LETTER

A Novel Design Approach for Contourlet Filter Banks

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SUMMARY This letter proposes a novel design approach for optimal contourlet filter banks based on the parametric 9/7 filter family. The Laplacian pyramid decomposition is replaced by optimal 9/7 filter banks with rational coefficients, and directional filter banks are activated using a pkva 12 filter in the contourlets. Moreover, based on this optimal 9/7 filter, we present an image denoising approach using a contourlet domain hidden Markov tree model. Finally, experimental results show that our approach in denoising images with texture detail is only 0.20 dB less compared to the method of Po and Do, and the visual quality is as good as for their method. Compared with the method of Po and Do, our approach has lower computational complexity and is more suitable for VLSI hardware implementation.

key words: *contourlet filter banks, 9/7 filter banks, directional filter banks, hidden Markov tree model, image denoising*

1. Introduction

Images can be effectively represented by wavelets [1], because wavelets possess multiscale and time-frequency localized characteristics. However, image representations in two-dimensional space using wavelets have only limited ability in capturing directional information. Therefore, multiscale and multi-directional representations have recently been presented for accurately capturing the intrinsic geometrical characteristics in an image. One of the most promising representations uses contourlets and is developed by Do and Vetterli [2]. Contourlets can efficiently represent the directional information of two-dimensional images, especially texture, contours and image details. The core issue of contourlets, which are composed of the Laplacian pyramid (LP) decomposition and directional filter banks (DFB) transform, is filter design. The contourlets not only possess the main features of wavelets, but also offer a high degree of directionality and anisotropy. Moreover, since the contourlets possess a rich set of basis functions, they can represent a smooth contour with fewer coefficients than wavelets.

However, the contourlets also have drawbacks, for example, the basis function regularity is not high enough, the locality of the time and frequency domain is not ideal, and spectrum aliasing also occurs. Therefore, the research on image denoising and image retrieval based on a hidden Markov tree (HMT) model in contourlet domain has been developed by Po and Do [3]. At present a few types of LP and DFB can be used for the contourlets, so that the existing wavelets and filter banks, among others the 9/7, 5/3, Haar and pkva filter, are utilized directly. In order to achieve practical application of contourlets for a wide class of image denoising, the new 9/7 filter banks with rational coefficients optimized in this letter are adopted in the LP decomposition stage, and the pkva 12 filter banks are used in the DFB transform stage [4].

This letter is organized as follows. In Sect. 2, we propose an optimal 9/7 filter with rational coefficients for image denoising using HMT model in the contourlet domain. In Sect. 3, we give the experimental results and their discussion. Finally, Sect. 4 states the conclusions and future work.

2. Design Approach of an Optimal 9/7 Filter for the Contourlets

According to Refs. [5], [6], and [7], we only need to design two lowpass filters, $H_0(z)$ and $G_0(z)$, of the 9/7 filter family on the analysis and synthesis stage, respectively. The 9/7 filter banks have 5 lowpass and 4 highpass filter coefficients, 4 lifting parameters, and 1 normalization constant. The polyphase matrix factorization for perfect reconstruction and the vanishing moment condition for the 9/7 filter banks (see [5], [6], and [7]) give us in total 13 independent equations for these 14 variables. Therefore, the variables can be described as functions of a single free parameter ξ. The filter coefficients are given in Eq. (1) and the lifting parameters and normalization constant in Eq. (2).

$$\begin{cases} h_0 = -(8\xi^3 - 18\xi^2 + 7\xi - 20)/(16\xi) \\ h_1 = (4\xi^3 - 11\xi^2 + 15\xi - 4)/(8\xi) \\ h_2 = (\xi - 2)/(4\xi) \\ h_3 = (4\xi^2 - 7\xi + 4)(\xi - 1)/(8\xi) \\ h_4 = (4\xi^2 - 7\xi + 4)(2\xi - 1)/(32\xi) \\ g_0 = (\xi + 1)/4 \\ g_1 = (2\xi + 7)/32 \\ g_2 = -(\xi - 1)/8 \\ g_3 = -(2\xi - 1)/32 \end{cases} \tag{1}$$

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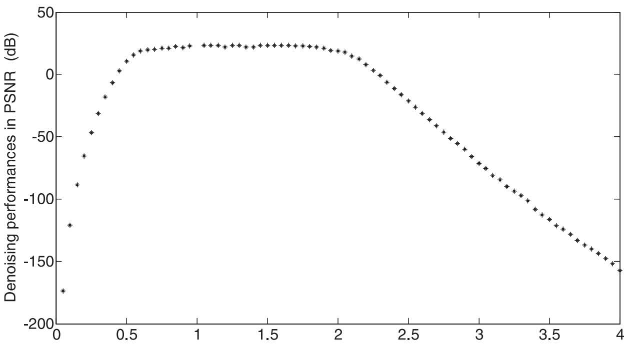


Fig. 1 Image denoising performances in different variable ξ.

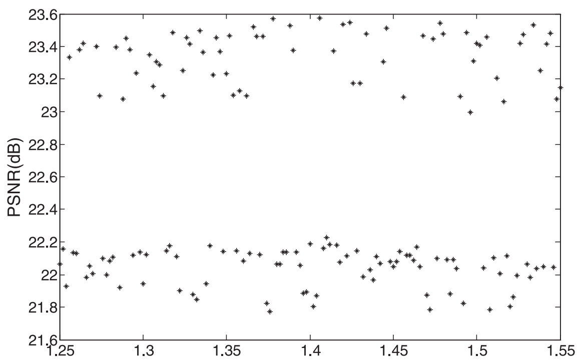


Fig. 2 Figure 1 zoomed in on range (1.25, 1.55).

$$
\begin{cases}
\alpha_1 = (1 - 2\xi)/[4(\xi - 1)] \\
\alpha_2 = -(\xi - 1)^2 \\
\alpha_3 = 1/[4\xi(\xi - 1)] \\
\alpha_4 = \xi^3 - 7\xi^2/4 + \xi \\
K = 2/\xi
\end{cases}
\tag{2}
$$

In the contourlets, we partition the finest and second finest scales into eight directional subbands, and the two next coarser scales into four directional subbands, and obtain a frequency partition. The images vary from simple edge-dominant images to highly textured images such as Barbara, an image of resolution 512×512 and 8 bits per pixel. The experimental platform is the same as for Ref. [3]. Here, the free variable ξ of the 9/7 filter family is selected in the range from 0 to 4 with an interval of 0.1. In addition, there is an LP decomposition with free variable ξ using the HMT model in the contourlet domain for image denoising [3], [8]. We carried out the image denoising experiments to the test image Barbara and the results are shown in Fig. 1.

In order to obtain optimal filter banks using contourlet HMT model for image denoising, we choose the free variable ξ from the range (1.25, 1.55) with interval 0.001. Figure 2 shows the results of Fig. 1 for this range. From Fig. 2 we can see that the results form two randomly ordered layers about denoising performance, so it is impossible to get the optimal point directly. However, we can get the optimal point statistically by data fitting. This letter chooses five datasets, formed by the values on the upper layers in Fig. 2, to complete the data fitting using a fourth order polynomial

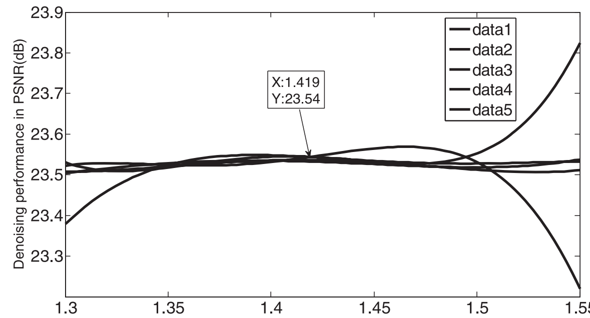


Fig. 3 Data fitting result about optimal samples.

Table 1 All the coefficients for optimal 9/7 filter banks.

	Analysis Filters		Synthesis Filters	
k	Lowpass Filter $\tilde{h}_k$	Highpass Filter $\tilde{g}_k$	Lowpass Filter h_k	Highpass Filter g_k
0	121/200	366389/355000	366389/355000	121/200
±1	123/400	−410797/710000	410797/710000	−123/400
±2	21/400	−29/284	−29/284	21/400
±3	−23/400	−55797/710000	55797/710000	23/400
±4		25139/710000	25139/710000	

for each dataset; the resulting curves are shown in Fig. 3. From this figure we can see that the five fitting curves almost focus on a single point at 1.419; we take 1.42 as the optimal point: $\xi_{opt} = 1.42$. Given ξ_{opt} all the coefficients and the lifting parameters of the optimal 9/7 filter banks are rational numbers and are determined completely through Eqs. (1) and (2). Moreover, if the above optimal filter banks are used in the contourlet HMT model and on the experimental platform presented by Po and Do [3], the image denoising performance in terms of the PSNR is maximum and equals 23.54 dB. Finally this denoising result with noise level of $\sigma = 50$ is only 0.2 dB less than the method of Po and Do in Ref. [3].

Finally, given ξ_{opt} the optimal 9/7 filter banks and their lifting scheme implementation can be calculated from Eqs. (1) and (2). All the coefficients of the analysis and synthesis filter can be completely determined, and they are shown in Table 1.

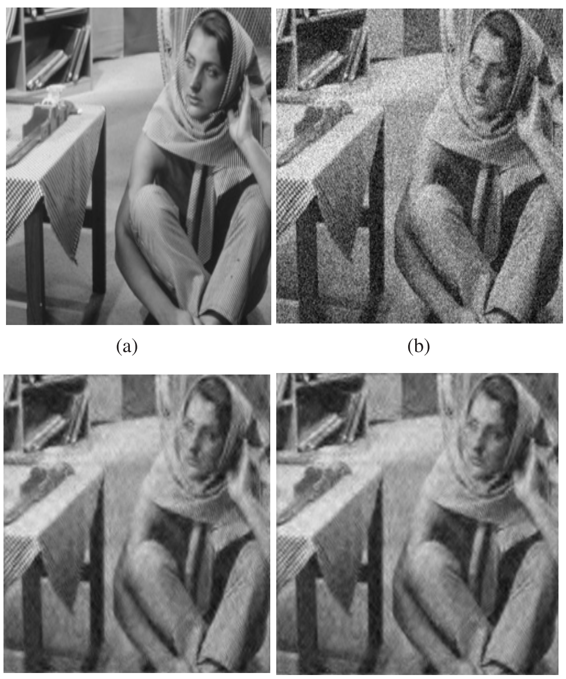
3. Results and Discussion

In Refs. [3], [6], Po and Do used the CDF 9/7 filter on the LP decomposition stage and pkva 12 filter on the DFB transform stage. This letter differs from [3] in the use of an optimal 9/7 filter banks with rational coefficients on the LP decomposition stage. As an objective comparison between the two methods, the image denoising effects for different images on various noise levels are shown in Table 2.

From Table 2 we conclude that our approach improved image denoising with 0.15 and 0.36 dB compared to Ref. [3] for images Lena and Zelda, respectively; it is only 0.27 dB less compared with the result of Ref. [3] for the Barbara im-

Table 2 Denoising effects comparison for different images on various noise levels (PSNR/dB).

Image	Noise level σ	Po and Do's method	our method
Lena	30	28.18	28.22
	40	27.00	27.18
	50	26.04	26.19
Barbara	30	25.27	25.12
	40	24.79	24.23
	50	23.74	23.47
Zelda	30	30.00	29.95
	40	28.29	28.72
	50	27.07	27.43



(a) (b) (c) (d)

Fig. 4 Subjective comparison for denoising performance (a) original image (b) noise image (c) Po and Do's result (d) our result.

age denoising while the noise level σ equals 50.

Finally, we present the subjective comparison using the optimal 9/7 filter banks with rational coefficients mentioned above on the LP decomposition stage and the pkva 12 filter on the DFB transform stage in the contourlet domain HMT model; the results are shown in Fig. 4. In Fig. 4 (b), Gaussian noise with $\sigma = 50$ is added to the original image. Comparing Fig. 4 (c) and (d) we can see that the visual quality using the image denoising approach of this letter is as good as the result of Ref. [3].

4. Conclusions

The optimal 9/7 filter banks designed in this letter have only rational coefficients, and thus the computational complexity can greatly be reduced and VLSI hardware implementation becomes easier, compared to the CDF 9/7 filter banks with irrational coefficients given by Po and Do. Although the optimal 9/7 filter banks above are designed based on the Barbara image, optimal 9/7 filter banks for other images can quickly be designed using our approach. For further research, we intend to design optimal filter banks for DFB transform stage in the contourlet domain HMT model.

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