\frac{1}{Z_B}+\frac{1}{Z_C}$$

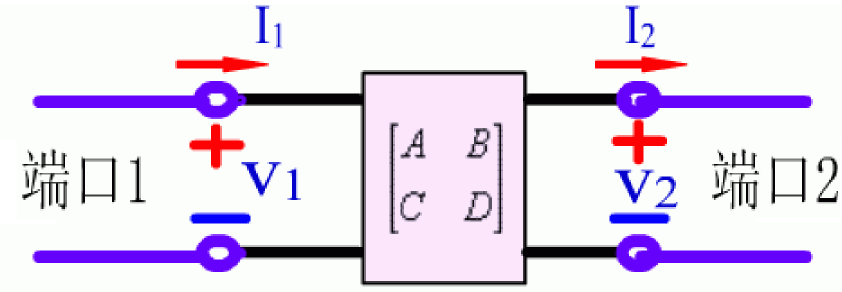
结论：通过假设网络端口为开路或短路状态，容易测得全部参数，且互易。

4.1 基本定义

转移参量 [A]

A矩阵

$$\begin{cases} v_1 = Av_2 + Bi_2 \\ i_1 = Cv_2 + Di_2 \end{cases} \qquad \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$



电流i_2的方向?

$$a = \left.\frac{v_1}{v_2}\right|_{i_2=0}$$ 端口2开路，2端口到1端口的电压转移系数

$$d = \left.\frac{i_1}{i_2}\right|_{v_2=0}$$ 端口2短路，2端口到1端口的电流转移系数

$$b = \left.\frac{v_1}{i_2}\right|_{v_2=0}$$ 端口2短路，2端口到1端口的转移阻抗

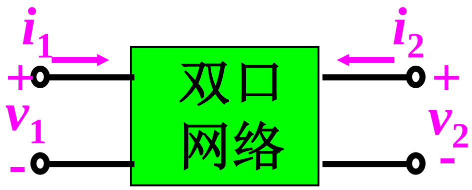
$$c = \left.\frac{i_1}{v_2}\right|_{i_2=0}$$ 端口2开路，2端口到1端口的转移导纳

4.1 基本定义

混合参量 [H]

H矩阵

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$



$$h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2=0}$$ 端口2短路，端口1的输入阻抗

$$h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1=0}$$ 端口1开路，端口2的输入导纳

$$h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1=0}$$ 端口1开路，2端口到1端口的电压传输系数（反馈）

$$h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2=0}$$ 端口2短路，1端口到2端口的电流传输系数（放大）

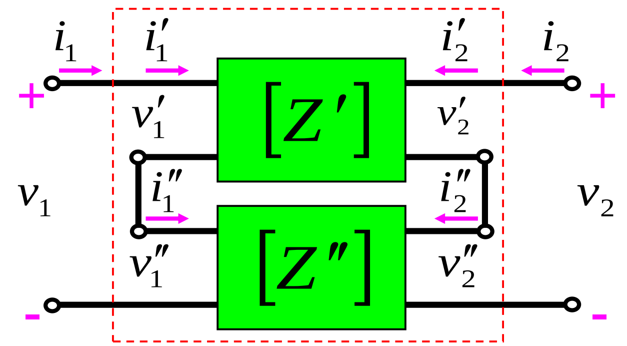
4.2 互联网络

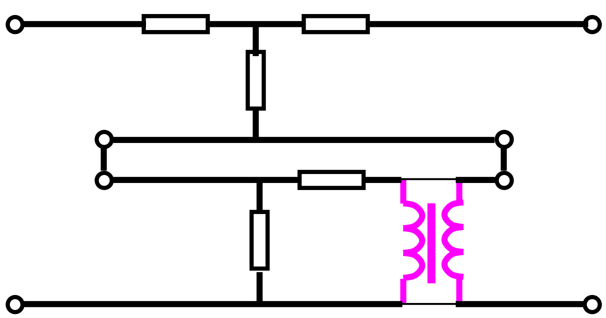
4.2.1 网络的串联

每个电压相互叠加而电流不变则用Z参数:

$$\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{bmatrix} v_1' + v_1'' \\ v_2' + v_2'' \end{bmatrix} = [Z]\begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix}$$

$$[Z] = [Z'] + [Z''] = \begin{bmatrix} Z_{11}' + Z_{11}'' & Z_{12}' + Z_{12}'' \\ Z_{21}' + Z_{21}'' & Z_{22}' + Z_{22}'' \end{bmatrix}$$

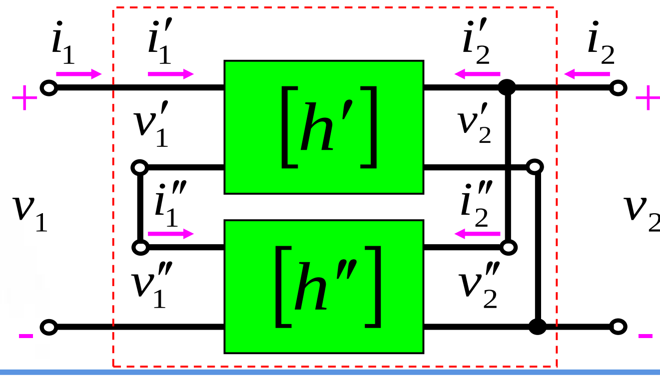




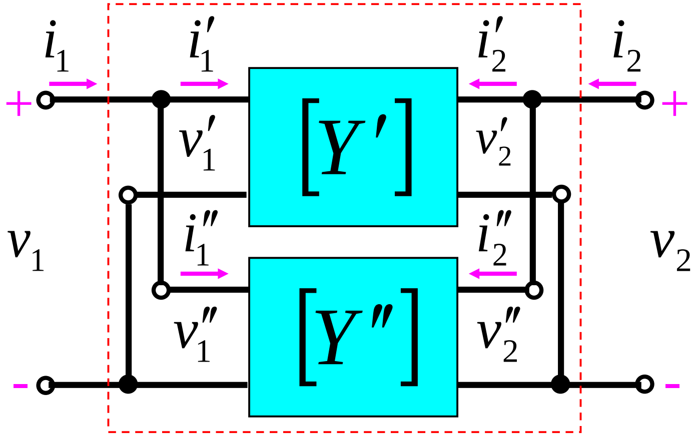
必须注意防止不加选择地将不同网络相连。

若输入电压及输出电流叠加，而输入电流及输出电压不变则用 h 参数:

$$\begin{Bmatrix} v_1 \\ i_2 \end{Bmatrix} = \begin{Bmatrix} v_1' + v_1'' \\ i_2' + i_2'' \end{Bmatrix} = \begin{bmatrix} h_{11}' + h_{11}'' & h_{12}' + h_{12}'' \\ h_{21}' + h_{21}'' & h_{22}' + h_{22}'' \end{bmatrix}\begin{Bmatrix} i_1 \\ v_2 \end{Bmatrix}$$



4.2.2 网络的并联

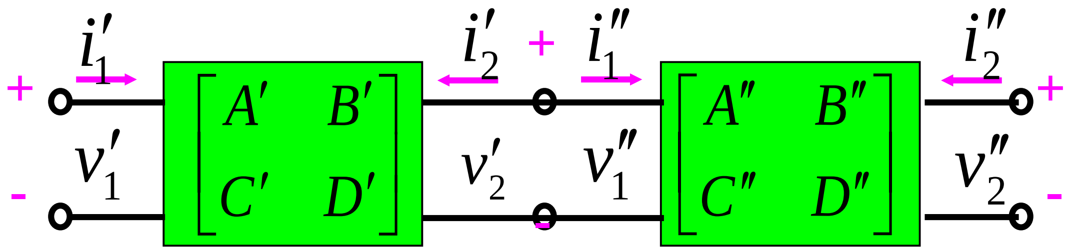


每个电流相互叠加而电压不变
则用Y参数：

$$\begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix} = \begin{Bmatrix} i_1' + i_1'' \\ i_2' + i_2'' \end{Bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

$$[Y] = [Y'] + [Y'']$$

4.2.3 级联网络

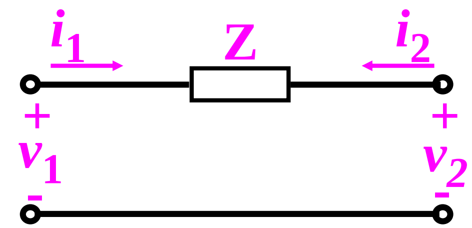


$$\begin{Bmatrix} v_1 \\ i_1 \end{Bmatrix} = \begin{Bmatrix} v_1' \\ i_1' \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} v_2' \\ -i_2' \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} v_1'' \\ i_1'' \end{Bmatrix}$$

$$= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{Bmatrix} v_1'' \\ -i_2'' \end{Bmatrix}$$

A参数特别适合级连网络

例4.2 求阻抗元件的ABCD参量。P105

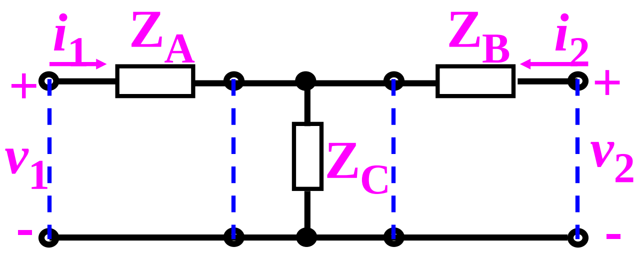


解：

$$A=\left.\frac{v_1}{v_2}\right|_{i_2=0}=1 \qquad B=\left.\frac{v_1}{-i_2}\right|_{v_2=0}=Z$$

$$C=\left.\frac{i_1}{v_2}\right|_{i_2=0}=0 \qquad D=\left.\frac{i_1}{-i_2}\right|_{v_2=0}=1$$

例4.3 求 T 形网络的ABCD参量。

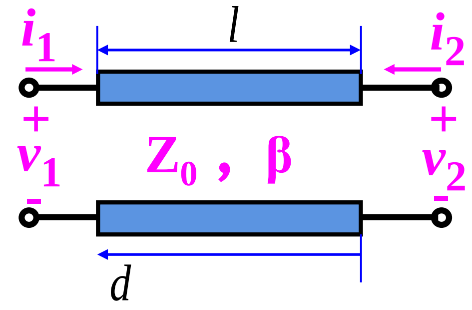


解：

$$[ABCD]=\begin{bmatrix}1 & Z_A\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ Z_C^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & Z_B\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+Z_A/Z_C & Z_A+Z_B+Z_AZ_B/Z_C\\ 1/Z_C & 1+Z_B/Z_C\end{bmatrix}$$

例4.4 求传输线段的A参量。P106



解： 当端口2 开路时，在端口1有

$$V(d)=2V^{+}\cos(\beta d),\ I(d)=\frac{2jV^{+}}{Z_0}\sin(\beta d)$$

当端口2 短路时，在端口1有

$$V(d)=2jV^{+}\sin(\beta d),\ I(d)=\frac{2V^{+}}{Z_0}\cos(\beta d)$$

$I(d)$流向负载，

$i_1=I(d),\ i_2=-I(d)$

$$A=\frac{v_1}{v_2}\bigg|_{i_2=0}=\frac{2V^{+}\cos(\beta l)}{2V^{+}}=\cos(\beta l)$$

$$C=\frac{i_1}{v_2}\bigg|_{i_2=0}=\frac{2jV^{+}\sin(\beta l)/Z_0}{2V^{+}}=j\sin(\beta l)/Z_0$$

$$B=\frac{v_1}{-i_2}\bigg|_{v_2=0}=\frac{2jV^{+}\sin(\beta l)}{2V^{+}/Z_0}=jZ_0\sin(\beta l)$$

$$D=\frac{i_1}{-i_2}\bigg|_{v_2=0}=\frac{2V^{+}\cos(\beta l)/Z_0}{2V^{+}/Z}=\cos(\beta l)$$

4.3.1

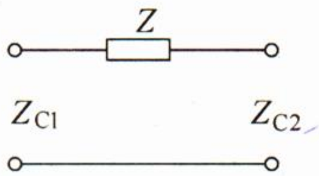
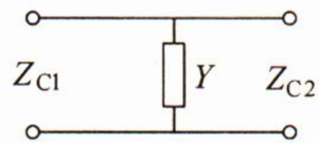
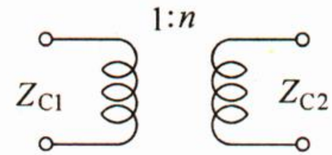
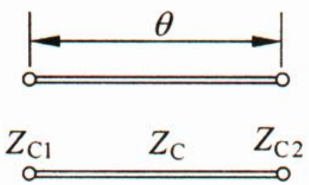
P109

表 4.2　不同网络参量之间的变换关系

	[Z]	[Y]	[h]	[A]
[Z]	$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$
[Y]	$\begin{bmatrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$	$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$	$\begin{bmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{bmatrix}$
[h]	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$
[A]	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta ABCD}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & -\frac{\Delta ABCD}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta ABCD}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

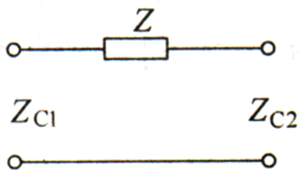
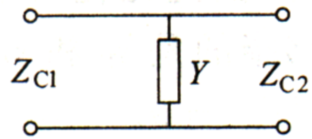
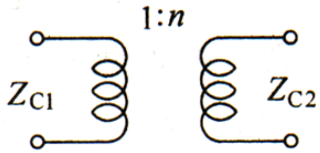
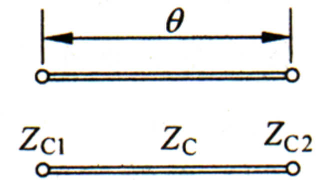
4.3 网络特性及其应用

表 4.2　二端口等效单元电路的阻抗矩阵、导纳矩阵和转移矩阵

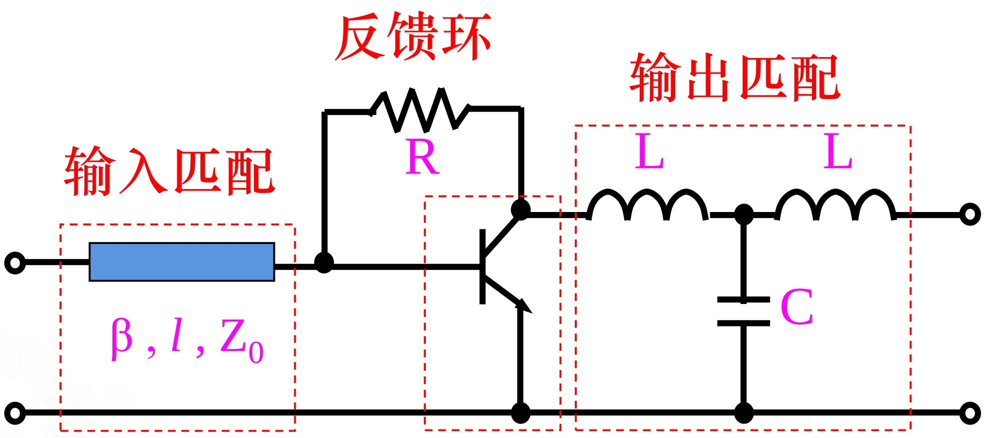
单元电路				
[Z]		$\begin{bmatrix} \frac{1}{Y} & \frac{1}{Y} \\ \frac{1}{Y} & \frac{1}{Y} \end{bmatrix}$		$\begin{bmatrix} -jZ_C\cot\theta & jZ_C\csc\theta \\ -jZ_C\csc\theta & -jZ_C\cot\theta \end{bmatrix}$
[z]		$\begin{bmatrix} \frac{1}{y} & \frac{1}{y\sqrt{r}} \\ \frac{1}{y\sqrt{r}} & \frac{1}{yr} \end{bmatrix}$		$\begin{bmatrix} -j\cot\theta & -j\csc\theta \\ -j\csc\theta & -j\cot\theta \end{bmatrix}$
[Y]	$\begin{bmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{bmatrix}$			$\begin{bmatrix} -\frac{j}{Z_C}\cot\theta & \frac{j}{Z_C}\csc\theta \\ \frac{j}{Z_C}\csc\theta & -\frac{j}{Z_C}\cot\theta \end{bmatrix}$

4.3 网络特性及其应用

续表

单元电路				
[y]	$\begin{bmatrix} \frac{1}{z} & -\frac{\sqrt{r}}{z} \\ -\frac{\sqrt{r}}{z} & \frac{r}{z} \end{bmatrix}$			$\begin{bmatrix} -\mathrm{j}\cot\theta & \mathrm{j}\csc\theta \\ \mathrm{j}\csc\theta & -\mathrm{j}\cot\theta \end{bmatrix}$
[A]	$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$	$\begin{bmatrix} \pm\frac{1}{n} & 0 \\ 0 & \pm n \end{bmatrix}$	$\begin{bmatrix} \cos\theta & \mathrm{j}Z_C\sin\theta \\ \frac{\mathrm{j}}{Z_C}\sin\theta & \cos\theta \end{bmatrix}$
[$\bar{a}$]	$\begin{bmatrix} \sqrt{r} & \frac{z}{\sqrt{r}} \\ 0 & \frac{1}{\sqrt{r}} \end{bmatrix}$	$\begin{bmatrix} \sqrt{r} & 0 \\ y\sqrt{r} & \frac{1}{\sqrt{r}} \end{bmatrix}$	$\begin{bmatrix} \pm\frac{\sqrt{r}}{n} & 0 \\ 0 & \pm\frac{n}{\sqrt{r}} \end{bmatrix}$	$\begin{bmatrix} \cos\theta & \mathrm{j}\sin\theta \\ \mathrm{j}\sin\theta & \cos\theta \end{bmatrix}$
说明：	$z=Z/Z_{C1}$ $r=Z_{C2}/Z_{C1}$	$y=YZ_{C1}$ $r=Z_{C2}/Z_{C1}$	$r=Z_{C2}/Z_{C1}$	$Z_{C1}=Z_{C2}=Z_C$

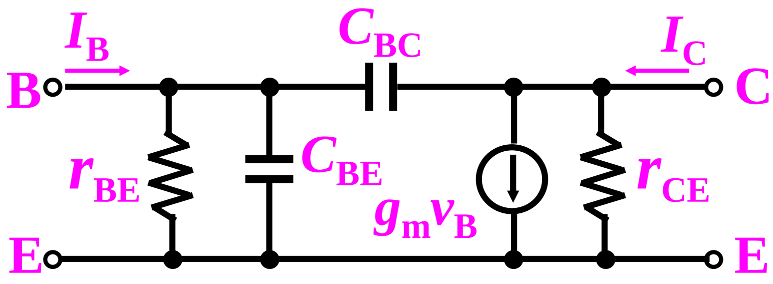
4.3.2 微波放大器分析



分析思路:

- 将h 参量变换为Y参量与反馈环并联
- 变换为A参量与匹配网络级连。

4.3 网络特性及其应用



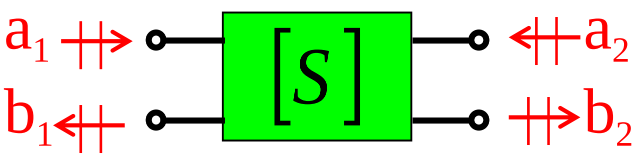
$$h_{11} = \frac{v_1}{i_1}\bigg|_{v_2=0} = \frac{v_1}{v_1\left(1/r_{BE} + j\omega C_{BE} + j\omega C_{BC}\right)} = \frac{r_{BE}}{1 + j\omega\left(C_{BE} + C_{BC}\right)r_{BE}}$$

$$h_{12} = \frac{v_1}{v_2}\bigg|_{i_1=0} = \frac{r_{BE}/\left(1 + j\omega r_{BE}C_{BE}\right)}{1/j\omega C_{BC} + r_{BE}/\left(1 + j\omega r_{BE}C_{BE}\right)} = \frac{j\omega C_{BC}r_{BE}}{1 + j\omega\left(C_{BE} + C_{BC}\right)r_{BE}}$$

$$h_{21} = \frac{i_2}{i_1}\bigg|_{v_2=0} = \frac{g_m v_1 - j\omega C_{BC}v_1}{v_1\left(1/r_{BE} + j\omega C_{BE} + j\omega C_{BC}\right)} = \frac{r_{BE}\left(g_m - j\omega C_{BC}\right)}{1 + j\omega\left(C_{BE} + C_{BC}\right)r_{BE}}$$

$$h_{22} = \frac{i_2}{v_2}\bigg|_{i_1=0} = \frac{1}{r_{CE}} + \frac{g_m j\omega C_{BC}r_{BE}}{1 + j\omega\left(C_{BE} + C_{BC}\right)r_{BE}} + \frac{j\omega C_{BC}\left(1 + j\omega C_{BE}r_{BE}\right)}{1 + j\omega\left(C_{BE} + C_{BC}\right)r_{BE}}$$

4.4.1 S 参量的定义



定义归一化入射电压波：$a_n = \dfrac{V_n + Z_0 I_n}{2\sqrt{Z_0}}$

定义归一化反射电压波：$b_n = \dfrac{V_n - Z_0 I_n}{2\sqrt{Z_0}}$

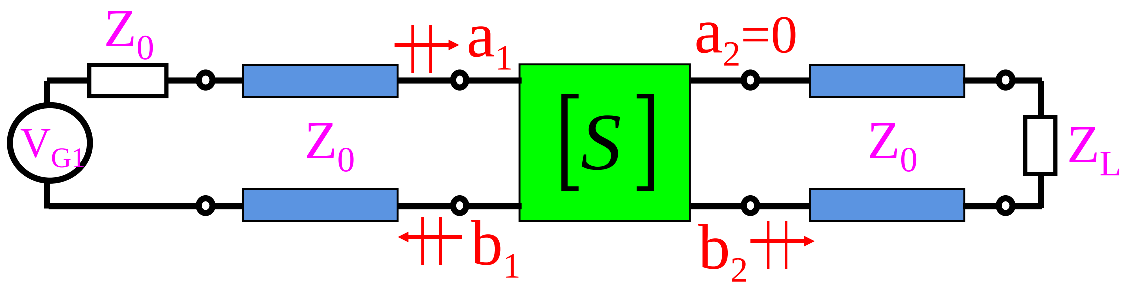
相加：$V_n = (a_n + b_n)\sqrt{Z_0}$

相减：$I_n = (a_n - b_n)/\sqrt{Z_0}$

所以：$a_n = V_n^+/\sqrt{Z_0} = I_n^+\sqrt{Z_0}$　　$b_n = V_n^-/\sqrt{Z_0} = -I_n^-\sqrt{Z_0}$

定义S 参量：$\begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$　其中：$S_{ij} = \dfrac{b_i}{a_j}\Big|_{a_n=0(n\neq j)}$

4.4.2 S 参量的物理意义

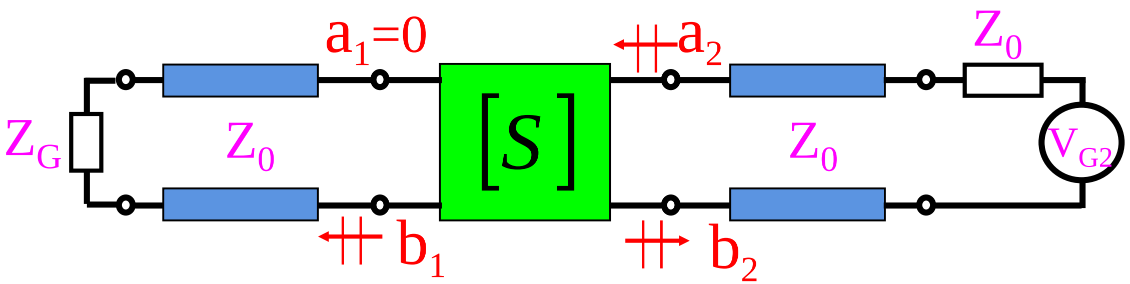


测量S_{11}和S_{21}, 为保证a_2= 0，必须使 $Z_L=Z_0$

则：$$S_{11}=\frac{b_1}{a_1}\bigg|_{a_2=0}=\frac{V_1^-}{V_1^+}=\Gamma_{in}=\frac{Z_{in}-Z_0}{Z_{in}+Z_0}$$

$$S_{21}=\frac{b_2}{a_1}\bigg|_{a_2=0}=\frac{V_2^-/\sqrt{Z_0}}{(V_1+Z_0I_1)/(2\sqrt{Z_0})}\bigg|_{I_2^+=V_2^+=0}=\frac{2V_2^-}{V_{G1}}=\frac{2V_2}{V_{G1}}$$ 正向电压增益

4.4.2 S 参量的物理意义

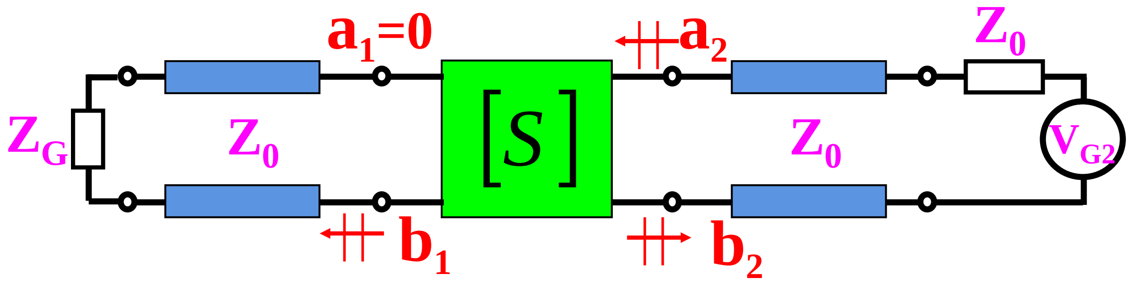


测量S_{22}和S_{12}, 为保证$a_1= 0$，必须使 $Z_G=Z_0$

则：$$S_{22}=\frac{b_2}{a_2}\bigg|_{a_1=0}=\frac{V_2^-}{V_2^+}=\Gamma_{out}=\frac{Z_{out}-Z_0}{Z_{out}+Z_0}$$

$$S_{12}=\frac{b_1}{a_2}\bigg|_{a_1=0}=\frac{V_1^-/\sqrt{Z_0}}{\left(V_2+Z_0I_2\right)/\left(2\sqrt{Z_0}\right)}\Bigg|_{I_1^+=V_1^+=0}=\frac{2V_1^-}{V_{G2}}=\frac{2V_1}{V_{G2}}$$ 反向电压增益

4.4.2 S 参量的物理意义



常用S参数名称

S11

前向反射系数

- 输入回波损耗
- 输入匹配
- VSWR

S21

前向传输系数

- 增益
- 损耗

S12

反向传输系数

- 反向隔离度

S22

反向反射系数

- 输出回波损耗
- 输出匹配

VSWR

4.4.2 S 参量的物理意义

电压驻波比（VSWR）与S参数关系

电压驻波比表示在端接任意负载的情况下，传输线Z0上可以测量到的最大电压与最小电压之比（驻波波峰电压与波谷电压的比值）。

对于输入端口 VSWR (s_{in}) 表示为

$$s_{in} = \frac{1+|S_{11}|}{1-|S_{11}|}$$

对于输出端口VSWR (s_{out})表示为

$$s_{out} = \frac{1+|S_{22}|}{1-|S_{22}|}$$

增益与S参数关系

网络增益与S参数之间关系如下

$$G = S_{21} = \frac{b_2}{a_1}$$

取标量可得

$$|G| = |S_{21}|$$

取对数表示则如下

$$g = 20\log_{10}|S_{21}| \text{ dB}$$

4.4.2 S 参量的物理意义

回波损耗（Return Loss）与S参数关系

$$RL(\text{dB}) = 10\log_{10}\frac{P_{\rm i}}{P_{\rm r}}$$

其中$P_{\rm i}$ 为输入功率，$P_{\rm r}$ 为反射功率。

对于二端口网络输入端口回波损耗为

$$RL_{\rm in} = 10\log_{10}\left|\frac{1}{S_{11}^2}\right| = -20\log_{10}|S_{11}| \quad \text{dB}$$

输出端口回波损耗为

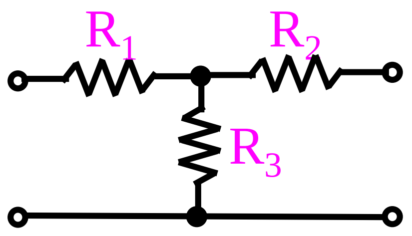
$$RL_{\rm out} = -20\log_{10}|S_{22}| \quad \text{dB}$$

插入损耗（Insert Loss）与S参数关系

$$IL = -20\log_{10}|S_{21}| \quad \text{dB.}$$

4.4.2 S 参量的物理意义

例4.5 假设一3dB衰减网络插入到 $Z_0 = 50\Omega$的传输线中，求该网络的S 参量和电阻。**P115**

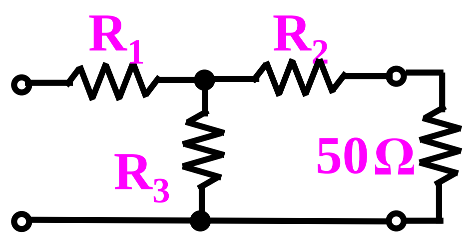


解： 网络匹配、对称： ⇨ $S_{11} = S_{22} = 0$

$R_1 = R_2 \quad V_1 = V_1^+ \quad V_2 = V_2^-$

$$Z_{in} = R_1 + R_3 \| (R_2 + 50) = R_1 + \frac{R_3(R_2+50)}{R_3+R_2+50} = 50\Omega$$

$$V_2 = \left(\frac{R_3 \| (R_2+50)}{R_1 + R_3 \| (R_2+50)} \right) \frac{50}{R_2+50} V_1$$

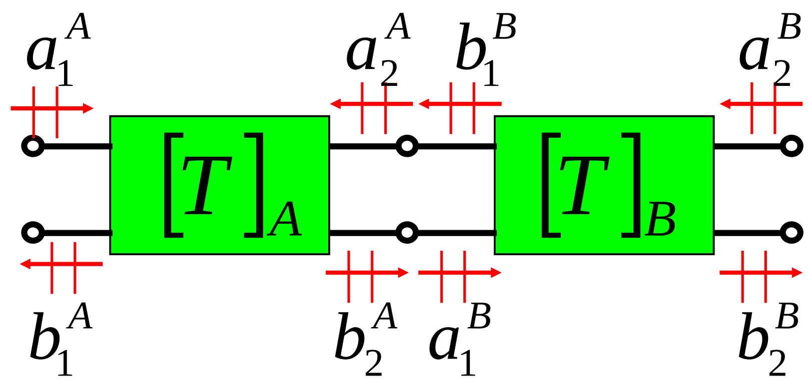


S_{11}和S_{21}测试电路

3dB衰减 ⇨ $S_{21} = \frac{V_2^-}{V_1^+} = \frac{V_2}{V_1} = \frac{1}{\sqrt{2}}$

得：$R_1 = R_2 = 8.58\ \Omega$， $R_3 = 141.4\ \Omega$

4.4.3 链式散射参量矩阵 [T]



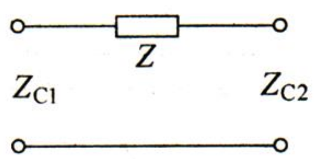
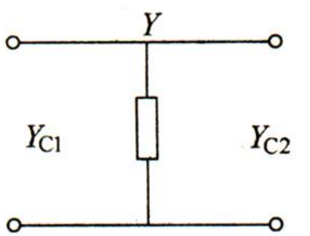
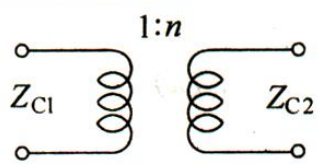
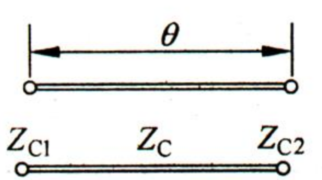
按输入输出口分类**重写电压波关系式**:

$$\begin{Bmatrix} a_1^A \\ b_1^A \end{Bmatrix} = \begin{bmatrix} T_{11}^A & T_{12}^A \\ T_{21}^A & T_{22}^A \end{bmatrix} \begin{bmatrix} T_{11}^B & T_{12}^B \\ T_{21}^B & T_{22}^B \end{bmatrix} \begin{Bmatrix} b_2^B \\ a_2^B \end{Bmatrix}$$

链形散射矩阵与A矩阵作用相同

4.4.4 S参量与其它网络参量的转换

表 4.3　二端口等效单元电路的散射矩阵和传输矩阵

单元电路	[s]	[t]	说明
	$\begin{bmatrix} \frac{z+r-1}{z+r+1} & \frac{2\sqrt{r}}{z+r+1} \\ \frac{2\sqrt{r}}{z+r+1} & \frac{z-r+1}{z+r+1} \end{bmatrix}$	$\begin{bmatrix} \frac{r+z+1}{2\sqrt{r}} & \frac{r-z-1}{2\sqrt{r}} \\ \frac{r+z-1}{2\sqrt{r}} & \frac{r-z+1}{2\sqrt{r}} \end{bmatrix}$	$z=Z/Z_{C1}$ $r=Z_{C2}/Z_{C1}$
	$\begin{bmatrix} \frac{1-y-1/r}{1+y+1/r} & \frac{2/\sqrt{r}}{1+y+1/r} \\ \frac{2/\sqrt{r}}{1+y+1/r} & -\frac{1+y-1/r}{1+y+1/r} \end{bmatrix}$	$\begin{bmatrix} \frac{1+y+1/r}{2/\sqrt{r}} & \frac{1+y-1/r}{2/\sqrt{r}} \\ \frac{1-y-1/r}{2/\sqrt{r}} & \frac{1-y+1/r}{2/\sqrt{r}} \end{bmatrix}$	$y=Y/Y_{C1}$ $r=Y_{C1}/Y_{C2}$
	$\begin{bmatrix} \frac{r-n^2}{r+n^2} & \frac{\pm 2n\sqrt{r}}{r+n^2} \\ \frac{\pm 2n\sqrt{r}}{r+n^2} & \frac{n^2-r}{r+n^2} \end{bmatrix}$	$\begin{bmatrix} \pm\frac{r+n^2}{2n\sqrt{r}} & \pm\frac{r-n^2}{2n\sqrt{r}} \end{bmatrix}$ $\begin{bmatrix} \pm\frac{r-n^2}{2n\sqrt{r}} & \pm\frac{r+n^2}{2n\sqrt{r}} \end{bmatrix}$	$r=Z_{C2}/Z_{C1}$
	$\begin{bmatrix} 0 & e^{-j\theta} \\ e^{-j\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{bmatrix}$	$Z_{C1}=Z_{C2}=Z_C$

4.4.4 S参量与其它网络参量的转换

矩阵 参量	用[s]表示	用[z]表示	用[y]表示	用[$\bar{a}$]表示
[s]	$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$	$[s]=([z]+[1])^{-1}\times([z]-[1])$ $s_{11}=\dfrac{\lvert z\rvert+z_{11}-z_{22}-1}{\lvert z\rvert+z_{11}+z_{22}+1}$ $s_{12}=\dfrac{2z_{12}}{\lvert z\rvert+z_{11}+z_{22}+1}$ $s_{21}=\dfrac{2z_{21}}{\lvert z\rvert+z_{11}+z_{22}+1}$ $s_{22}=\dfrac{\lvert z\rvert-z_{11}+z_{22}-1}{\lvert z\rvert+z_{11}+z_{22}+1}$	$[s]=([1]+[y])^{-1}\times([1]-[y])$ $s_{11}=\dfrac{1-y_{11}+y_{22}-\lvert y\rvert}{1+y_{11}+y_{22}+\lvert y\rvert}$ $s_{12}=\dfrac{-2y_{12}}{1+y_{11}+y_{22}+\lvert y\rvert}$ $s_{21}=\dfrac{-2y_{21}}{1+y_{11}+y_{22}+\lvert y\rvert}$ $s_{22}=\dfrac{1+y_{11}-y_{22}-\lvert y\rvert}{1+y_{11}+y_{22}+\lvert y\rvert}$	$s_{11}=\dfrac{\bar{a}+\bar{b}-\bar{c}-\bar{d}}{\bar{a}+\bar{b}+\bar{c}+\bar{d}}$ $s_{12}=\dfrac{2\lvert\bar{a}\rvert}{\bar{a}+\bar{b}+\bar{c}+\bar{d}}$ $s_{21}=\dfrac{2}{\bar{a}+\bar{b}+\bar{c}+\bar{d}}$ $s_{22}=\dfrac{-\bar{a}+\bar{b}-\bar{c}+\bar{d}}{\bar{a}+\bar{b}+\bar{c}+\bar{d}}$
[z]	$[z]=([1]-[s])^{-1}\times([1]+[s])$ $z_{11}=\dfrac{1+s_{11}-s_{22}-\lvert s\rvert}{1-s_{11}-s_{22}+\lvert s\rvert}$ $z_{12}=\dfrac{2s_{12}}{1-s_{11}-s_{22}+\lvert s\rvert}$ $z_{21}=\dfrac{2s_{21}}{1-s_{11}-s_{22}+\lvert s\rvert}$ $z_{22}=\dfrac{1-s_{11}+s_{22}-\lvert s\rvert}{1-s_{11}-s_{22}+\lvert s\rvert}$	$[z]=\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$[z]=[y]^{-1}$ $=\dfrac{1}{\lvert y\rvert}\times\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$	$[z]=\dfrac{1}{\bar{c}}\begin{bmatrix} \bar{a} & \lvert\bar{a}\rvert \\ 1 & \bar{d} \end{bmatrix}$

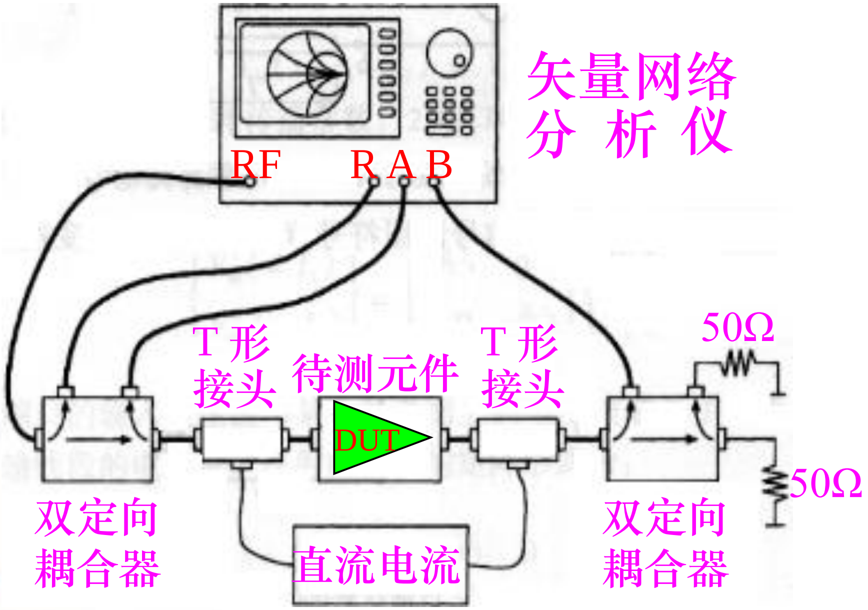
4.4.4 S参量与其它网络参量的转换

续表

矩阵参量	用[s]表示	用[z]表示	用[y]表示	用[$\bar{a}$]表示
[y]	$[y]=([1]-[s])\times([1]+[s])^{-1}$ $y_{11}=\dfrac{1-s_{11}+s_{22}-\vert s\vert}{1+s_{11}+s_{22}+\vert s\vert}$ $y_{12}=\dfrac{-2s_{12}}{1+s_{11}+s_{22}+\vert s\vert}$ $y_{21}=\dfrac{-2s_{21}}{1+s_{11}+s_{22}+\vert s\vert}$ $y_{22}=\dfrac{1+s_{11}-s_{22}-\vert s\vert}{1+s_{11}+s_{22}+\vert s\vert}$	$[y]=[z]^{-1}=\dfrac{1}{\vert z\vert}\times\begin{bmatrix}z_{22} & -z_{12}\\ -z_{21} & z_{11}\end{bmatrix}$	$[y]=\begin{bmatrix}y_{11} & y_{12}\\ y_{21} & y_{22}\end{bmatrix}$	$[y]=\dfrac{1}{\bar{b}}\times\begin{bmatrix}\bar{d} & -\vert\bar{a}\vert\\ -1 & \bar{a}\end{bmatrix}$
[$\bar{a}$]	$\bar{a}=\dfrac{1}{2s_{21}}(1+s_{11}-s_{22}-\vert s\vert)$ $\bar{b}=\dfrac{1}{2s_{21}}(1+s_{11}+s_{22}+\vert s\vert)$ $\bar{c}=\dfrac{1}{2s_{21}}(1-s_{11}-s_{22}+\vert s\vert)$ $\bar{d}=\dfrac{1}{2s_{21}}(1-s_{11}+s_{22}-\vert s\vert)$	$[\bar{a}]=\dfrac{1}{z_{21}}\begin{bmatrix}z_{11} & \vert z\vert\\ 1 & z_{22}\end{bmatrix}$	$[\bar{a}]=\dfrac{-1}{y_{21}}\begin{bmatrix}y_{22} & 1\\ \vert y\vert & y_{11}\end{bmatrix}$	$[\bar{a}]=\begin{bmatrix}\bar{a} & \bar{b}\\ \bar{c} & \bar{d}\end{bmatrix}$

4.4.7 S 参量的测量

- 射频源RF输出射频信号
- 测量通道R用于测量入射波，同时也作为参考端口。
- 通道A和B：测量反射波和传输波(S_{11}=A/R，S_{21}=B/R)。
- 若要测量S_{12}和S_{22}，将待测元件反接。



测量S_{11}和S_{21}的实验系统

4.4.7 S 参量的测量

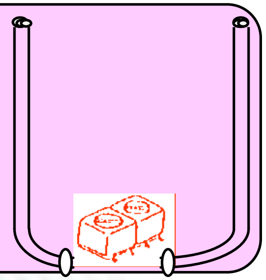
为什么要进行校正测量

A ＝

待测元件特性

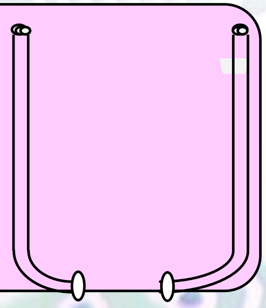
B —

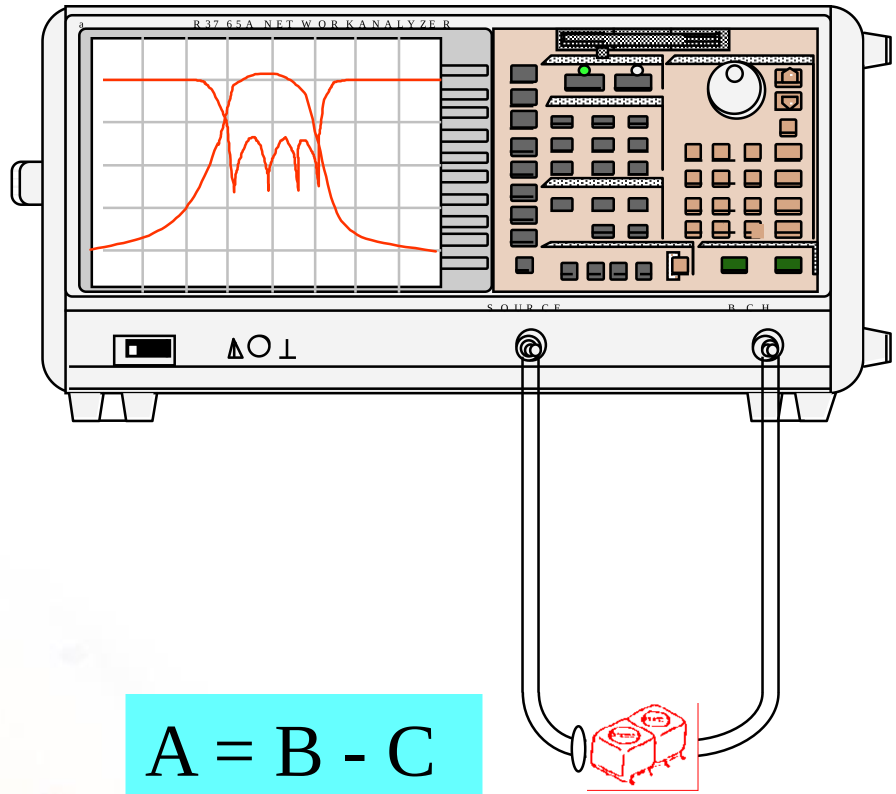
全体的特性



C

测定系统的特性





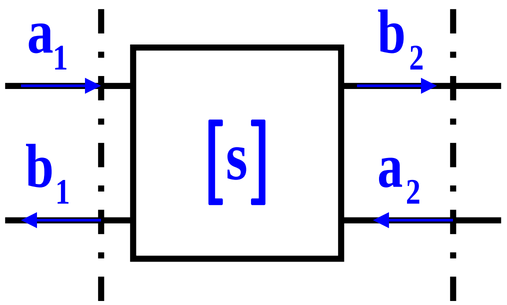
A = B - C

4.4.5 信号流图模型

一、微波网络的信号流图

$$b_1 = s_{11}a_1 + s_{12}a_2$$

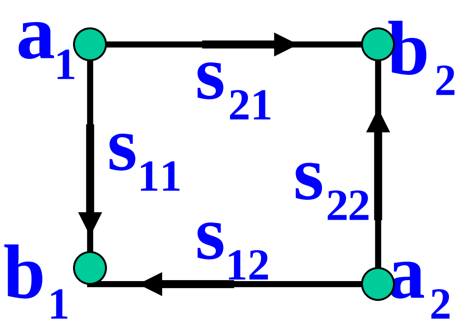
$$b_2 = s_{21}a_1 + s_{22}a_2$$



信号流图 线性方程组对应的拓扑图（结点与方向支线）

结点：

支线系数：

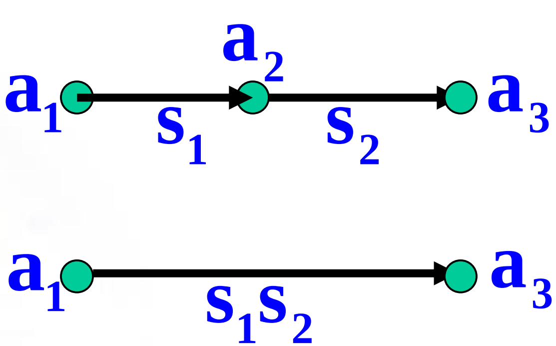


二、信号流图简化法则

1. 串联支线合并法则

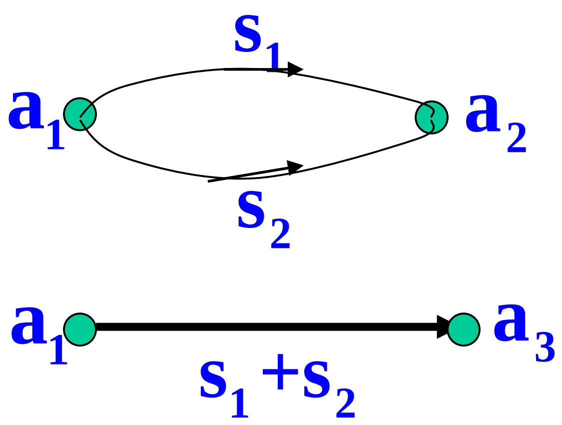
$$a_2 = s_1 a_1 \quad a_3 = s_2 a_2$$

$$a_3 = s_1 s_2 a_1$$



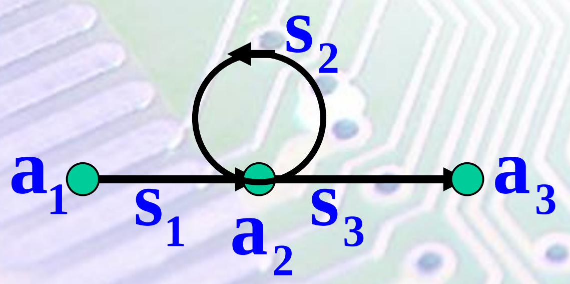
2. 并联支线合并法则

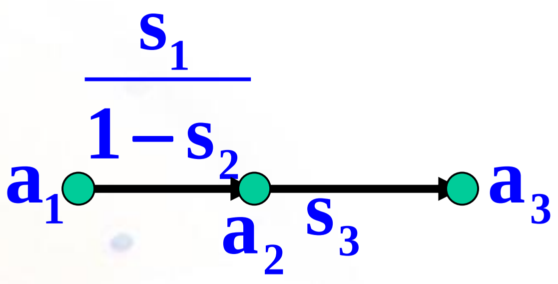
$$a_2 = s_1 a_1 + s_2 a_1 = (s_1 + s_2) a_1$$



3. 自闭环消除法则

$$a_2 = s_1 a_1 + s_2 a_2 \qquad a_2 = \frac{s_1}{1 - s_2} a_1$$

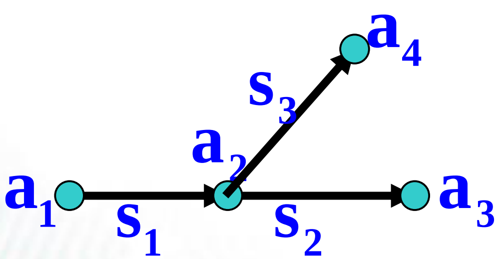


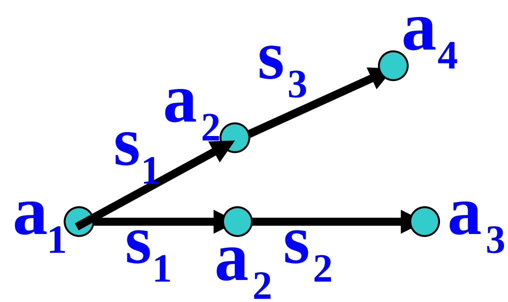


4. 结点分裂法则

$$a_2 = s_1 a_1 \quad a_3 = s_2 a_2 \quad a_4 = s_3 a_2$$

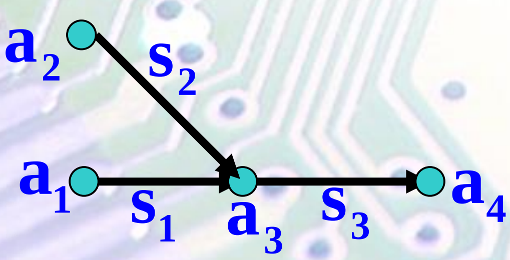
$$a_3 = s_1 s_2 a_1 \quad a_4 = s_1 s_3 a_1$$

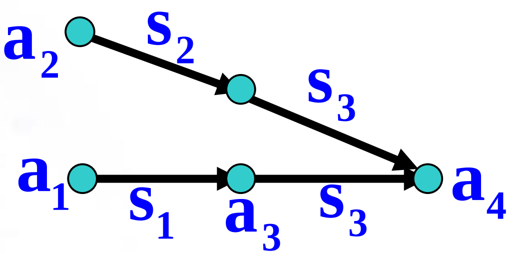




$$a_3 = s_1 a_1 + s_2 a_2 \quad a_4 = s_3 a_3$$

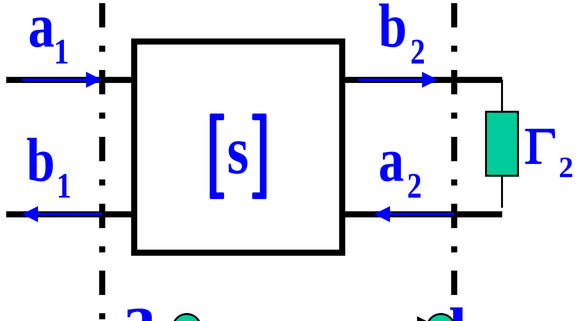
$$a_4 = s_1 s_3 a_1 + s_2 s_3 a_2$$

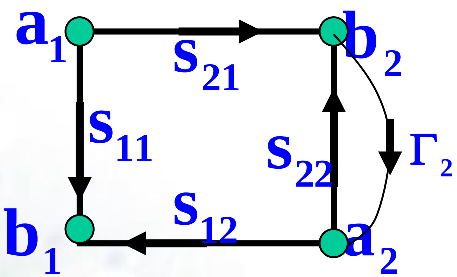




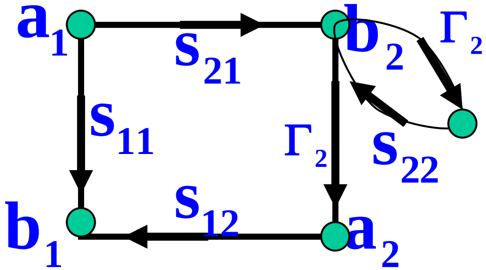
4.4.5 信号流图模型

例4.6　求：$\Gamma_{in}=\dfrac{b_1}{a_1}$

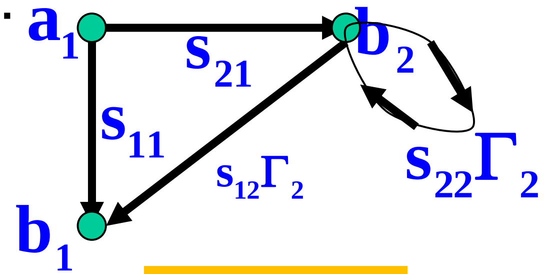




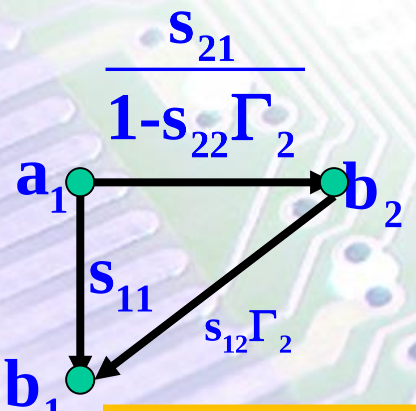
信号流图



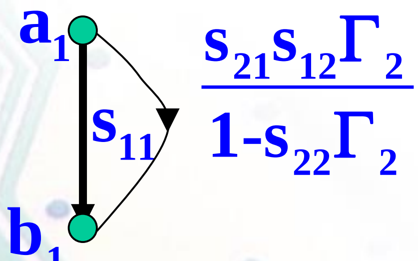
分裂b2



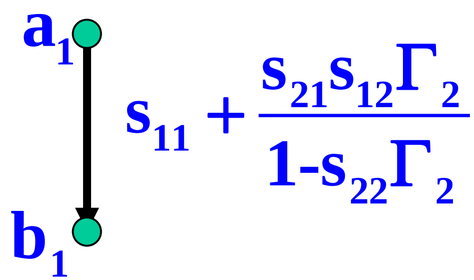
串联消除



消除自闭环



消除串联



消除并联

4.4.5 信号流图模型

端接短路、开路和匹配负载（$\Gamma_L = -1$、1、0），测得的输入端反射系数分别为Γ_s、Γ_o和Γ_m，代入公式可得：

$$\begin{cases} \Gamma_s = s_{11} - \dfrac{s_{12}^2}{1+s_{22}} \\ \Gamma_m = s_{11} \\ \Gamma_o = s_{11} + \dfrac{s_{12}^2}{1-s_{22}} \end{cases} \Rightarrow \begin{cases} s_{12}^2 = \dfrac{2(\Gamma_0-\Gamma_m)(\Gamma_s-\Gamma_m)}{\Gamma_s-\Gamma_0} \\ s_{11} = \Gamma_m \\ s_{22} = \dfrac{2\Gamma_m-\Gamma_0-\Gamma_s}{\Gamma_s-\Gamma_0} \end{cases}$$

$$\Gamma_{in} = s_{11} + \frac{s_{21}s_{12}\Gamma_L}{1-s_{22}\Gamma_L}$$

若进行直通测量可直接测得S_{21}

该测量方法叫：SOLT，是常用的校准法

矢量网络分析仪的测量精度很大程度上依赖于校准原始误差（校准前）2%~80% 剩余误差（校准后）0.1%~2%

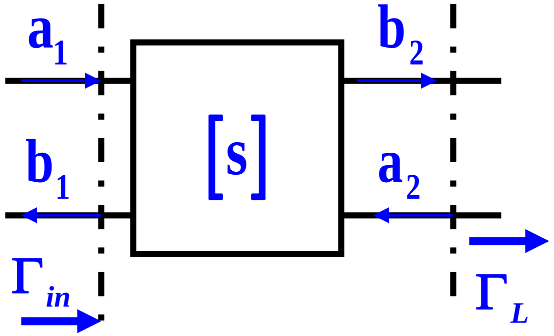
接头是一个重要的误差来源，特别在较高的频率

本章作业

附加题1：已知放大器输入、输出端口的驻波系数分别为 VSWR=2 和 VSWR=3，求输入、输出端口反射系数的模。若采用S_{11}和S_{22}表示计算结果，其物理含义是什么？

附加题2：利用散射参量方程推导（互易）

$$\Gamma_{in} = s_{11} + \frac{s_{12}^2 \Gamma_L}{1 - s_{22}\Gamma_L}$$



习题4.13、4.21、4.32