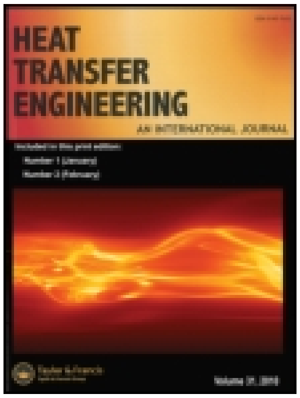

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A Novel Model of Turbulent Convective Heat Transfer in Round Tubes at Supercritical Pressures

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A Novel Model of Turbulent Convective Heat Transfer in Round Tubes at Supercritical Pressures

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In the present study, a novel model was established to investigate the enhanced heat transfer to turbulent pipe flow of supercritical pressure fluids. The governing equations for the steady turbulent compressible pipe flow were simplified into the one-dimensional nondimensionalized forms based on the boundary layer theory. A conventional mixing length turbulence model for constant-property pipe flows was modified by introducing the effect of density fluctuations into the equations of turbulent transport, and the modified turbulence model was applicable to both constant-property and variable-property pipe flows. With the suggested model, which was a combination of the nondimensional governing equations and the modified turbulence model, the numerical calculations were carried out for the turbulent convective heat transfer of water in round tubes at supercritical pressures. The results showed that the present model can provide a relatively precise prediction about the effect of pressure, mass flux, and wall heat flux on heat transfer for supercritical fluid flows and greatly reduce the calculation workload. The modified turbulence model showed a much better agreement with the experimental results than the original turbulence model.

INTRODUCTION

Owing to their unique physical and chemical properties, supercritical fluids are commonly seen in engineering applications, for example, the supercritical water used as the working fluid in thermal power plants, the supercritical helium as the coolant in cryogenics systems, and the supercritical carbon dioxide as the extractant in extraction processes. Therefore, a study on the heat transfer of supercritical fluid flows is of great significance for the design and improvement of the design of those systems operating at the supercritical pressures. The recent developments in supercritical fluid technologies, e.g., the supercritical water-cooled reactor (SCWR) [1], the supercritical water oxidation (SCWO) for organic waste disposal [2], and the supercritical water gasification (SCWG) of biomass for hydrogen production [3], also necessitate the study.

The peculiarity of the heat transfer to the supercritical (pressure) fluid flow is primarily attributed to the great variation in the physical properties of the fluid in the pseudocritical region. Over the past several decades, a number of experimental investigations on heat transfer at supercritical pressures were carried out [4–14], the majority of which were on the turbulent flows in round tubes. According to these investigations, either enhanced or deteriorated heat transfer regime occurred in the supercritical fluid flows depending on the controlling parameters. The heat transfer coefficient was remarkably increased in the pseudocritical region and had a maximum value in the vicinity of the pseudocritical point at low or moderate heat flux. The variation of the heat transfer coefficient with the bulk fluid temperature resembled the way the isobaric specific heat varied with the temperature. As the heat flux was increased, the enhancement of the heat transfer was suppressed gradually, and eventually the heat transfer deterioration occurred in the pseudocritical region. The deterioration in the heat transfer at supercritical pressures was generally interpreted qualitatively in terms of the buoyancy effect and the acceleration effect [15, 16].

Since the fluid stays in a single-phase state under the supercritical pressure, it is convenient to develop a mathematical model to simulate the supercritical fluid flows. Therefore, many numerical studies on the turbulent convective heat transfer of the supercritical pressure fluids in various flow channels were performed [12, 13, 17–26], especially those recent studies with the help of the commercial CFD code [13, 21–26]. The major

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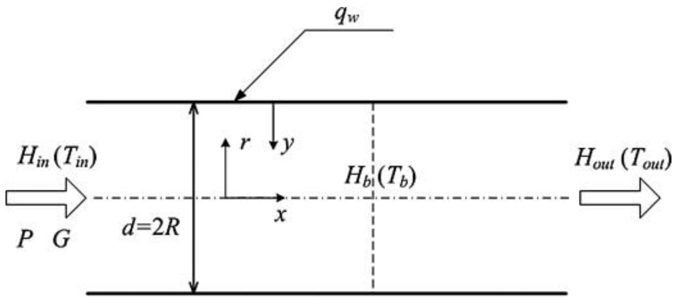

Figure 1 Schematic diagram of flow geometry for simulation.

difficulty in the numerical calculation is the proper selection of the turbulence model. The prevalent turbulence models are developed exclusively for constant property flows, and their direct application to variable property flows is in doubt. The applicability of different turbulence models in predicting supercritical pressure heat transfer was examined by many researchers [20–26], but the reported results were not in agreement with one another. Generally, for the enhanced heat transfer regime, the performance of the turbulence models would get worse in the pseudocritical region when the pressure was close to the critical pressure or the heat flux was relatively high; for the deteriorated heat transfer regime, none of the models was able to predict quantitatively the onset of the heat transfer deterioration. Furthermore, in the former studies, the two- or three-dimensional governing equations were usually solved with the SIMPLE-type algorithms. However, it is also doubtful whether the conventional SIMPLE-type algorithms that are originally developed for the incompressible flow can be applied directly to the supercritical fluid flows because of the considerable variations in fluid density near the pseudocritical point.

Up to now, few studies were carried out to improve the conventional turbulence models by taking into account the influence of the property variation on turbulent transport. As a pioneer work, Bellmore and Reid [17] suggested a modified mixing length model by including density fluctuations into the equations of turbulent transport. The additional terms owing to density fluctuations into the time-averaged turbulent continuity equation added to the difficulty in solving the flow field. So based on the equations for the steady turbulent compressible axial symmetric boundary layer flow, Bellmore and Reid introduced a turbulent stream function into the calculations. The density fluctuating products in the time-averaged momentum equation then were grouped into two kinds: one introduced into the turbulent stream function and the other into the turbulent shear stress. The model by Bellmore and Reid was used in terms of the turbulent stream function and was not widely applied because of its complexity in calculation.

The present paper focuses on developing a novel model to study the enhanced heat transfer under the supercritical pressure. First, based on the assumption that the effects of buoyancy and acceleration are negligible, the convection transport equations for the steady turbulent compressible pipe flows are reduced into

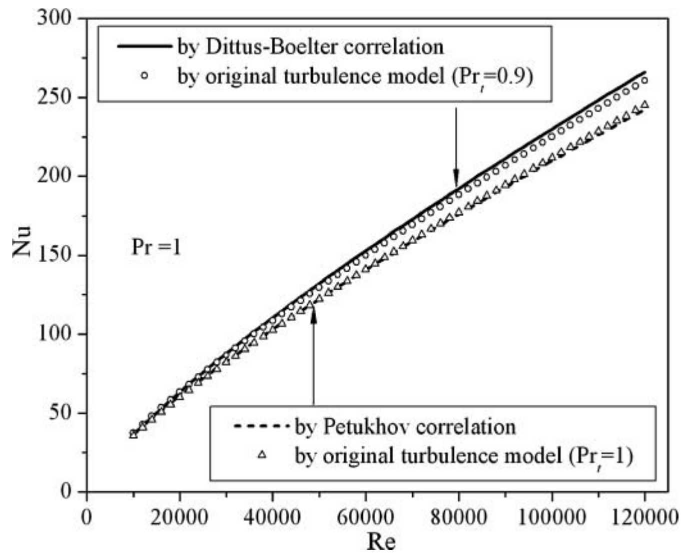

Figure 2 Comparison of simulation with empirical correlations for incompressible flows at a fluid Prandtl number of 1.

the one-dimensional boundary-layer equations and then nondimensionalized. Second, on the basis of the work by Bellmore and Reid [17], a new approach to include density fluctuations in the equations of turbulent transport is suggested, and then a modified mixing length turbulence model is developed for the variable property flow. Finally, with these mathematical models, numerical calculations are performed to simulate the turbulent convective heat transfer of water flowing in round tubes at supercritical pressures.

MATHEMATICAL MODELS

Governing Equations

Figure 1 shows the simplified diagram of the flow system in a round tube for the numerical simulation. The supercritical pressure fluid enters the uniformly heated section of the test tube at a constant heat flux. The heated test section is preceded by a relatively long entrance section. The idea supported by various researchers [11, 12] that the supercritical fluid flow far from the entrance region can be taken as fully developed is accepted in the present study. Under a supercritical condition with a given pressure P, mass velocity G, wall heat flux q_w and tube inner diameter d, the heat transfer coefficient h or the wall temperature T_w at different bulk enthalpies H_b within a given range (H_{in}–H_{out}) is to be calculated.

Based on the boundary layer theory, a one-dimensional (1-D) mathematical model is established to calculate the local heat transfer rate to the turbulent flow of water in the round tube at the supercritical pressure. The governing equations for the steady, fully developed, axial symmetric, buoyancy unaffected, and variable-property turbulent pipe flow can be written as follows:

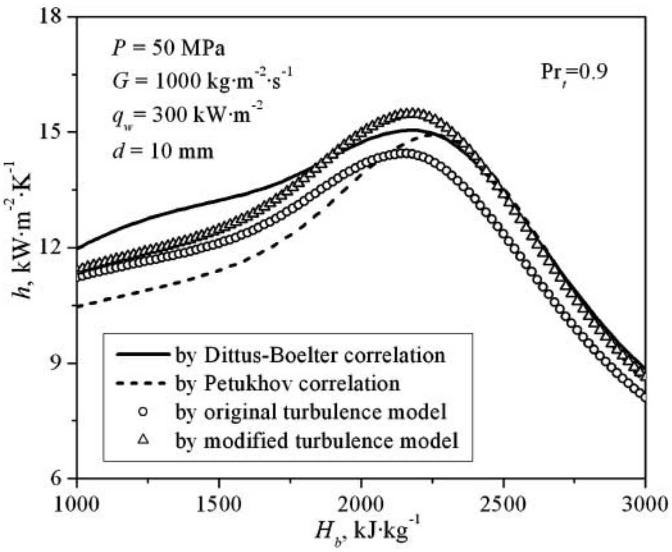

Figure 3 Comparison of simulation with empirical correlations for ultra-supercritical fluid flows at a pressure of 50 MPa.

Momentum equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu+\mu_t\right)\frac{\partial u}{\partial r}\right]+\left(-\frac{\mathrm{d}P}{\mathrm{d}x}\right)=0 \quad (1)$$

where the pressure gradient along the flow direction is assumed to be constant in a given flow cross section.

Energy equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\lambda+\lambda_t\right)\frac{\partial T}{\partial r}\right]-\frac{\rho}{\rho_b}\frac{c_p}{c_{pb}}\frac{u}{u_m}\frac{4q_w}{d}=0 \quad (2)$$

The dimensionless parameters are defined as follows:

$$\eta=\frac{r}{R},\ U=\frac{\mu_b u}{\left(-\mathrm{d}P/\mathrm{d}x\right)R^2},\Theta=\frac{T_w-T}{q_w d/\lambda_b} \quad (3)$$

As a result, the momentum equation (1) and the energy equation (2) can be nondimensionalized as the following forms, respectively:

$$\frac{1}{\eta}\frac{\partial}{\partial \eta}\left[\eta\frac{\mu}{\mu_b}\left(1+\frac{\mu_t}{\mu}\right)\frac{\partial U}{\partial \eta}\right]+1=0 \quad (4)$$

$$\frac{1}{\eta}\frac{\partial}{\partial \eta}\left[\eta\frac{\lambda}{\lambda_b}\left(1+\frac{\mu_t}{\mu}\frac{\mathrm{Pr}}{\mathrm{Pr}_t}\right)\frac{\partial \Theta}{\partial \eta}\right]+\frac{\rho}{\rho_b}\frac{c_p}{c_{pb}}\frac{U}{U_m}=0 \quad (5)$$

The difference between the constant-property flow and the variable-property flow is thus outstanding according to the dimensionless governing equations (4) and (5). The boundary conditions for Eq. (4) and Eq. (5) have the same expressions as follows:

$$\frac{\partial \phi}{\partial \eta}=0\,\mathrm{at}\,\eta=0;\ \phi=0\,\mathrm{at}\,\eta=1.\ \phi=U\ \mathrm{or}\ \Theta \quad (6)$$

The relation between the local bulk Nusselt number Nu_b and the local bulk dimensionless excess temperature Θ_b is as follows:

$$Nu_b=\frac{hd}{\lambda_b}=\frac{1}{\Theta_b} \quad (7)$$

During the iterative procedure, the temperature field is renewed based on the following equation:

$$T(I)=T_b+\frac{q_w d\left[\Theta_b-\Theta(I)\right]}{\lambda_b} \quad (8)$$

Then the thermal–physical properties are updated according to the latest temperature field.

The Modified Mixing Length Turbulence Model

For a steady, two-dimensional (2-D) axial symmetric (*x*-*u*, *r*-*v*), compressible turbulent boundary layer flow, the time-averaged momentum equation taking density fluctuations into account is expressed as

$$\frac{\partial\left(\overline{\rho u}\cdot\bar{u}\right)}{\partial x}+\frac{1}{r}\frac{\partial\left(r\overline{\rho v}\cdot\bar{u}\right)}{\partial r}=-\frac{\partial P}{\partial x}$$
$$+\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu\frac{\partial \bar{u}}{\partial r}-\bar{\rho}\overline{v'u'}-\bar{v}\overline{\rho'u'}-\bar{u}\overline{\rho'v'}-\overline{\rho'u'v'}\right)\right] \quad (9)$$

By introducing all the additional terms owing to density fluctuations into the turbulent shear stress and assuming that for boundary layer flows, $\bar{v}+\overline{\rho'v'}/\bar{\rho}\approx 0$, the turbulent shear stress is expressed as

$$\tau_t=\left|\bar{\rho}\overline{v'u'}+\bar{u}\overline{\rho'v'}+\overline{\rho'u'v'}-\overline{\rho'u'}\cdot\overline{\rho'v'}/\bar{\rho}\right| \quad (10)$$

According to the classical thermodynamics theory, the fluctuating density is determined by

$$\rho'\approx-\bar{\rho}\beta T',\beta=-\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p \quad (11)$$

Thus, Eq. (10) is transformed as follows:

$$\tau_t=\left|\bar{\rho}\overline{v'u'}\right|\left|1-\beta\bar{u}\frac{\overline{v'T'}}{\overline{v'u'}}-\beta\frac{\overline{u'v'T'}}{\overline{v'u'}}-\beta^2\frac{\overline{u'T'}\cdot\overline{v'T'}}{\overline{v'u'}}\right| \quad (12)$$

For a turbulent boundary layer flow under heating conditions, eddies that occur with a positive value of v' give rise to the positive u' and the negative T'; eddies that occur with a negative value of v' are related to the negative u' and the positive T'. Hence, $\overline{u'v'}$ has a positive value, while $\overline{u'T'}$ and $\overline{v'T'}$ have a negative value. The sign of the three-order turbulent fluctuating product $\overline{u'v'T'}$ is uncertain and is temporarily assumed to be negative. Based on the mixing length theory, the time-averaged products of the fluctuating parameters in Eq. (12) are defined as follows:

$$\begin{cases} \overline{u'v'} = l_m^2 \left(\dfrac{\partial \bar{u}}{\partial r}\right)^2 \\ \overline{u'T'} \approx \overline{v'T'} = -\dfrac{l_m^2}{\mathrm{Pr}_t} \left|\dfrac{\partial \bar{T}}{\partial r} \cdot \dfrac{\partial \bar{u}}{\partial r}\right| \\ \overline{u'v'T'} = -\dfrac{l_m^3}{\mathrm{Pr}_t} \left(\dfrac{\partial \bar{u}}{\partial r}\right)^2 \left|\dfrac{\partial \bar{T}}{\partial r}\right| \end{cases} \tag{13}$$

Substituting Eq. (13) into Eq. (12) yields

$$\tau_t = \rho l_m^2 \left(\frac{\partial u}{\partial r}\right)^2 \left[1 + \frac{\beta u}{\mathrm{Pr}_t}\left|\frac{\partial T/\partial u}{\partial u/\partial r}\right| + \frac{\beta l_m}{\mathrm{Pr}_t}\left|\frac{\partial T}{\partial r}\right| - \left(\frac{\beta l_m}{\mathrm{Pr}_t}\frac{\partial T}{\partial r}\right)^2\right] \tag{14}$$

where the overbars on the basic time-averaged variables are omitted for the sake of simplicity.

According to Eq. (14), the dynamical turbulent viscosity for compressible variable property flows is expressed as

$$\mu_t = \rho l_m^2 \left|\frac{\partial u}{\partial r}\right| (1 + F_m + F_{BR}) \tag{15}$$

where F_m and F_{BR} are defined as follows:

$$F_m = \frac{\beta u}{\mathrm{Pr}_t}\left|\frac{\partial T/\partial u}{\partial r/\partial r}\right| = \frac{\beta q_w d}{\lambda_b}\frac{U}{\mathrm{Pr}_t}\left(\frac{|\partial\Theta/\partial U|}{|\partial\eta/\partial\eta}\right) \tag{16}$$

$$F_{BR} = \frac{\beta l_m}{\mathrm{Pr}_t}\left|\frac{\partial T}{\partial r}\right| - \left(\frac{\beta l_m}{\mathrm{Pr}_t}\frac{\partial T}{\partial r}\right)^2 = \frac{\beta q_w d}{\lambda_b}\frac{l_m}{R}\frac{1}{\mathrm{Pr}_t}\left|\frac{\partial\Theta}{\partial\eta}\right| - \left(\frac{\beta q_w d}{\lambda_b}\frac{l_m}{R}\frac{1}{\mathrm{Pr}_t}\left|\frac{\partial\Theta}{\partial\eta}\right|\right)^2 \tag{17}$$

F_m describes the effect of $\bar{u}\overline{\rho'v'}$ on the turbulent transport; F_{BR} describes the effect of $\bar{v}\overline{\rho'u'}$ and $\overline{\rho'u'v'}$ on the turbulent transport. Bellmore and Reid [17] introduced the influence of F_m into the turbulent stream function during the calculation, and the dynamical turbulent viscosity defined by them is as follows:

$$\mu_{t,BR} = \rho l_m^2 \left|\frac{\partial u}{\partial r}\right| \left[1 + \frac{\beta l_m}{\mathrm{Pr}_t}\left|\frac{\partial T}{\partial r}\right| - \left(\frac{\beta l_m}{\mathrm{Pr}_t}\frac{\partial T}{\partial r}\right)^2\right] = \rho l_m^2 \left|\frac{\partial u}{\partial r}\right| (1 + F_{BR}) \tag{18}$$

Hence, the model suggested by Bellmore and Reid is used with the turbulent stream function, which makes the computation quite complicated.

For the enhanced heat transfer regime at supercritical pressures, in the present study the effect of F_{BR} on heat transfer is assumed to be negligible compared with the effect of F_m on heat transfer, which will be verified in the following part of the present paper. According to the assumption, Eq. (15) is reduced as follows:

$$\mu_t = \rho l_m^2 \left|\frac{\partial u}{\partial r}\right| (1 + F_m) = \mu_{t,0}(1 + F_m) \tag{19}$$

where $\mu_{t,0}$ is the dynamical turbulent viscosity calculated by the original mixing length model. Equation (19) is the modified mixing length model proposed by the present study and its dimensionless form can be expressed as

$$\frac{\mu_t}{\mu} = \frac{\mu_b}{\mu}\frac{\rho}{\rho_b}\frac{\mathrm{Re}_b}{2}\left(\frac{l_m}{R}\right)^2 \left|\frac{\partial(U/U_m)}{\partial\eta}\right| (1 + F_m) \tag{20}$$

Based on the definition of the turbulent shear stress, there is a more convenient way to take into account the effect of density variation on the turbulent transport. If the density is variable, the turbulent shear stress could be expressed as follows:

$$\tau_t = \nu_{t,0}\left|\frac{\partial(\rho u)}{\partial r}\right| = \rho\nu_{t,0}\left|\frac{\partial u}{\partial r}\right| \left|1 + u\frac{\partial\rho}{\partial r}/\rho\frac{\partial u}{\partial r}\right| \tag{21}$$

According to Eq. (21) the dynamical turbulent viscosity for variable property flows under heating conditions is determined by

$$\mu_t = \mu_{t,0}\left(1 + F'_m\right) \tag{22}$$

where F'_m is defined as follows:

$$F'_m = \left(u\frac{\partial\rho}{\partial r}\right) \Big/ \left(\rho\frac{\partial u}{\partial r}\right) \approx \beta u \left|\frac{\partial T}{\partial r}\Big/\frac{\partial u}{\partial r}\right| \tag{23}$$

If the turbulent Prandtl number is 1, $F_m = F'_m$, and Eq. (19) is equivalent to Eq. (22).

The Nikuradse–Van Driest mixing length model [27] for the constant-property pipe flow serves as the original model, which is expressed as follows:

$$\frac{l_m}{R} = (DF)_V \cdot \frac{(l_m)_N}{R} = \left[1 - \exp\left(-\frac{y^+}{26}\right)\right] \cdot \left(0.14 - 0.08\eta^2 - 0.06\eta^4\right) \tag{24}$$

The dimensionless distance y^+ in Eq. (24) is determined by its standard definition or by Goldmann's definition [28], which are given next.

Standard definition:

$$y^+ = \frac{y\sqrt{\tau_w/\rho}}{\nu} = (1-\eta)\sqrt{\frac{\rho}{\rho_b}\frac{\mu_w\mu_b}{\mu^2}\frac{\mathrm{Re}_b}{2}\left|\frac{\partial(U/U_m)}{\partial\eta}\right|_{\eta=1}} = \frac{1-\eta}{2}\frac{\mu_b}{\mu}\sqrt{\frac{\rho}{\rho_b}\frac{\mathrm{Re}_b}{U_m}} \tag{25}$$

Goldmann's definition:

$$y^+ = \int_0^y \frac{\sqrt{\tau_w/\rho}}{\nu}\mathrm{d}y = \int_\eta^1 \sqrt{\frac{\rho}{\rho_b}\frac{\mu_w\mu_b}{\mu^2}\frac{\mathrm{Re}_b}{2}\left|\frac{\partial(U/U_m)}{\partial\eta}\right|_{\eta=1}}\mathrm{d}\eta = \int_\eta^1 \left(\frac{1}{2}\frac{\mu_b}{\mu}\sqrt{\frac{\rho}{\rho_b}\frac{\mathrm{Re}_b}{U_m}}\right)\mathrm{d}\eta \tag{26}$$

For constant property flows, these two definitions are equivalent to each other. Goldmann's definition is applied to the supercritical fluid flows in the present study.

Numerical Method

The dimensionless governing equations of momentum (4) and energy (5) are discretized according to the finite-volume method. By using the cell-centered scheme, the computational domain is discretized into a nonuniform mesh, with the grid compressed at a constant ratio (about 1.08–1.1) toward the wall. In the present paper, the grid number in the radial direction is set to be about 150–200 to ensure that the dimensionless distance y^+ of the first node next to the wall is smaller than 0.1.

During the iterative procedure, the thermodynamic and transport properties of water are updated according to the source code of the IAPWS-95 formulation [29]. Owing to the great variation of the properties of water in the pseudocritical region, relaxation factors ranged from 0.5 to 0.8 are introduced for U, Θ, μ_t, and the thermophysical properties, to guarantee the stability of convergence. The convergence criteria are as follows:

$$\left|\frac{\phi^{i+1}-\phi^{i}}{\phi^{i}}\right| \leq 10^{-6}, \quad \phi = U \text{ or } \Theta \tag{27}$$

NUMERICAL SIMULATION FOR CONSTANT-PROPERTY FLOWS

First, numerical simulation is performed for constant-property pipe flows to check the validity of the present mathematical models. The computational results are compared with the following Dittus–Boelter correlation and the Petukhov correlation [30]:

Dittus–Boelter correlation:

$$\mathrm{Nu} = 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^{0.4} \tag{28}$$

Petukhov correlation:

$$\mathrm{Nu} = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7\,(f/8)^{1/2}\left(\mathrm{Pr}^{2/3}-1\right)} \tag{29}$$

where the Darcy friction coefficient is calculated by the Filonenko correlation:

$$f = (1.82\lg\mathrm{Re} - 1.64)^{-2} \tag{30}$$

For incompressible flows, both F_m and F_{BR} are equal to 0, and the Nusselt number Nu is related to the Reynolds number Re alone if the Prandtl number Pr is given. Figure 2 shows the simulation for incompressible flows at Re = 10,000–120,000 with Pr = 1. It can be seen that the turbulent Prandtl number Pr_t has influence on the computational results. As the Pr_t is increased, the Nu is decreased at a fixed Re. When the Pr_t is set to be 0.9, the computational results agree well with the Dittus–Boelter correlation, with a discrepancy less than 3%. When the Pr_t is set to be 1, the computational results agree well with the Petukhov correlation, also with a discrepancy less than 3%.

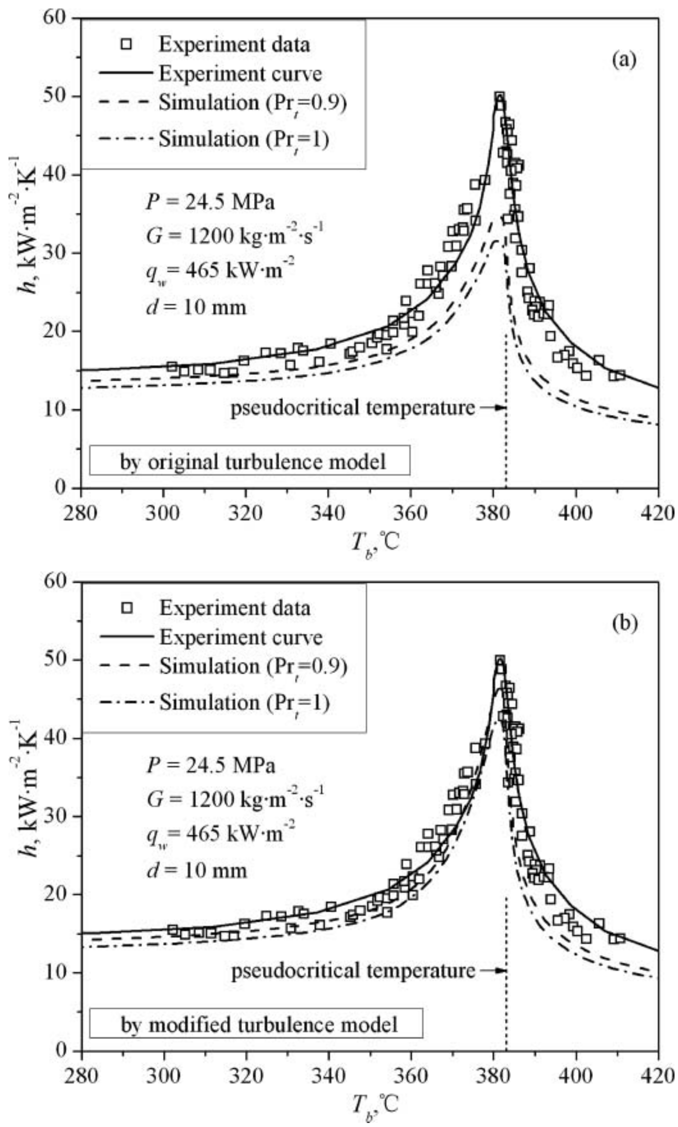

Figure 4 Comparison of simulation with the experimental data by Yamagata et al. [9] for a typical supercritical heat transfer: (a) calculated by the original turbulence model; (b) calculated by the modified turbulence model.

When the pressure is far from the critical pressure (about 22.064 MPa for water), the variation in thermophysical properties is small and has little effect on heat transfer. Figure 3 shows the simulation for ultra-supercritical fluid flows at a pressure P of 50 MPa, a mass velocity G of 1000 kg m^{-2} s^{-1}, a wall heat flux q_w of 300 kW m^{-2}, and a tube diameter d of 10 mm. The Pr_t is set to be 0.9 in the calculation. The figure shows that in the low-enthalpy region the computational results lie between the results of the Dittus–Boelter correlation and the results of the Petukhov correlation; in the high-enthalpy region, the computational results agree well with the correlations; the results calculated by the original turbulent model are a little smaller

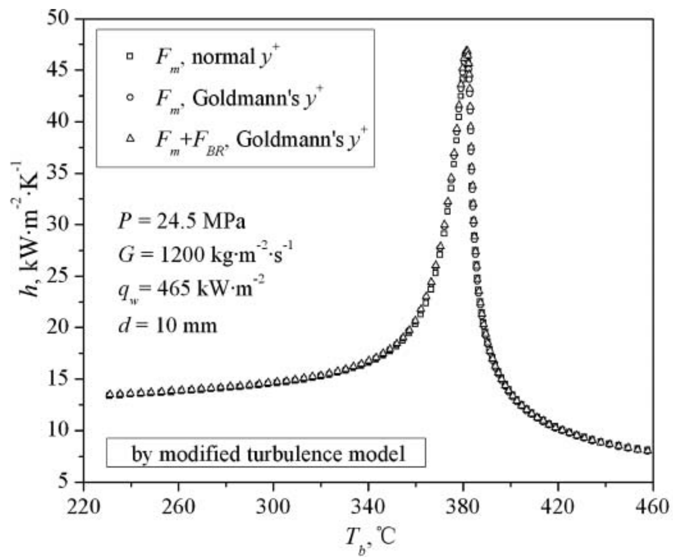

Figure 5 Influence of F_{BR} and y^+ on computational results.

than those calculated by the modified model.

NUMERICAL SIMULATION FOR SUPERCRITICAL FLUID FLOWS

Comparison With a Typical Supercritical Heat Transfer in Vertical Upward Flows

A typical supercritical heat transfer ($P = 24.5$ MPa, $G = 1200$ kg m^{-2} s^{-1}, $q_w = 465$ kW m^{-2}, $d = 10$ mm) in vertical upward flows is simulated first. Compared with the experimental data of Yamagata et al. [9], the calculated heat transfer coefficients by using the original turbulence model and the modified turbulence model are shown against bulk enthalpy in Figure 4a and b, respectively. As can be seen from the figure, the heat transfer is enhanced remarkably in the pseudocritical region, and the heat transfer coefficient reaches a maximum value near the pseudocritical point, which is similar to the way the isobaric specific heat varies with the temperature. The original turbulent model significantly underestimates the heat transfer coefficient near and above the pseudocritical temperature compared with the experimental results; however, the results by the present modified model are in good agreement with the experimental results. It is also concluded that when $\mathrm{Pr}_t = 0.9$, the simulation for the supercritical fluid flows is more precise than when $\mathrm{Pr}_t = 1$. Hence, the Pr_t is set to be 0.9 for the following simulation in the present paper.

As shown in Figure 5, different definitions (standard definition and Goldmann's definition) of the dimensionless distance y^+ in the Van Driest damp function exert little influence on the computational results. Meanwhile, the influence of F_{BR} on the heat transfer can also be neglected, which proves the validity of the assumption mentioned earlier in the present paper.

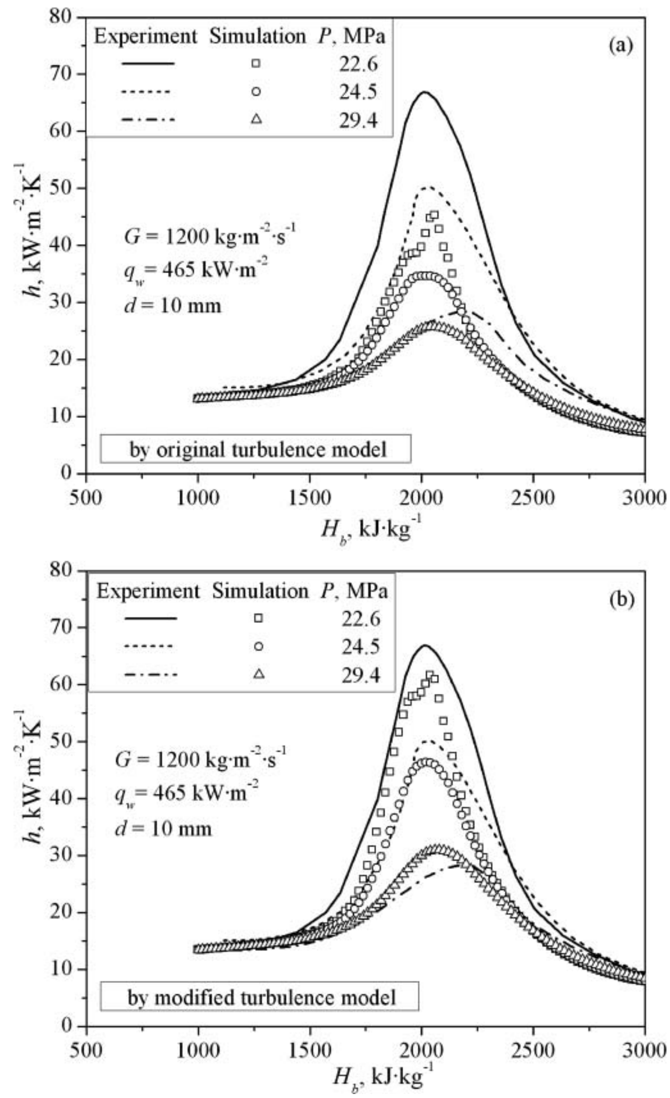

Figure 6 Effect of pressure on heat transfer (heat transfer coefficients versus bulk enthalpy): (a) calculated by the original turbulence model; (b) calculated by the modified turbulence model.

Effect of Pressure on Heat Transfer

Figure 6 shows the simulation of heat transfer for supercritical fluid flows at pressures P of 22.6 MPa, 24.5 MPa, and 29.4 MPa, with mass velocity G of 1200 kg m^{-2} s^{-1}, wall heat flux q_w of 465 kW m^{-2}, and tube diameter d of 10 mm. The results are compared with the experimental curve of vertical upward flows by Yamagata et al. [9]. As is shown in the figure, the pressure exerts significant effect on the heat transfer at supercritical pressures. The heat transfer coefficient (including its maximum value) near the pseudocritical point is increasing as the pressure is getting closer to the critical pressure, which is similar to the way the isobaric specific heat varies with the pressure. Therefore, it is reasonable to conclude that the enhancement of the heat transfer in the pseudocritical region is primarily attributed to the rapid increase of the isobaric specific heat. Figure

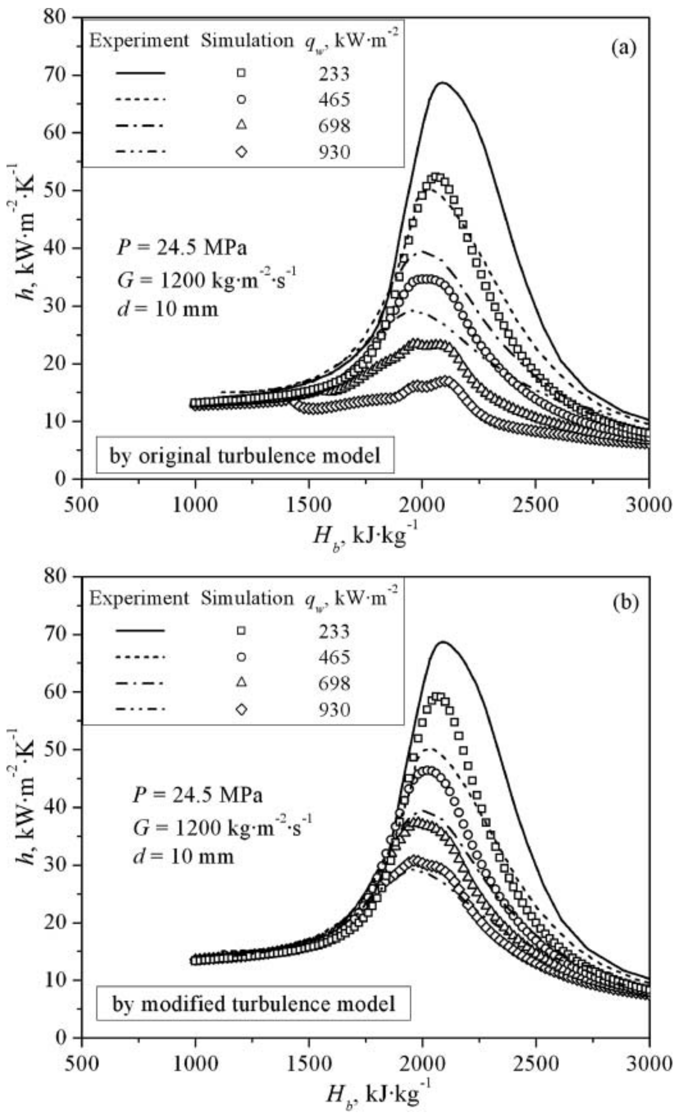

Figure 7 Effect of wall heat flux on heat transfer (heat transfer coefficients vs. bulk enthalpy): (a) calculated by the original turbulence model; (b) calculated by the modified turbulence model.

6a shows that the heat transfer coefficient in the pseudocritical region is obviously underestimated by the original turbulence model; however, the simulation in Figure 6b presents that the proposed model produces a much more precise prediction compared to the original model. When pressure is getting closer to the critical pressure, the thermophysical properties exhibit more rapid variation with the change of temperature and pressure, which usually leads to worse predictions by both of the turbulence models.

Effect of Wall Heat Flux on Heat Transfer

The effect of wall heat flux on heat transfer for supercritical fluid flows is simulated by the original and modified turbulence models at a pressure of 24.5 MPa, a mass velocity of 1200 kg

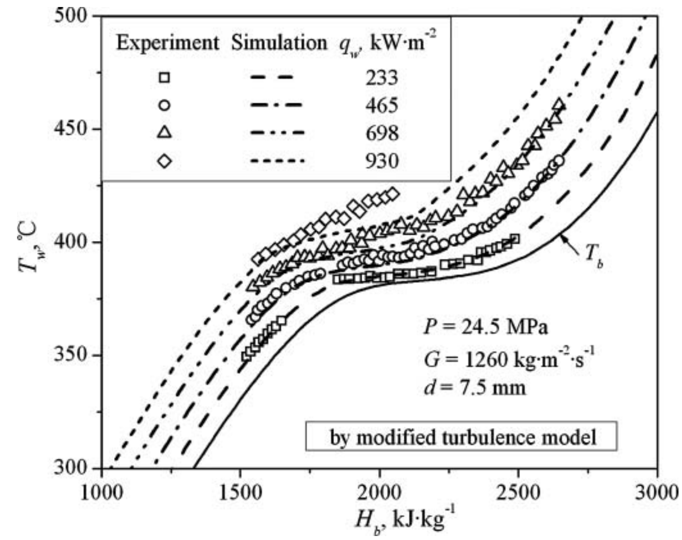

Figure 8 Effect of wall heat flux on heat transfer (wall temperature vs. bulk enthalpy).

m^{-2} s^{-1}, a tube diameter of 10 mm, and heat fluxes of 233 kW m^{-2}, 465 kW m^{-2}, 698 kW m^{-2}, and 930 kW m^{-2}. In Figure 7 a comparison is made with the experimental curve of the vertical upward flows produced by Yamagata et al. [9]. It is evident from the figure that the heat transfer coefficient in the pseudocritical region decreases with increasing heat flux, i.e., enhancement of heat transfer is gradually suppressed as the heat flux is increased. Figure 7a shows that the predicted results by the original model are much smaller than the experimental results and the prediction accuracy decreases rapidly with the increasing heat flux, which is similar to the results of the Jones–Launder low Re k–ε turbulence model [18]. As can be seen from Figure 7b, the prediction accuracy of the present modified model is significantly improved compared to that of the original model, especially in the case of heat transfer at relatively high heat flux. The phenomenon that the peak of the heat transfer coefficient decreases and moves to the lower bulk enthalpy as the heat flux increases is precisely simulated.

The modified model is also used to calculate the wall temperature distributions against the bulk enthalpy at heat fluxes q_w of 233 kW m^{-2}, 465 kW m^{-2}, 698 kW m^{-2}, and 930 kW m^{-2}, with pressure P of 24.5 MPa, mass velocity G of 1260 kg m^{-2} s^{-1}, and tube diameter d of 7.5 mm. Figure 8 shows the comparison between the calculated results and the experimental data about the vertical upward flows by Yamagata et al. [9]. The results agree well with the experimental data at low and moderate heat flux. When the heat flux is 698 kW m^{-2}, the numerical results are slightly smaller than the experimental data near the pseudocritical point. When the heat flux reaches 930 kW m^{-2}, the numerical results are a little smaller. This behavior at high heat flux is contrary to that of the Hassid–Poreh two-layer k-ε turbulence model and the standard high Re k-ε turbulence model [23]. The prediction accuracy of the present model is a little greater than that of the Hassid–Poreh two-layer model and of

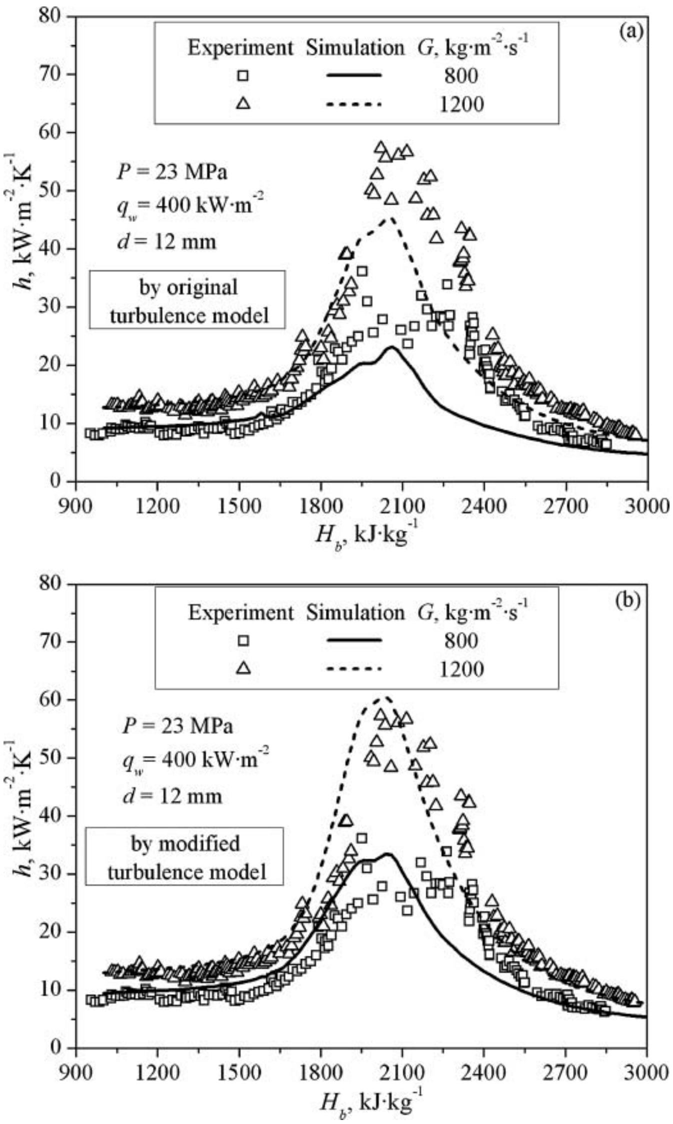

Figure 9 Effect of mass velocity on heat transfer (heat transfer coefficients vs. bulk enthalpy): (a) calculated by the original turbulence model; (b) calculated by the modified turbulence model.

the standard high Re model, and is much greater than that of some low Re k-ε turbulence models [26].

According to Figure 7b, the biggest discrepancies between the predictions and the experiment in the tube with a diameter of 10 cm occur at the lowest heat flux of 233 kW m^{-2}. However, according to Figure 8, the biggest discrepancies in the tube with a diameter of 7.5 cm occur at the highest heat flux of 930 kW m^{-2}. The possible reason for this phenomenon is the uncertainty of the experimental errors in the pseudocritical region, especially near the pseudocritical point. In the experiment, when the inlet or outlet temperature is close to the pseudocritical temperature, a small error of 1 or 2°C in measuring the bulk temperature can lead to significant miscalculations of enthalpies, and hence the miscalculations of the heat transfer coefficients. In fact, the experimental data about the heat transfer in the pseudocritical region obtained by many researchers are largely scattered.

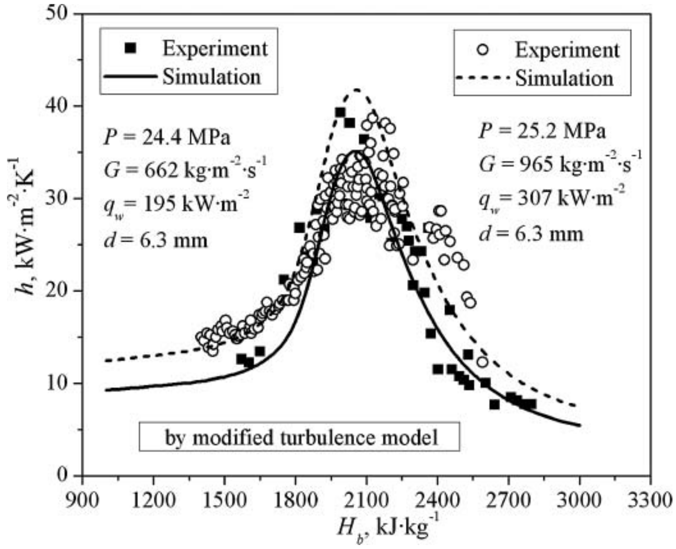

Figure 10 Comparison of simulation with experimental data by Bazargan [12] for horizontal flows.

Effect of Mass Velocity on Heat Transfer

Figure 9a and b show the calculated heat transfer against the bulk enthalpy at mass velocities G of 800 kg m^{-2} s^{-1} and 1200 kg m^{-2} s^{-1}, with $P = 23$ MPa, $q_w = 400$ kW m^{-2}, and $d = 12$ mm, by using the original turbulence model and the modified turbulence model, respectively. Comparison is made with the experimental data of vertically upward flows by Xu [13]. The figure illustrates that the effect of the mass velocity on the heat transfer at supercritical pressures is similar to what happens under constant-property conditions. The higher the mass velocity is, the higher the convective heat transfer coefficient is. When the bulk enthalpy is less than 1800 kJ kg^{-1}, the prediction by the original turbulence model agrees well with the experimental data. But the original model greatly underestimates the heat transfer coefficient in the pseudocritical region with high bulk enthalpy, especially at the lower mass velocity. The modified model is able to produce a more precise prediction in the whole enthalpy region, and the prediction accuracy is relatively free of the effect of the mass flux.

Comparison With Experimental Data for Horizontal Flows

The modified turbulence model in the present paper is also applicable to the buoyancy-unaffected horizontal flow at supercritical pressures, which usually occurs at low or moderate heat flux and relatively high mass velocity. Figure 10 presents the comparison of the simulation with the experimental data by Bazargan [12] in the two cases of the heat transfer in a horizontal tube. As is shown in the figure, the predictions by the present model are in good agreement with the experimental data.

CONCLUSIONS

For the supercritical fluid pipe flows unaffected by buoyancy and acceleration, a 1-D numerical model based on the boundary theory is developed, which not only simulates the heat transfer of the variable-property flows but also makes the computation more efficient and straightforward. Taking into account the density fluctuations in the equations of turbulent transport with the mixing length theory, a new modified turbulence model based on a conventional model for constant property pipe flows is proposed, which can be applied to both constant-property flows and variable-property flows.

Numerical simulation by the proposed mathematic models is carried out for water flowing in circular tubes at supercritical pressure. The computational results show that the conventional turbulence model applicable to the incompressible pipe flows cannot be directly applied to the supercritical fluid flows, as it usually significantly underpredicts the heat transfer coefficient in the pseudocritical region, especially as the pressure is getting closer to the critical pressure or the heat flux is relatively high. However, the predictions by the present modified turbulence model are in good agreement with the experimental data provided by other researchers. The results also indicate that the calculated heat transfer coefficients are somewhat affected by the value of the turbulent Prandtl number, which slightly decrease with the increasing turbulent Prandtl number. For the enhanced heat transfer regime at the supercritical pressures, different definitions of the dimensionless distance y^+ (the standard definition and the Goldmann's definition) have little effect on the calculated results, and the effect of F_{BR} (the correction factor of density fluctuations by Bellmore and Reid) on heat transfer is negligible.

The present mathematic models prove to be capable of accurately predicting the heat transfer in supercritical fluid flows unaffected by the buoyancy and the acceleration. However, to properly account for the more complicated cases where the effects of buoyancy and/or acceleration cannot be neglected, a 2-D or three-dimensional (3-D) mathematic model is imperative. There are two questions to be faced. First, the conventional complicated turbulence models (e.g., the k-ε models) should be improved for their application to the variable property flows. Second, the conventional SIMPLE-type algorithms need to be modified; the compressibility effects (i.e., density variations) have to be taken into account in the pressure correction equations. With these two problems solved, the deteriorated heat transfer at supercritical pressures would be accurately predicted.

NOMENCLATURE

c_p specific isobaric heat capacity, J kg^{-1} K^{-1}
d tube diameter, m
f Darcy friction coefficient
F_m correction factor of density fluctuations
F_{BR} correction factor of density fluctuations by Bellmore and Reid
G mass velocity, kg m^{-2} s^{-1}
h heat transfer coefficient, W m^{-2} K^{-1}
H specific enthalpy, J kg^{-1}
l_m mixing length, m
Nu Nusselt number
P pressure, Pa
Pr Prandtl number
q heat flux, W m^{-2}
r radial coordinate, m
R tube radius, m
Re Reynolds number
T temperature, K
u axial velocity, m s^{-1}
u_m cross-sectional mean velocity, m s^{-1}
v radial velocity, m s^{-1}
U dimensionless axial velocity
U_m dimensionless cross-sectional mean velocity
x axial coordinate, m
y distance from the wall, m
y^+ dimensionless distance

Greek Symbols

Θ dimensionless excess temperature
β isobaric thermal expansion coefficient, K^{-1}
η dimensionless radius
λ thermal conductivity, W m^{-1} K^{-1}
μ dynamic viscosity, N s m^{-2}
ν kinematic viscosity, m^2 s^{-1}
ρ density, kg m^{-3}
τ shear stress, N m^{-2}

Subscripts

b bulk condition
t turbulent condition
w wall condition

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