

# An Image Coding Approach Using Wavelet-Based Adaptive Contourlet Transform

Guoan Yang, Zhiqiang Tian, Chongyuan Bi, Yuzhen Yan

Institute of Artificial Intelligence and Robotics, Xi'an Jiaotong University, Xi'an 710049, China  
gayang@mail.xjtu.edu.cn

## Abstract

*In this paper, we propose an image coding approach using wavelet-based adaptive contourlet transform. The approach adopts the wavelet transform to implement subband decomposition to replace Laplace pyramid decomposition in contourlet transform, and thus the redundancy caused by the Laplace pyramid decomposition can be eliminated. Moreover, it chooses the directional filter banks as a key technique, combine SPIHT code, and realizes image compression coding called wavelet-based adaptive contourlet transform. The experiment results show that our approach is comparatively more effective than wavelet, contourlet and wavelet-based contourlet transform in terms of accurately capturing the characteristics of the texture and the edge of the image.*

## 1. Introduction

Wavelet transform has achieved great success in the field of image compression [1-3]. However, due to the isotropic of wavelet transform, there are still disadvantages existing in representing the directions of image information and the details of texture. A method named multi-scale geometric analysis (MGA) has been developed in recent years, and this method has the demonstrational function of anisotropy, which is a more effective method in sparse representation for multi-dimensional discrete signals [4-5]. Moreover, the method sufficiently utilizes the geometric features of the images. Therefore it can better represent edges, textures and structures for image compression, image enhancement, image restoration and noise removal etc [6-8]. Thereinto, contourlet transform is one of the most typical approaches in MGA.

Although the contourlet transform has many advantages than the wavelet, it has some inadequacies, for example, the SPIHT coding based on contourlet transform, with the code rate increasing, sometimes the peak signal to noise ratios (PSNR) is dropping. This is largely because the Laplacian pyramid (LP)

decomposition of contourlet transform has some redundancy. Although LP decomposition and direction filter banks (DFB) can perfectly reconstruct the original signal, the pyramid direction filter banks (PDFB) which include LP decomposition, is of overabundance sampling and has exactly  $4/3$  degrees of redundancy. Therefore the final contourlet transform also inevitably has  $4/3$  degrees of redundancy. This causes the coefficients increment after the contourlet transform, which is a great disadvantage to image compression coding [9]. Clearly the wavelet transform is a kind of critical sample, which will not produce any redundancy, hence the wavelet-based contourlet transform called BWCT has been developed in recent years [10-12]. In this paper, we have proposed a new approach based on the BWCT that with wavelet instead of LP, and with adaptive DFB (ADFB) instead of DFB in the contourlet transform, thus, form the basic arithmetic called wavelet-based adaptive contourlet transform. We name it the WBACT.

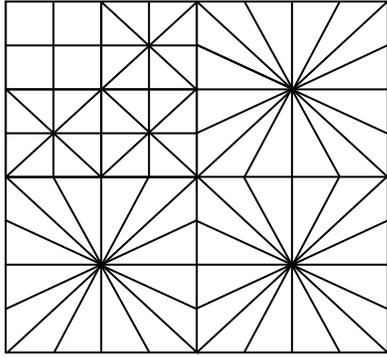
## 2. The WBACT

We know that it is hard to accurately capture the characteristics of the texture and the edge of an image with wavelet transform; also the contourlet transform will bring several problems such as redundancy, frequency aliasing and so on. To solve the problem above and improve the coding efficiency, the WBACT decomposes the original image into  $J$  levels using wavelet transform to replace LP in contourlet transform, and then it implements the adaptive direction transform of the subband using the DFB for primal two levels with 6 subbands. Then, according to the character of the wavelet decomposition and direction filter, it can construct the expanded structure of the space direction-tree. The implementation approach is as below.

For a wavelet decomposition of level  $J$ , we choose the high frequency subbands LH, LH and HH, which contain detailed information to implement the object of direction decomposition. In order to further improve

the coding efficiency, the paper selects the extremely high frequency and hypo-high frequency subbands. Direction transform of the same decomposition number is implemented at the same levels. To satisfy the anisotropic filter, we can implement direction transform of the most decomposition number at the highest-level  $J$  of wavelet decomposition. Figure 1 indicates 3 levels of wavelet decomposition and directional decomposition in different levels.

We have done directional decomposition of 3 levels to the subband of the highest level, adding up to 8 direction subbands, and 2 levels to the subband of the hypo-highest level, adding up to 4 direction sub-bands.



**Figure 1. The schematic plot of the WBACT using 3 dyadic wavelet levels and 8 directions at the finest level.**

The WBACT's directional characteristics of multi resolution and its construction in continuous area are as follows: for the separable and orthogonal wavelet, they have the characteristic of separable two-dimension multiresolution:  $V_{j-1}^2 = V_j^2 \oplus W_j^2$ ,  $W_j^2 = W_j \otimes W_j$

where  $W_j^2$  is the orthogonal complement subspace of  $V_{j-1}^2$  and  $V_j^2$ , which represent the detail weight of the image. Family  $\{\Psi_{j,n}^1, \Psi_{j,n}^2, \Psi_{j,n}^3\}_{n \in \mathbb{Z}^2}$  is  $W_j^2$ 's one sets of orthogonal basis. By implementing  $\ell_j$  levels direction decomposition to the detailed subspace  $W_j^2$ , we can get  $2^{\ell_j}$  direction subband of  $W_j^2$ :

$$W_j^2 = \bigoplus_{k=0}^{2^{\ell_j-1}} W_{j,k}^{2,(\ell_j)}$$

We define

$$\eta_{j,k,n}^{i,(\ell_j)} = \sum_{m \in \mathbb{Z}^2} g_k^{(\ell_j)} [m - S_k^{(\ell_j)} n] \Psi_{j,m}^i$$

thus  $\{\eta_{j,k,n}^{1,(\ell_j)}, \eta_{j,k,n}^{2,(\ell_j)}, \eta_{j,k,n}^{3,(\ell_j)}\}$  is the basis of directional

subspace  $W_{j,k}^{2,(\ell_j)}$ .

For the wavelet, it has the features of local time-frequency. Therefore, there is similarity among the wavelet transform coefficients when the WBACT describe the same space position of an image in different scales, which is called the self-similarity between different scales. Because of the structure of multiresolution, wavelet coefficients at the same space position have the quad-tree structure between the low frequency subband and the neighboring high frequency one in the same direction, which can effectively describe the local time-frequency characteristic in the wavelet transform domain. Thus, if we assume that the threshold value is  $T$ , it can be considered correct that the grandson in a larger scale of an unimportant coefficient of wavelet  $a_{i,j}$  ( $|a_{i,j}| < T$ ) is also an unimportant coefficient. In the other scale transforms, there is also a self-similarity of inter-scales with wavelet. So there is also a corresponding tree structure between the subbands.

To contourlet transform, the LP decomposition is dyadic both in horizontal and vertical direction, so the coefficients of the low frequency subband have four sons in the neighboring high frequency one. According to the different numbers of directional subband, there will be different mapping relations between the father coefficients and the son coefficients. If the subband has the same decomposition number, then the sub coefficient of low frequency subband will be mapped to the relevant directional subband of high frequency. In addition, the directional subband of high frequency has as many as twice of the number of low frequency, while the number of the directional decomposition of the neighboring high frequency subband is one more than that of the low frequency one. Then, the sub coefficient of low frequency subband will be mapped to the two neighboring directional subbands of high frequency one. The two directional subbands can be gotten from the further decomposing of the father coefficient's directional subband. In addition, the WBACT makes direction decomposing to the wavelet's subband, so the mapping relations between the father coefficient and the son coefficient of LH, HL, HH subband are similar to the contourlet transform. The tree structure is shown in Figure 2.

### 3. WBACT-Based SPIHT Coding

Since WBACT adopts wavelet to implement the multiscale decomposition, then it has multiresolution structure similar to wavelet, just different in the mapping of its parent or child coefficients. Therefore, we put the coefficients of WBACT in order to make

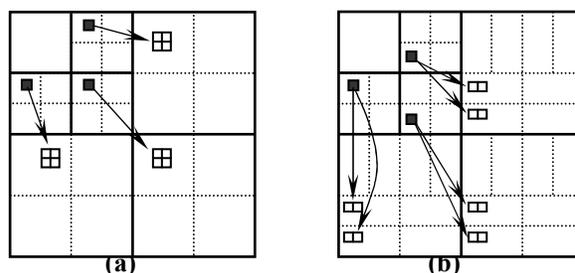
them in the same quad-tree relationship similar to wavelet; then we can encode the WBACT coefficients with the SPIHT coding method. The Figure 3 shows the ordered subband of WBACT decomposed eight-dimensionally. When the decomposition numbers of the above level is 4, Figure 3(a) shows that every two neighboring vertical or horizontal subbands of the 8 subbands corresponding to one vertical or one horizontal subband of the above level. So the positions of child-coefficients are different from the wavelet transform. Take a  $512 \times 512 \times 8$  image for example. The LH subband of highest frequency is  $256 \times 256$ . The child coefficient of (1,1) of the second highest frequency is mapped into 4 spots: (1,1), (1,2), (128,1) and (128,2) of the highest frequency. In order to get the position distribution of child-coefficients, as the Figure 3(b) shows, we just need to exchange the coefficient positions of two subbands to aggregate spots with the same parent-coefficient.

An example of repositioning a radial subband in the WBACT having 8 directional subbands assuming its coarser subband is at first level and has 4 directional decompositions. In this process we combine each two neighboring directional subbands by interlacing the columns of horizontal directional subbands and the rows of vertical directional subbands.

As mentioned above, the coefficients of WBACT need to be put in order before the encoding of WBACT. Finally, the SPIHT encoding method is adopted to complete the image compressing. Figure 4 shows the flow chart. The SPIHT-like encoding method here is an improvement of SPHIT coding method on wavelet, which is more suitable for the characteristics of coding of WBACT.

#### 4. Experiment Results and Analysis

The WBACT experiment of the wavelet transform still chooses biorthogonal wavelet CDF 9/7, and it is decomposed into 5 levels, as the directional information of the image mainly exists in the subband of high frequency. At the same time, for the sake of the improvement in the calculation speed, this paper only makes directional decomposition for those highest or the second highest frequencies after multiscale decomposition. The directional decomposition number of the second highest frequency is 4, and the highest one is 8. Here, we still adopt PSNR as the objective criterion of the quality evaluation. The test image Barbara and Lena of the  $512 \times 512 \times 8$  standard gray image is chosen as the examples; meanwhile, the SPIHT compression coding based on wavelet compared with the mixed transform coding based on wavelet-contourlet also has 5 levels in decomposition.



- (a) the number of directional subbands are the same at the adjacent two wavelet scales. Here we have 4 directions at each wavelet subbands.
- (b) the numbers of directional subbands in the finer wavelet scale is twice as many as those in the coarser wavelet scale.

Figure 2. Two possible parent-child relationship in the WBACT

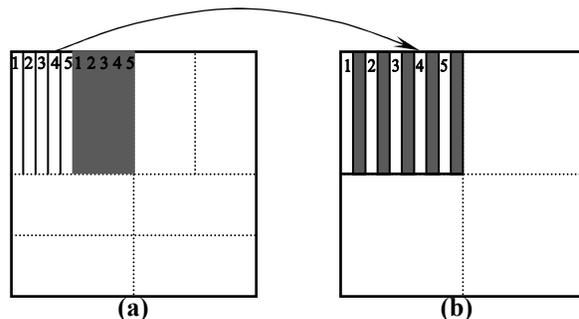


Figure 3. Ordered sub-band of WBACT

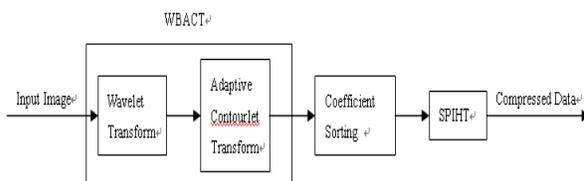
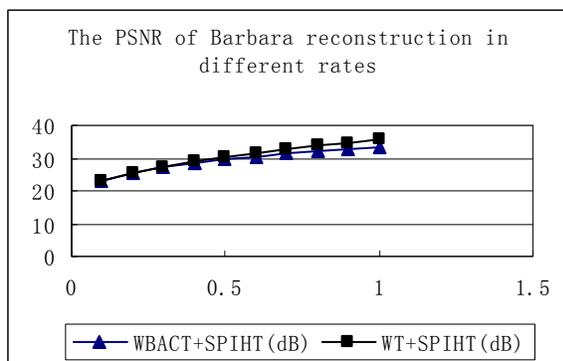


Figure 4. Flow chart of image coding using WBACT

Table 1. The PSNR of Barbara reconstruction in different rates

Rates	WBACT+SPIHT	WT+SPIHT
0.1	23.2277	23.2456
0.2	25.4622	25.4342
0.3	27.1599	27.2204
0.4	28.2254	28.9126
0.5	29.4636	30.3316
0.6	30.4220	31.4723
0.7	31.2362	32.5747

0.8	31.9774	33.6574
0.9	32.6841	34.6166
1.0	33.6022	35.5250



**Figure 5. Comparison with WBACT and Wavelet coding**

Table 1 shows the PSNR comparison between the SPIHT coding based on WBACT and the one based on wavelet. From this image, we can see that the PSNR of WBACT is a little higher than that of using wavelet in both intermediate and high frequency, and with the increment of bit rate, the PSNR of wavelet exceeded that of WBACT. Or rather, when the compression rate is somewhat higher, the result of the reconstructed image based on WBACT is better than that based on wavelet. Furthermore, an important result is demonstrated in the experiment of the SPIHT coding based on contourlet transform, i.e., with the bit rate increasing, yet the PSNR drop is completely disappeared. It also has proved that the construction of WBACT resolved the problem of the redundancy in LP decomposition, which has become arithmetic in deed applied on the coding of compressed image. As mentioned above, we evaluate the reconstructing effect by the value of PSNR, which is a relatively objective way to some degree. However, from the visual perspective, we can obviously find from Figure 6, whatever bit rate it is, the image reconstructing effect based on WBACT is better than that based on wavelet, and the details in the table cloth, scarf, and pant etc. appear more obvious. In Figure 7 we intentionally choose images of 0.25bpp to compare the details under two different kinds of image coding method. From Figure 7, we can see that if we adopt the SPIHT encoding method based on wavelet, the texture of the table cloth is very obscure, only some sparse spots remained. But, if we adopt the method based on WBACT presented in this paper, the result is satisfying. Not only is the texture of tablecloth very integrated, but also it is very clear. It proves that the

construction of WBACT not only gets over the defect of redundancy by the contourlet transform, but also it keeps the advantage of contourlet transform which is good at describing the texture and the edge of the image. Moreover, we find that other kinds of image also follow similar rules.



**(a) Original Image**

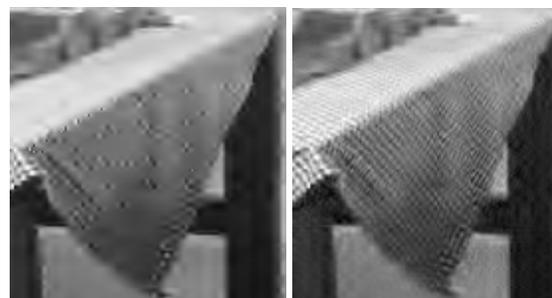


**(b) Left: Wavelet, Right: WBACT (0.25bpp)**



**(c) Left: Wavelet, Right: WBACT (0.50bpp)**

**Figure 6. Barbara reconstructed image in different rates**



**Left: Wavelet; Right: WBACT (0.25bpp)**

**Figure 7. Barbara reconstruction compare**

## 5. Conclusion

This paper presents a WBACT approach and the tree structures of the coefficients in the WBACT transform. After the careful analysis of the distribution of the coefficients in the WBACT transform, we put forward the method of SPIHT compression encoding based on WBACT by rearranging the transformed coefficients, which obtains better effect, based on the feature of the coefficient distribution. The experiment results show that if the same bit rate is used, this method can renew more information of the image details compared with the SPIHT encoding based on wavelet. At the same time, the PSNR is more similar to the latter one. Above all, it fully indicates that the WBACT transform is superior in the area of reconstructing the texture information from the compressed image.

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