A mathematical model and numerical simulation of pressure wave in horizontal gas-liquid bubbly flow*

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Abstract By using an ensemble-averaged two fluid model, with valid closure conditions of interfacial momentum exchange due to virtual mass force, viscous shear stress and drag force, a model for pressure wave propagation in a horizontal gas-liquid bubbly flow is proposed. According to the small perturbation theory and solvable condition of one-order linear uniform equations, a dispersion equation of pressure wave is induced. The pressure wave speed calculated from the model is compared and in good agreement with existing data. According to the dispersion equation, the propagation and attenuation of pressure wave are investigated systemically. The factors affecting pressure wave, such as void fraction, pressure, wall shear stress, perturbation frequency, virtual mass force and drag force, are analyzed. The result shows that the decrease in system pressure, the increase in void fraction and the existence of wall shear stress, will cause a decrease in pressure wave speed and an increase in the attenuation coefficient in the horizontal gas-liquid bubbly flow. The effects of perturbation frequency, virtual mass and drag force on pressure wave in the horizontal gas-liquid bubbly flow at low perturbation frequency are different from that at high perturbation frequency.

Keywords: pressure wave, two-fluid model, gas-liquid bubbly flow, dispersion, speed, attenuation.

In gas-liquid two-phase bubbly flow, pressure wave is an important factor affecting the flow instability and occurs frequently in many processes in nature and industries, such as electrical power station, nuclear reactor power station, petroleum and chemical engineering, and so on. For example, the loss of coolant accident (LOCA) caused by some small crevasses in nuclear reactor and the pressure vibration in petroleum transportation pipe systems are directly related to pressure wave propagation. Another kind of application of pressure wave is to apply the character of pressure wave to the measurement of void fraction, interfacial area and flow rate in two-phase flow[1,2]. Therefore, it is necessary to analyze deeply the mechanism of pressure wave in gas-liquid bubbly flow.

Owing to the compressibility of gas phase, the change of interfacial, and the exchange of momentum and energy in gas-liquid flow, the pressure wave propagation is very complicated. Some pressure wave models, such as elasticity model, homogeneous mixture model and single-phase model[3~5], and so on, were proposed in literature. But the above-mentioned models could not describe exactly the pressure wave in gas-liquid bubble flow because of some ideal assumptions applied. Some other researchers[6~8] investigated the character of pressure wave by the perfect two-fluid model, which is a rigorous model presented so far. And yet a little analysis of the attenuation of pressure wave was made in their research. Many researchers in China investigated the pressure wave propagation in gas-liquid flow by two-fluid model[9~11]; however, the wall shear stress, slip velocity and interface pressure gradient were ignored in their work, and only the pressure wave speed was considered without any discussion on the attenuation of pressure wave.

As a whole, there exist many pressure wave models in gas-liquid flow presented in literature, among which the two-fluid model is a comparatively perfect one. But the closure conditions in the two-fluid model and the system analysis on dispersion equation are not sufficient. In this paper, a model for pressure wave propagation in horizontal gas-liquid two-phase flow is proposed from an ensemble-averaged two-fluid model by considering matching valid closure conditions of interfacial momentum exchange and integrating virtual mass force, viscous shear stress and drag force. The effects of some factors or parameters on the propagation velocity and attenuation of pressure wave, such as void fraction, pres-

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sure, wall shear stress, perturbation frequency, virtual mass force and drag force, are investigated systematically.

1 Mathematical model

1.1 Basic equations

Assuming the pressure wave propagates in horizontal gas-liquid bubble flow under following idealized conditions:

(i) No heat transfers between the phases;

(ii) no mass transfers between the phases;

(iii) the bubbly liquid is treated as a continuum fluid, consisting of a suspension of bubbles which are sufficiently small compared to the characteristic length of the bubbly liquid such as wavelength;

(iv) the bubble is assumed to retain its spherical shape.

The one-dimensional integrated equations of two-fluid models are as follows[12].

Continuity equations:

$$\frac{\partial}{\partial t}(\alpha\rho_G)+\frac{\partial}{\partial x}(\alpha\rho_G u_G)=0, \quad (1)$$

$$\frac{\partial}{\partial t}[(1-\alpha)\rho_L]+\frac{\partial}{\partial x}[(1-\alpha)\rho_L u_L]=0. \quad (2)$$

Momentum equations:

$$\frac{\partial}{\partial t}(\alpha\rho_G u_G)+\frac{\partial}{\partial x}(\alpha\rho_G u_G^2)=-\frac{\partial}{\partial x}(\alpha p_G)+\frac{\partial}{\partial x}[\alpha(\tau_G^{fr}+\tau_G^{Re})]+M_{Gi}-4\frac{\tau_{GW}}{D}, \quad (3)$$

$$\frac{\partial}{\partial t}[(1-\alpha)\rho_L u_L]+\frac{\partial}{\partial x}[(1-\alpha)\rho_L u_L^2]=-\frac{\partial}{\partial x}[(1-\alpha)p_L]+\frac{\partial}{\partial x}[(1-\alpha)(\tau_L^{fr}+\tau_L^{Re})]+M_{Li}-4\frac{\tau_{LW}}{D}, \quad (4)$$

where, the interfacial momentum source for the gas and liquid phases can be written as

$$M_{Gi}=-M_{Li}^{(nd)}-M_{Li}^{(d)}+(\tau_{Li}^{fr}+\tau_{Li}^{Re})\frac{\partial}{\partial x}(1-\alpha)+\frac{\partial}{\partial x}(\alpha\sigma_S)+\frac{\partial}{\partial x}(\alpha p_{Gi})-\alpha\frac{\partial p_{Li}}{\partial x}, \quad (5)$$

$$M_{Li}=M_{Li}^{(nd)}+M_{Li}^{(d)}+p_{Li}\frac{\partial}{\partial x}(1-\alpha)-(\tau_{Li}^{fr}+\tau_{Li}^{Re})\frac{\partial}{\partial x}(1-\alpha), \quad (6)$$

in which $M_{Li}^{(nd)}$ is the liquid non-drag interfacial momentum exchange and $M_{Li}^{(d)}$ the liquid drag interfacial momentum exchange.

1.2 Closure conditions

In order to solve the problem of pressure wave by using the above two-phase model, additional information is required to complete the formulation. This implies that the conservation equations are necessary but not sufficient for describing completely the dynamics of a given system. The supplement information needed includes state equation of ideal gas, constitutive equations as well as boundary and initial conditions.

Assuming that the gas phase is ideal in the bubble flow, the state equation of ideal gas can be written as

$$\rho_G=p_G/(RT). \quad (7)$$

1.2.1 Stress description Because the shear stress inside the gas phase and liquid phase is very small in comparison with Reynolds stress, and the shear stress in liquid interfacial for different Reynolds number is very small too, the shear stress may be ignored. Therefore, an approximate formulation is taken as

$$\tau_G^{fr}\approx\tau_L^{fr}\approx\tau_{Li}^{fr}\approx 0. \quad (8)$$

For gas-liquid bubbly flows under low pressure where $\rho_G/\rho_L\ll 1$, it is generally assumed that the Reynolds stress is negligible compared to the pressure gradient and interfacial forces in the gas phase momentum equation, so

$$\tau_G^{Re}\approx 0. \quad (9)$$

The Reynolds stress and interfacial averaged Reynolds stress for the liquid are given by

$$\tau_L^{Re}=-c_r\rho_L\frac{\alpha}{1-\alpha}|u_r|u_r, \quad (10)$$

$$\tau_{Li}^{Re}=-c_r\rho_L u_r^2, \quad (11)$$

$$u_r=u_G-u_L. \quad (12)$$

Here, c_r is a coefficient, given as 1/5.

The source of the average interfacial stress can be written as[12]:

$$\sigma_s=-c_i\rho_L u_r^2. \quad (13)$$

The coefficient c_i is 3/10.

Assuming that the gas phase is dispersed within the liquid phase, the wall shear stress on the gas phase can be neglected so that the liquid phase wall shear stress is[13]

$$\tau_{Lw} = \frac{1}{2} f_{Lw} \rho_L u_L |u_L|, \quad (14)$$

$$f_{Lw} = 0.046 \left(\frac{u_L D}{\nu_L} \right)^{-0.2}, \quad (15)$$

where, f_{Lw} is the liquid-wall shear stress coefficient and ν_L the kinematic viscosity of liquid.

1.2.2 Pressure According to the transient Bernoulli equation, Arnold1) proposed a correlation for the calculation of corresponding liquid phase pressure as follows:

$$p_L = p - c_p \rho_L \alpha u_r^2, \quad (16)$$

where, p is the system pressure and the coefficient c_p is 1/4.

Park utilized a potential flow solution around a single isolated spherical bubble to postulate the difference between the liquid phase interfacial pressure and the pressure of liquid phase in gas-liquid flows[12]

$$p_{Li} - p_L = -c_p \rho_L (1-\alpha) u_r^2. \quad (17)$$

The difference between the gas phase interfacial pressure and the pressure of gas phase in gas-liquid flows is generally taken to be zero since there exists no meaningful pressure gradient in the relatively small regions occupied by gas phase,

$$p_G - p_{Gi} \approx 0. \quad (18)$$

1.2.3 Interfacial extra momentum source Because pressure wave is an instant phenomenon, the interfacial momentum exchange induced by virtual mass and drag force plays an important role. The momentum exchange term induced by non-drag forces is[12]

$$M_{Li}^{(nd)} = c_{vm} \alpha \rho_L a_{vm} - c_{m1} \alpha \rho_L u_r \frac{\partial u_r}{\partial x} - c_{m2} \rho_L u_r^2 \frac{\partial \alpha}{\partial x}, \quad (19)$$

$$a_{vm} = \left(\frac{\partial u_G}{\partial t} + u_G \frac{\partial u_G}{\partial x} \right) - \left(\frac{\partial u_L}{\partial t} + u_L \frac{\partial u_L}{\partial x} \right), \quad (20)$$

$$c_{vm} = 0.5 \frac{1 + 2\alpha}{1 - \alpha}, \quad (21)$$

where, the coefficients c_{m1} and c_{m2} are given as 1/10.

For the uniform dispersed bubbly flow, the drag force can be taken as[14]:

$$M_{Li}^{(d)} = \frac{3}{8} \frac{c_D}{r} \rho_L \alpha |u_r| u_r, \quad (22)$$

$$c_D = \frac{4}{3} r \sqrt{\frac{g(\rho_L - \rho_G)}{\sigma}} \left\{ \frac{1 + 17.67 [f(\alpha)]^{\frac{6}{7}}}{18.67 f(\alpha)} \right\}^2, \quad (23)$$

$$f(\alpha) = (1 - \alpha)^{1.5}. \quad (24)$$

2 Analysis

By introducing the following equations,

$$dp_L = a_L^2 d\rho_L, \quad (25)$$

$$dp_G = a_G^2 d\rho_G, \quad (26)$$

Eqs. (1)~(4) can be rewritten as follows,

$$\rho_G \frac{\partial \alpha}{\partial t} + \frac{\alpha}{a_G^2} \frac{\partial p_G}{\partial t} + \rho_G u_G \frac{\partial \alpha}{\partial x} + \alpha \frac{u_G}{a_G^2} \frac{\partial p_G}{\partial x} + \alpha \rho_G \frac{\partial u_G}{\partial x} = 0, \quad (27)$$

$$-\rho_L \frac{\partial (1-\alpha)}{\partial t} + \frac{(1-\alpha)}{a_L^2} \frac{\partial p_L}{\partial t} - \rho_L u_L \frac{\partial (1-\alpha)}{\partial x} + (1-\alpha) \frac{u_L}{a_L^2} \frac{\partial p_L}{\partial x} + (1-\alpha) \rho_L \frac{\partial u_L}{\partial x} = 0, \quad (28)$$

$$\begin{aligned} &\alpha(\rho_G + c_{vm}\rho_L) \frac{\partial u_G}{\partial t} - c_{vm} \alpha \rho_L \frac{\partial u_L}{\partial t} \\ &+ \rho_L u_r^2 (\alpha c_p - c_r + c_i - c_{m2}) \frac{\partial \alpha}{\partial x} \\ &+ \alpha \left[1 - \frac{c_p (1-\alpha) u_r^2}{a_L^2} + \frac{c_i u_r^2}{a_L^2} \right] \frac{\partial p_L}{\partial x} \\ &+ \alpha [\rho_G u_G - 2 c_p (1-\alpha) \rho_L u_r + 2 c_i \rho_L u_r \\ &+ c_{vm} \rho_L u_G - c_{m1} \rho_L u_r] \frac{\partial u_G}{\partial x} \\ &+ \alpha \rho_L [2 c_p (1-\alpha) u_r - 2 c_i u_r - c_{vm} u_L \\ &+ c_{m1} u_r] \frac{\partial u_L}{\partial x} = -M_{Li}^{(d)} - 4 \frac{\tau_{Gw}}{D}, \end{aligned} \quad (29)$$

$$\begin{aligned} &- c_{vm} \alpha \rho_L \frac{\partial u_G}{\partial t} + \rho_L [(1-\alpha) + c_{vm} \alpha] \frac{\partial u_L}{\partial t} \\ &+ \rho_L u_r^2 [-c_p (1-\alpha) + 2 c_r + c_{m2}] \frac{\partial \alpha}{\partial x} \\ &+ \left((1-\alpha) + \frac{c_r \alpha u_r^2}{a_L^2} \right) \frac{\partial p_L}{\partial x} \\ &+ \alpha \rho_L (2 c_r u_r - c_{vm} u_G + c_{m1} u_r) \frac{\partial u_G}{\partial x} \\ &+ \rho_L [(1-\alpha) u_L - 2 c_r \alpha u_r + c_{vm} \alpha u_L \\ &- c_{m1} \alpha u_r] \frac{\partial u_L}{\partial x} = M_{Li}^{(d)} - 4 \frac{\tau_{Lw}}{D}. \end{aligned} \quad (30)$$

1) Arnold, G. S. Entropy and objectivity as constraints upon constitutive equations for two-fluid modeling of multiphase flow. PH.D. Thesis, Rensselaer Polytechnic Institute, Troy, New York, 1998.

Assuming that the gas-liquid system is on a steady state before we put small perturbations on it, we can write any variable $\xi(\alpha, p, u_G, u_L)$ when small perturbations are imposed on the fluid as

$$\xi = \xi_0 + \delta\xi \cdot \exp[\mathrm{i}(wt - kx)], \qquad (31)$$

where, k and w are wave number and angular frequency respectively, and $\delta\xi \ll \xi_0$. Linearizing Eqs. (27)~(30), we can obtain one-order linear uniform equations on $(\delta\alpha, \delta p, \delta u_G, \delta u_L)^T$ by omitting the higher order of small disturbance. According to the solvable condition of one-order linear uniform equations, that is, $\det(A) = 0$, a dispersion equation of pressure wave in horizontal gas-liquid bubbly flow is deduced as follows:

$$\begin{vmatrix} \left(\rho_G + c_p\alpha\rho_L \frac{u_r^2}{a_G^2}\right)w & \frac{\alpha}{a_G^2}\left[1 - c_p(1-\alpha)\frac{u_r^2}{a_L^2}\right]w & -\left[\alpha\rho_G k + 2c_p\alpha(1-\alpha)\rho_L \frac{u_r}{a_G^2}w\right] & 2c_p\alpha(1-\alpha)\rho_L \frac{u_r}{a_G^2}w \\ -\rho_L w & \frac{1-\alpha}{a_L^2}w & 0 & -k(1-\alpha)\rho_L \\ \rho_L u_r^2 k(-\alpha c_p + c_r - c_i + c_{m2}) & -\alpha k\left[1 - (1-\alpha)\frac{c_p u_r^2}{a_L^2} + c_i \frac{u_r^2}{a_L^2}\right] & E & F \\ \rho_L u_r^2 k[(1-\alpha)c_p - 2c_r - c_{m2}] & -k\left(1 - \alpha + c_r\alpha \frac{u_r^2}{a_L^2}\right) & F & H \end{vmatrix} = 0, \qquad (32)$$

where,

$$E = \alpha(\rho_G + c_{vm}\rho_L)w - \mathrm{i}\left(\frac{3}{4}\frac{c_D}{r}\rho_L\alpha u_r + \frac{4}{D}f_{GW}\rho_G u_G\right), \qquad (33)$$

$$F = -c_{vm}\alpha\rho_L w + \mathrm{i}\frac{3}{4}\frac{c_D}{r}\rho_L\alpha u_r, \qquad (34)$$

$$H = \rho_L[(1-\alpha) + c_{vm}\alpha]w - \mathrm{i}\left(\frac{3}{4}\frac{c_D}{r}\rho_L\alpha u_r + \frac{4}{D}f_{LW}\rho_L u_L\right). \qquad (35)$$

Eq. (32) is a biquadratic equation with complex coefficients. It gives four roots for each value of angular frequency w. Since the wavelengths associated with these roots are too short to allow the two-fluid medium to be treated as a continuum when the frequencies of pressure waves are very high and highly attenuated, two of those roots are neglected. One of the remaining two roots has a positive real part, which implies that the pressure wave travels with the flow. On the contrary, the other one has a negative real part and travels downwind with the flow. From the latter two roots, the speeds of pressure wave and spatial attenuation coefficient can be defined as[7]

$$c = \frac{\left|\left[\frac{w}{\mathrm{Re}(k)}\right]^+ - \left[\frac{w}{\mathrm{Re}(k)}\right]^-\right|}{2}, \qquad (36)$$

$$\eta = \frac{|\mathrm{Im}(k)^+ - \mathrm{Im}(k-)^-|}{2}, \qquad (37)$$

where, $\mathrm{Re}(k)$ is the real part and $\mathrm{Im}(k)$ the imaginary part of wave number k.

To examine the correction and accuracy of the present model, the numerical results from the present model are compared with the experimental data of pressure wave speed measured by Henry et al.[15] Fig. 1 shows that the calculated wave speed is in good agreement with the experimental value.

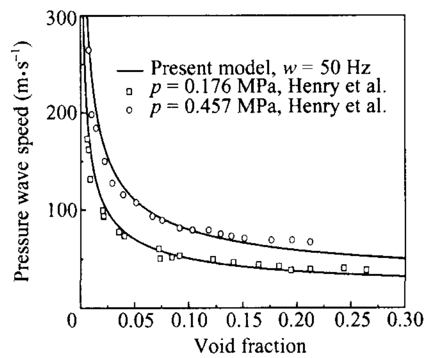

Fig. 1. Pressure wave speed compared with Henry et al.'s[15].

3 Results and discussion

Fig. 2 exhibits the effect of the system pressure on pressure wave in the gas-liquid bubbly flow. It is seen that the pressure wave speed increases quickly but the attenuation coefficient decreases sharply with the increase in system pressure within a relatively low value range. With the further increasing of system pressure, the curves of pressure wave speed and attenuation coefficient versus pressure vary more and more slowly. According to the gas state equation, $\rho_G = p_G/(R \cdot T)$, raising the system pressure will cause the increase in density of gas phase and the decrease in fluid compressibility in a constant temperature. As a result, the pressure wave has a much higher propagation velocity and much smaller attenuation under a high system pressure. When the system pressure reaches a very high value, the curve of pressure wave speed and attenuation coefficient will tend to approach a certain constant because of the little compressibility in the gas phase.

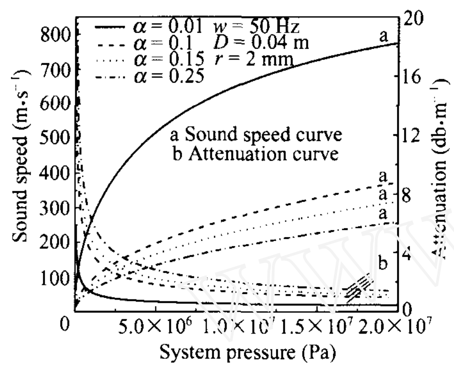

Fig. 2. Effect of the system pressure on pressure wave in the gas-liquid bubbly flow.

Fig. 3 presents the effect of the wall shear stress on pressure wave in the gas-liquid bubbly flow. From this figure we can find that the wall shear stress decreases the speed of pressure wave. As the void fraction increases, the difference between two pressure wave speeds of existence and inexistence of wall shear stress will arrive at about 10 m/s under the atmosphere pressure. The wall shear stress also enhances greatly the attenuation of pressure wave and the attenuation curve ascends continuously with the increase in void fraction. Fig. 3 also shows that the curve of pressure wave speed changes very sharply when the void fraction is close to zero; however, the changes are slow when the void fraction is high comparatively. The reason is that the density of fluid varies slowly but the compressibility of fluid increases sharply when there are a few bubbles in the pure water. Consequently, there is a clear decrease in the pressure wave propagation velocity. With further increasing void fraction, the pressure wave speeds will

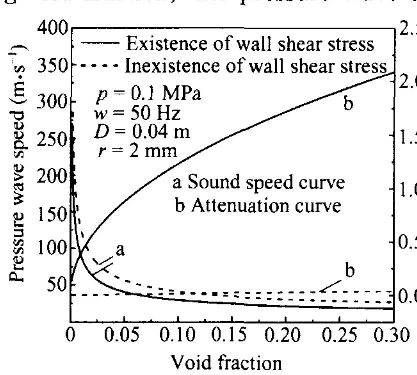

Fig. 3. Effect of the wall shear stress on pressure wave in the gas-liquid bubbly flow.

have a comparatively smooth value because the changes in the fluid density and compressibility are small. When the value of void fraction enters into a high value range, the pressure wave speed decreases and the attenuation increases because the fluid has a great compressibility and energy dissipation.

The effect of the disturbance frequency on pressure wave in the gas-liquid bubbly flow is shown in Fig. 4. That the different pressure wave speed corresponds to different disturbance frequency is the dispersion phenomenon of pressure wave. The curves of pressure wave speed and attenuation go up slowly as the frequency increases for the small void frequency. When the disturbance frequency goes up to about 400 Hz, the pressure wave speed and attenuation will arrive at a certain constant. As a result the dispersion of pressure wave will disappear at the same time. The reason for these is that the interfacial momentum exchange is greatly sufficient in the region of low frequencies and so the pressure wave speed and attenuation are relatively small. With increasing the frequency, there is not enough time for the interfacial momentum exchange especially for higher frequencies, so that the dispersion phenomenon of pressure wave disappears.

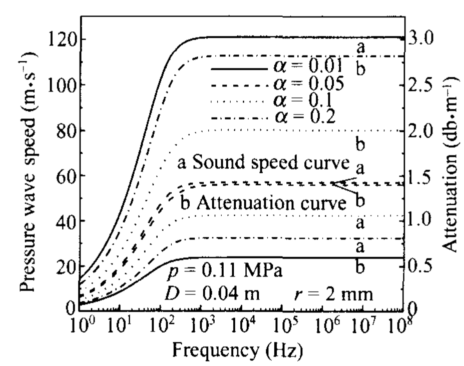

Fig. 4. Effect of the frequency on pressure wave in the gas-liquid bubbly flow.

Figs. 5 and 6 show that the virtual mass force and drag force have great effects on pressure wave in the bubbly flow. If these forces are neglected simultaneously, the pressure wave speed can approach about 350 m/s, which is just close to the sound velocity of pure gas under the atmosphere pressure and there is almost no attenuation. Fig. 5 shows that the virtual mass stress lowers the pressure wave speed being consistent with the result of Chung et al. [16]. Moreover it enhances the attenuation of pressure wave at high disturbance frequencies if the drag force is neglected.

Similarly, the points of acute change on the curve of pressure wave speeds and attenuation move to a high frequency with the increase in virtual mass force. Fig. 6 reveals the effect of drag force when ignoring the virtual mass force. The drag force reduces the wave velocity at low frequencies, but increases enormously the wave attenuation at high frequencies. The reason is that the virtual mass force and drag force increase the interfacial momentum exchange so that they decrease the pressure wave propagation velocity and enhance the attenuation.

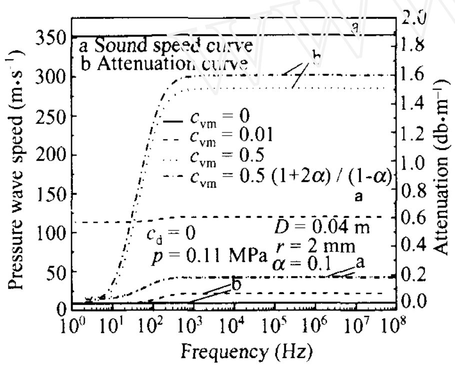

Fig. 5. Effect of the virtual mass force on pressure wave in the gas-liquid bubbly flow.

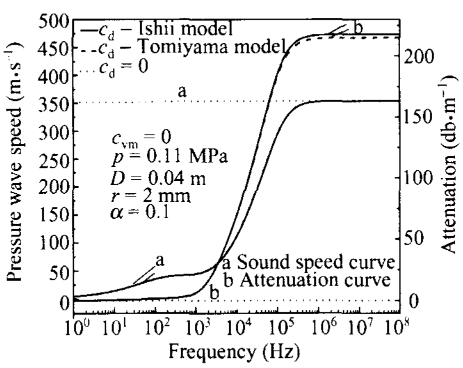

Fig. 6. Effect of the drag force on pressure wave in the gas-liquid bubbly flow.

4 Conclusions

(i) The increase in system pressure enhances the pressure wave propagation velocity, but it decreases the pressure wave attenuation. If the system pressure is much higher, the pressure wave speed and attenuation coefficient will almost remain at a constant value.

(ii) The wall shear force reduces the pressure wave propagation velocity and enhances the pressure wave attenuation.

(iii) The effect of disturbance frequency on pressure wave is very complicated. At a low frequency, both the speed and attenuation of pressure wave are small and change slowly. When the frequency reaches about 400 Hz, the pressure wave speed and attenuation will tend to a certain high value.

(iv) The virtual mass force reduces the pressure wave propagation velocity but increases the attenuation in high frequencies. Moreover it causes the acute change region of pressure wave speed and attenuation to move towards the high frequency region.

(v) The drag force decreases the pressure wave propagation velocity in the region of low frequency and increases the pressure wave attenuation in the region of high frequency.

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