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A mixture Weibull proportional hazard model for mechanical system failure prediction utilising lifetime and monitoring data

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ABSTRACT

As mechanical systems increase in complexity, it is becoming more and more common to observe multiple failure modes. The system failure can be regarded as the result of interaction and competition between different failure modes. It is therefore necessary to combine multiple failure modes when analysing the failure of an overall system. In this paper, a mixture Weibull proportional hazard model (MWPHM) is proposed to predict the failure of a mechanical system with multiple failure modes. The mixed model parameters are estimated by combining historical lifetime and monitoring data of all failure modes. In addition, the system failure probability density is obtained by proportionally mixing the failure probability density of multiple failure modes. Monitoring data are input into the MWPHM to estimate the system reliability and predict the system failure time. A simulated sample set is used to verify the ability of the MWPHM to model multiple failure modes. Finally, the MWPHM and the traditional Weibull proportional hazard model (WPHM) are applied to a high-pressure water descaling pump, which has two failure modes: sealing ring wear and thrust bearing damage. Results show that the MWPHM is greatly superior in system failure prediction to the WPHM.

1. Introduction

The failure of mechanical systems is a developing process involving the load action and damage accumulation. There are two types of data that are often used to predict failure: lifetime data and condition monitoring data. For most traditional reliability analysis methods, the failure distribution function is estimated by historical lifetime data, and the failure probability at any given time can be obtained. However, the lifetime data, which is the length of service time prior to failure, only present the final result from the failure and are not suitable for modelling the failure process under various operating conditions. The fault diagnosis method extracts the failure features from the monitoring data to indicate the failure occurrence. If condition monitoring is implemented through the lifetime of a mechanical system, a trend model for failure features can be built to predict the failure time in which the feature value would reach a predefined threshold. However, the relationship between the degree of the failure and predetermined threshold value has not been satisfactorily determined,

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and current fault diagnosis methods still fall short of providing accurate estimates on remaining system life. To achieve proper failure prediction, a more effective strategy that combines lifetime data and monitoring data is required.

Many attempts have been made to relate the failure probability to both historical service lifetime and condition monitoring variables. The proportional hazard model (PHM) is the most widely accepted of these attempts [1–6]. For this time-dependent model, failure prediction is treated as estimating the remaining lifetime for a system with regard to a specific hazard level under the current conditions. In the PHM developed by Jardine et al. [2], the Weibull distribution parameters are estimated using lifetime data from aircraft engines and marine gas turbines, and the metal particle level is used as the monitoring variable to provide the condition information. Results show that the PHM well interprets the actual influence of monitoring variable on system residual life. Moreover, the PHM is also remodelled to proportional intensity model (PIM) and proportional covariate model (PCM). Volk et al. [3] studied the application of the PIM to analyse the inter-arrival failure times and vibration data acquired from bearings. Sun et al. [4] proposed the PCM, utilising the data from accelerated life tests to estimate mechanical system hazards for the case of sparse or even no historical failure data.

All the above models aim to assume a single failure mode, in which all the lifetime data follow population distribution and condition monitoring variables take influence on system life in only one single way. In practice, mechanical systems are composed of multiple parts with various failure mechanisms. For example, a gearbox failure may result from individual failures in the gears, bearings or shafts and include fatigue cracks, teeth breakage, wear, bearing spalling, etc. If each failure form in these different parts is regarded as an independent failure mode with an individual life distribution, occurrence frequency and monitoring data presentation, multiple PHMs can be separately constructed to predict failure. However, the correlation between the different failure modes should not be neglected due to the interaction of mechanical parts, the propagation of failure and the competition between failure modes. The mixture model approach, which has been applied in the field of medicine [7–9], is a good reference to improve the PHM and analyse the failure of mechanical systems with multiple failure modes.

In this article, we present a mixture Weibull proportional hazard model (MWPHM) for mechanical system failure prediction. It combines lifetime and monitoring data of multiple failure modes to estimate the system lifetime. The rest of the paper is organized as follows. We briefly introduce the fundamental theory of the Weibull proportional hazard model in Section 2.1. In the remaining part of Section 2, the methodologies of the MWPHM are described. The failure prediction strategy based on the MWPHM is discussed in Section 3. Section 4 shows the results from the simulation verification. Next, a case study demonstrating the application of the MWPHM on a high-pressure water descaling pump is presented in Section 5. Finally, the conclusions are presented in Section 6.

2. Mixture Weibull proportional hazard model

2.1. Weibull proportional hazard model

The PHM, which was first introduced by Cox [10], has become an important statistical regression model and has been applied in many studies of mechanical system reliability. The basic assumption of the PHM is that the hazard rate of a system consists of two multiplicative factors, the baseline hazard rate and an exponential function including the effects of the monitoring variables. The hazard rate at time t is written as:

$$h(t,\mathbf{z}_t) = h_0(t)\exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t), \tag{1}$$

where $h_0(t)$ is the baseline hazard rate that is dependent on the service time, $\mathbf{z}_t$ is a row vector composed of monitoring values at time t, and $\boldsymbol{\gamma}$ is a column vector composed of the regression parameters corresponding to the monitoring variables. In the PHM, $\mathbf{z}_t$ is regarded as a vector of covariates that increases or decreases the system hazard rate proportionally, and the coefficient vector $\boldsymbol{\gamma}$ defines the influence of the monitoring variables on the failure process.

The Weibull distribution is frequently used to model the failure time of mechanical systems. The hazard rate function of the Weibull distribution is commonly selected as the baseline hazard rate of the PHM:

$$h_0(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}, \tag{2}$$

where $\beta>0$ and $\eta>0$ are the shape and scale parameter of the Weibull distribution, respectively. The PHM with the Weibull baseline function is called the Weibull proportional hazard model (WPHM). Then, the hazard function of the WPHM is defined as:

$$h(t,\mathbf{z}_t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t). \tag{3}$$

According to the principle of reliability analysis [11], the reliability and the failure probability density are respectively estimated as:

$$R(t,\mathbf{z}_t) = \exp\left[-\int_0^t h(t,\mathbf{z}_t)dt\right] = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)\right], \tag{4}$$

$$f(t,\mathbf{z}_t)=h(t,\mathbf{z}_t)R(t,\mathbf{z}_t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)\exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)\right]. \tag{5}$$

The maximum likelihood method is commonly applied to estimate the unknown parameters of the WPHM. In practice, a mechanical system will sometimes run to failure, and at other times the system will be repaired prior to failure. Therefore, the lifetime data usually contains the failure times and the suspension times. To deal with both types of data, a likelihood function is defined as:

$$L(\beta,\eta,\gamma)=\prod_{i=1}^{n}f(t_i,\mathbf{z}_t)\prod_{s=1}^{m}R(t_j,\mathbf{z}_t), \tag{6}$$

where i indexes the failure times, s indexes the suspension times, n is the number of failure samples and m is the number of suspension samples. Substituting Eqs. (4) and (5) into Eq. (6), the likelihood function can be rewritten as:

$$L(\beta,\eta,\gamma)=\prod_{i=1}^{n}\frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}t_i)\prod_{j=1}^{n+m}\exp\left[-\left(\frac{t_j}{\eta}\right)^{\beta}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}t_i)\right], \tag{7}$$

where j indexes both the failure times and the suspension times. The log-likelihood function:

$$\ln\left[L(\beta,\eta,\gamma)\right]=n\ln\left(\frac{\beta}{\eta}\right)+\sum_{i=1}^{n}\ln\left(\frac{t_i}{\eta}\right)^{\beta-1}+\sum_{i=1}^{n}\boldsymbol{\gamma}\cdot\mathbf{z}_{t_i}-\sum_{j=1}^{n+m}\left(\frac{t_j}{\eta}\right)^{\beta}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}_{t_i}), \tag{8}$$

is numerically more tractable than the likelihood function. Therefore, $\ln[L(\beta,\eta,\gamma)]$ is commonly used as the target function of maximisation. By setting the partial derivatives of Eq. (8) with respect to the parameters β, η and $\boldsymbol{\gamma}$ equal to zero, an optimal estimation of $\hat{\beta}$, $\hat{\eta}$ and $\hat{\boldsymbol{\gamma}}$ can be obtained.

In essence, the WPHM is a fully parameterised model. The historical lifetime data and condition monitoring data are combined to fit the model so the reliability and the failure probability density at the service time t can be estimated.

2.2. Mixture Weibull proportional hazard model

The traditional WPHM is a hazard estimation method for a single failure mode. The single parameter family of the WPHM is inadequate to model the failure process with the interaction and competition of multiple failure modes. Thus, we made the following assumptions to build a new mixture Weibull proportional hazard model:

(a) The failure modes of a mechanical system are various, but only a limited number of failure modes frequently occur.
(b) The failure of a mechanical system is the result of interaction and competition between failure modes.
(c) The lifetime of each failure mode is subject to the Weibull distribution. Only the shape and scale parameters of the Weibull distribution are different.
(d) The monitoring variables affect each failure mode.

Based on these assumptions, the probability density function of a system failure is defined by mixing the probability density function of the multiple dominant failure modes as:

$$f(t,\mathbf{z}_t)=\sum_{g=1}^{p}\lambda_g f_g(t,\mathbf{z}_t), \tag{9}$$

where $f_g(t,\mathbf{z}_t)$ is the density function of the gth dominant failure mode, λ_g is the proportion of system failures belonging to the gth dominant failure mode and p is the number of dominant failure modes. Considering the probability dominance of the analysed failure modes, the summation of $\lambda_g, g=1,\cdots,p$ is approximately equal to one. In this mixture model, the time to failure for each failure modes is still subject to the Weibull distribution. Therefore, the hazard function for the gth failure mode has the same form as Eq. (3):

$$h_g(t,\mathbf{z}_t)=\frac{\beta_g}{\eta_g}\left(\frac{t}{\eta_g}\right)^{\beta_g-1}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t). \tag{10}$$

The reliability and the failure probability density of the gth failure mode, denoted as $R_g(t,\mathbf{z}_t)$ and $f_g(t,\mathbf{z}_t)$, are calculated from Eqs. (4) and (5), respectively. Substituting $f_g(t,\mathbf{z}_t)$ into Eq. (9), the probability density function of a system failure can be rewritten as:

$$f(t,\mathbf{z}_t)=\sum_{g=1}^{p}\lambda_g\frac{\beta_g}{\eta_g}\left(\frac{t}{\eta_g}\right)^{\beta_g-1}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)\exp\left[-\left(\frac{t}{\eta_g}\right)^{\beta_g}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)\right]. \tag{11}$$

Specially, the system reliability is calculated not by combining the reliability of multiple failure modes, but by derivation from $f(t, \mathbf{z}_t)$:

$$R(t, \mathbf{z}_t) = 1 - \int_0^t f(t, z_t) dt. \tag{12}$$

To build the MWPHM, the likelihood function is defined as:

$$L(\beta, \eta, \gamma, \lambda) = \prod_{g=1}^{p} \left(\prod_{i=1}^{n_g} \lambda_g f_g(t_i, \mathbf{z}_t) \prod_{s=1}^{m_g} \lambda_g R_g(t_j, \mathbf{z}_t) \right), \tag{13}$$

where n_g and m_g are the number of failure samples and suspension samples of the gth failure mode, respectively. The log-likelihood of the MWPHM is:

$$\ln [L(\beta, \eta, \gamma, \lambda)] = \sum_{g=1}^{p} \left[(n_g + m_g) \left(\ln \lambda_g + \ln \frac{\beta_g}{\eta_g} \right) + \sum_{i=1}^{n_g} \ln \left(\frac{t_i}{\eta_g} \right)^{\beta_g - 1} + \sum_{i=1}^{n_g} \boldsymbol{\gamma} \cdot \mathbf{z}_{t_i} - \sum_{j=1}^{n_g+m_g} \left(\frac{t_j}{\eta_g} \right)^{\beta} \exp(\boldsymbol{\gamma} \cdot \mathbf{z}_{t_j}) \right]. \tag{14}$$

Unlike the WPHM, the unknown parameters of the MWPHM include $\lambda_g, \beta_g, \eta_g, g = 1, \cdots, p$ and $\boldsymbol{\gamma}$. The complexity of the maximum likelihood estimation is greatly increased. Therefore, an iterative algorithm, the Nelder–Mead method [12,13], is applied to approximately estimate these mixed parameters.

In the MWPHM, the system failure density is obtained by proportionally accumulating the probability density of multiple failure modes. Compared with the single Weibull distribution, the baseline distribution of the MWPHM provides more detailed information about the lifetime, and is closer to actual distribution. The monitoring data $\mathbf{z}_t$ is regarded as the response to the system failure instead of the individual failure mode. For a specific service time t, the hazard rates of all failure modes are influenced by $\mathbf{z}_t$ with the form of the covariate function $\exp(\boldsymbol{\gamma} \cdot \mathbf{z}_t)$. As the coefficient vector $\boldsymbol{\gamma}$ is estimated with all of the lifetime and monitoring data, it actually reflects the compromise of influence of the monitoring variables on all the failure modes. To ensure accuracy of the MWPHM, the monitoring variables, which just indicate the individual failure mode, should not be chosen as covariates.

3. Failure prediction based on MWPHM

Unlike failures in electrical systems, most failures in mechanical systems are gradual processes, rather than sudden occurrences. The monitoring data, which are continuously or periodically acquired, can be used to reflect the operating condition and reveal the failure process. When a mechanical system fails, it is usually repaired or overhauled to restore the system to working order. The lifetime data, which is equal to the time interval between adjacent failures, can be collected during working, failure and recovery cycles. If the system was restored to near to the initial state after repairs, and continued working with consistent operating parameters, the lifetime data can be regarded as independent and identically distributed. In such a situation, the failure distribution can be accurately estimated with sufficient failure data. However, repairs or overhauls generally cannot return the system to its initial state. Practically, the lifetimes of repaired systems show a significant amount of uncertainty. Moreover, the operating parameters may well be adjusted to slow down the failure development after the abnormal condition of a system is detected. Therefore, the monitoring data, which describe the condition of a system or the conditions under which a system operates, should be used to provide supplementary information for the lifetime data analysis.

If a failure occurs in a monitored mechanical system, we can figure out the lifetime, identify the failure mode and acquire the monitoring data at the failure time or for a period of time before the failure. After many incidences of failure in a system, or many failures in identical types of systems, lifetime and monitoring data are accumulated and classified by failure mode. The lifetime data of each failure mode can be used to estimate the failure distribution, and the monitoring data indicate the operating condition for certain failure modes in failure time. In the MWPHM, the two types of data are combined. The hazard function of each failure mode is modelled as a product of the baseline hazard rate and deduced from the lifetime distribution and the covariate function, reflecting the influence of the monitoring data. The system failure distribution is estimated by proportionally mixing the probability distribution of all the failure modes. For a working system, suppose the monitoring data at time T is $\mathbf{z}_T$. The system reliability function, $R(t, \mathbf{z}_T)$, can be obtained according to Eq. (12). If the determined value for $R(t, \mathbf{z}_T)$ falls below the reliability threshold, the system is expected to fail. The failure time is predicted as follows:

$$T'(R_0) = \inf\{t : R(t, \mathbf{z}_T) = R_0, t > 0\}, \tag{15}$$

where R_0 is the reliability threshold. As long as the system continues functioning, the monitoring data are updated and the failure time can be estimated in succession. The prediction process is also shown in Fig. 1.

4. Simulation result

To test the modelling ability of our method for multiple failure modes, we constructed a simulated sample set, including lifetime and monitoring data. It was assumed that the simulated mechanical system has two different failure modes and the

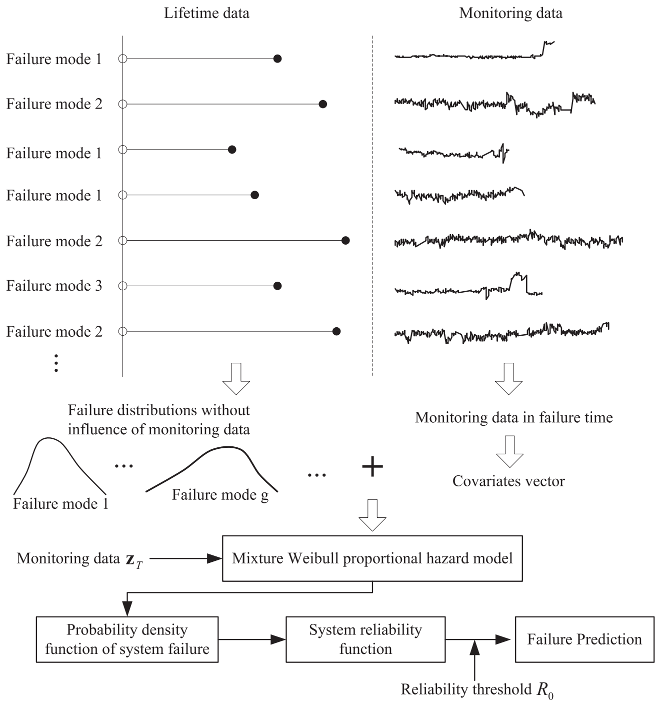

Fig. 1. Failure prediction based on the MWPHM.

probability density of system failure has the form:

$$f(t,\mathbf{z}_t) = \lambda f_1(t,\mathbf{z}_t) + (1-\lambda) f_2(t,\mathbf{z}_t), \tag{16}$$

where $f_g(t,\mathbf{z}_t), g=1,2$ are the failure density functions of the two failure modes and λ is the occurrence proportion of failure mode 1. To simplify the process of sample generation, $f_g(t,\mathbf{z}_t)$ is defined as:

$$\begin{aligned} f_g(t,\mathbf{z}_t) &= \frac{\beta_g}{\eta_g}\left(\frac{t}{\eta_g}\right)^{\beta_g-1} \exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)\exp\left[-\left(\frac{t}{\eta_g}\right)^{\beta_g}\exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)\right] \\ &= \frac{\beta_g}{\eta_g \exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)^{-(1/\beta_g)}}\left(\frac{t}{\eta_g \exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)^{-(1/\beta_g)}}\right)^{\beta_g-1}\exp\left[-\left(\frac{t}{\eta_g \exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)^{-(1/\beta_g)}}\right)^{\beta_g}\right]. \end{aligned} \tag{17}$$

for $\eta_g' = \eta_g \exp(\boldsymbol{\gamma}\cdot\mathbf{z}_t)^{-(1/\beta_g)}$, Eq. (17) is converted to:

$$f_g(t) = \frac{\beta_g}{\eta_g'}\left(\frac{t}{\eta_g'}\right)^{\beta_g-1}\exp\left[-\left(\frac{t}{\eta_g'}\right)^{\beta_g}\right]. \tag{18}$$

Eq. (18) has the same form as the probability density function of the Weibull distribution without the influence of the covariates. If β_g and η_g' are given, the lifetime samples following the Weibull distribution can be randomly generated. Accordingly, monitoring samples exert their influence only by changing the scale parameter of the Weibull distribution, and the two types of samples need not be generated simultaneously. Taking advantage of this definition, the simulated sample set is constructed as follows:

(1) Generate three random numbers between 0 and 1, which are denoted as z_1,z_2 and z_3 separately, to simulate the normalised monitoring data. The monitoring sample is defined as $\mathbf{z} = [z_1, z_2, z_3]$.
(2) Calculate η_g' under the given β_g, η_g, $\boldsymbol{\gamma}$ and $\mathbf{z}$, where $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \gamma_3]$.
(3) Generate a random number μ between 0 and 1.
(4) If $\mu \le \lambda$, a lifetime sample is randomly generated using the parameters β_1 and η_1'. Otherwise, the sample is generated using the parameters β_2 and η_2'.
(5) Repeat steps 1 to 4 until the simulated sample set is completed.

Following these steps, a simulated sample set of 1000 lifetime samples and monitoring samples was constructed. The simulation parameters are listed in Table 1. Both the MWPHM and the WPHM are used to model the simulated sample set.

The model parameters estimated from these two methods are also presented in Table 1. By comparison, it is clear that the parameters estimated by the MWPHM are consistent with the simulation parameters. To get a view of this comparison, Fig. 2 shows the failure probability density curves of the theoretical distribution, the estimated distribution of the MWPHM and the estimated distribution of the WPHM, in which $\mathbf{z}_t$ is equal to the mean of monitoring samples.

According to the simulation results, we can draw the following conclusions:

(a) The MWPHM has the ability to accurately estimate the system failure probability density of multiple failure modes.
(b) Due to inadequate model parameters, the traditional WPHM is not suitable for modelling the failure process of a system with multiple failure modes.

5. Case study

A high-pressure water descaling pump, shown in Fig. 3, was selected for a case study. It is equipped with an 11-level centrifugal pump to provide high pressure and large water flow rates for removing oxide scale on stainless steel surface. Due to working continuously in a high-pressure environment, the descaling pump frequently breaks down from two failure modes: sealing ring wear and thrust bearing damage. To ensure safe operation, the outlet pressure, input-end vibration and thrust bearing temperature are regularly monitored. The lifetime data from the years 2005 to 2008 and the monitoring data in the failure times are collected and listed in Table 2.

The outlet pressure, input-end vibration and thrust bearing temperature are denoted as z_1, z_2 and z_3, respectively. For these monitoring variables, z_2 and z_3 increase as failures arise, but z_1 decreases when the descaling pump malfunctions. To enable a comparison of the influence of the monitoring variables on the system failure, normalised operations are executed as follows:

$$z_1' = 1 - \frac{z_1 - z_{1\,\min}}{z_{1\,\max} - z_{1\,\min}}, \tag{19}$$

$$z_i' = \frac{z_i - z_{i\,\min}}{z_{i\,\max} - z_{i\,\min}} \quad i = 2, 3, \tag{20}$$

where $z_{1\,\max}$, $z_{1\,\min}$, $z_{2\,\max}$, $z_{2\,\min}$, $z_{3\,\max}$ and $z_{3\,\min}$ are set as 230 bar, 210 bar, 25 mm/s, 2 mm/s, 100 °C and 40 °C. The sealing ring wear and thrust bearing damage are defined as failure modes 1 and 2, respectively. By combining the lifetime data and the normalised monitoring data, our method is applied to model the failure of the high-pressure water descaling pump. For comparison, the estimation result of the WPHM is also examined. The data of failure #1–#14 are selected as training samples. Table 3 shows the estimated parameters of the MWPHM and the WPHM. When z_1', z_2' and z_3' are equal to their mean values, the failure probability density curves are respectively calculated by the two models and are presented in Fig. 4.

Table 1
Comparisons of the model parameters.

Parameters	β_1	η_1	β_2	η_2	λ	γ_1	γ_2	γ_3
Simulation	2	15	2	70	0.2	0.3	0.6	0.4
MWPHM	2.01	15.05	2.00	70.56	0.20	0.29	0.64	0.38
WPHM	1.48	58.99	/	/	/	0.22	0.48	0.31

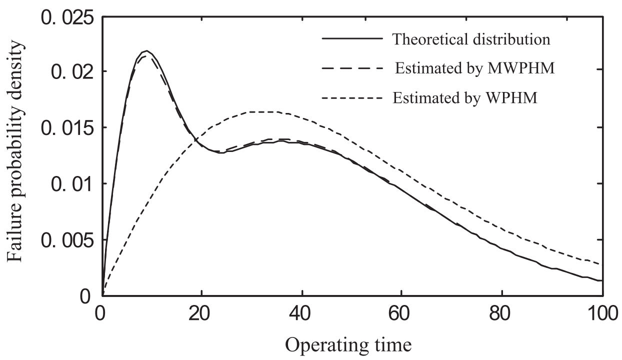

Fig. 2. Comparisons of the failure probability density curves.

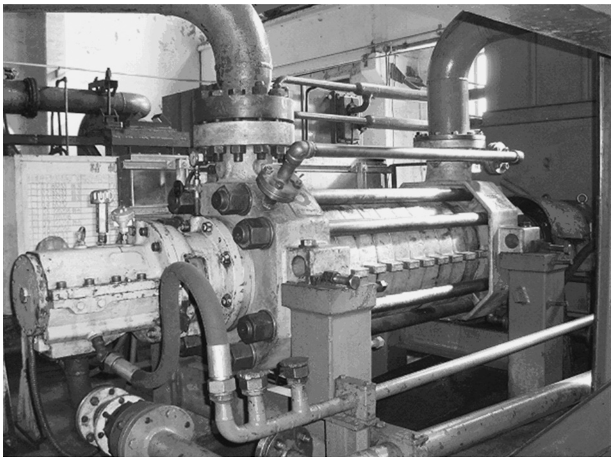

Fig. 3. High-pressure water descaling pump chosen for the case study.

Table 2
Historical lifetime and monitoring data for the selected high-pressure water descaling pump.

Failure no.	Failure Mode	Lifetime (h)	Monitoring data in the failure time		
			Outlet pressure (bar)	Input-end vibration (mm/s)	Thrust bearing temperature (∘C)
1	Sealing ring wear	136	217.2	6.4	83.3
2	Sealing ring wear	387	215.6	24.7	88.7
3	Sealing ring wear	22	213.4	20.6	95.5
4	Sealing ring wear	698	217.2	20.8	98
5	Sealing ring wear	772	217.1	11.4	78.5
6	Sealing ring wear	1495	220.5	11.4	76.5
7	Sealing ring wear	324	219.4	17.3	82.4
8	Sealing ring wear[a]	1357	212.2	11.3	92.5
9	Sealing ring wear	86	217.1	4.5	78.1
10	Thrust bearing damage	545	213.4	20.6	95.5
11	Thrust bearing damage	324	219.4	17.3	82.4
12	Thrust bearing damage[a]	1357	212.2	11.3	92.5
13	Thrust bearing damage	1709	220.3	5.2	84.6
14	Thrust bearing damage	1622	218.4	5.5	91.3
15	Thrust bearing damage	1692	213.8	10.2	91.9

[a] Two failure modes occurred at the same time.

Table 3
Model parameters of the MWPHM and the WPHM applied to the high-pressure water descaling pump.

Parameters	β_1	η_1	β_2	η_2	λ	γ_1	γ_2	γ_3
MWPHM	1.13	622.31	2.61	887.40	0.64	1.44	2.80	−3.26
WPHM	1.39	683.41	/	/	/	0.84	2.37	−2.42

It can be observed that the WPHM provides an excessive smooth density curve, in which the detailed distribution information of two failure modes is lost. Conversely, the density curve estimated by the MWPHM exhibits the mixture of two failure modes properly.

The monitoring data of failure #15 are used to assess the ability of the MWPHM to estimate the system reliability. In this service cycle, the descaling pump operated for 1692 h before malfunction. During overhaul, it was found that the thrust bearing had been damaged. The monitoring data from the beginning of operation to failure is shown in Fig. 5(a)–(c). It can be seen that there are significant changes in the monitoring data after 1500 h. The monitoring data at intervals of 1 h are input into the MWPHM whose parameters have been listed in Table 3, and the system reliability is estimated and presented in Fig. 5(d). The reliability curve shows the decreasing process of system reliability which is under the influence of monitoring data. After 1500 h, the decreasing speed of system reliability is accelerated. When the descaling pump malfunctioned, the system reliability dropped to about 0.1.

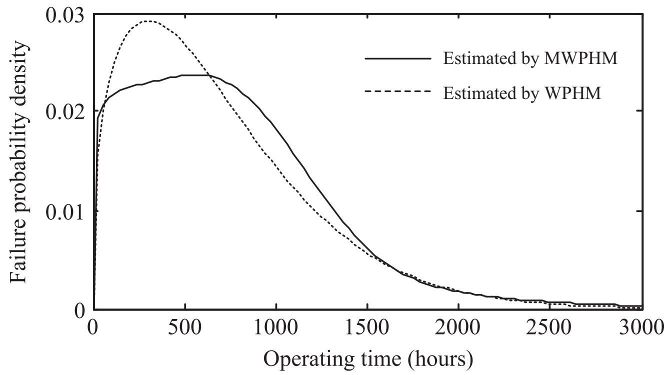

Fig. 4. Failure probability density of the descaling pump estimated by the MWPHM and the WPHM.

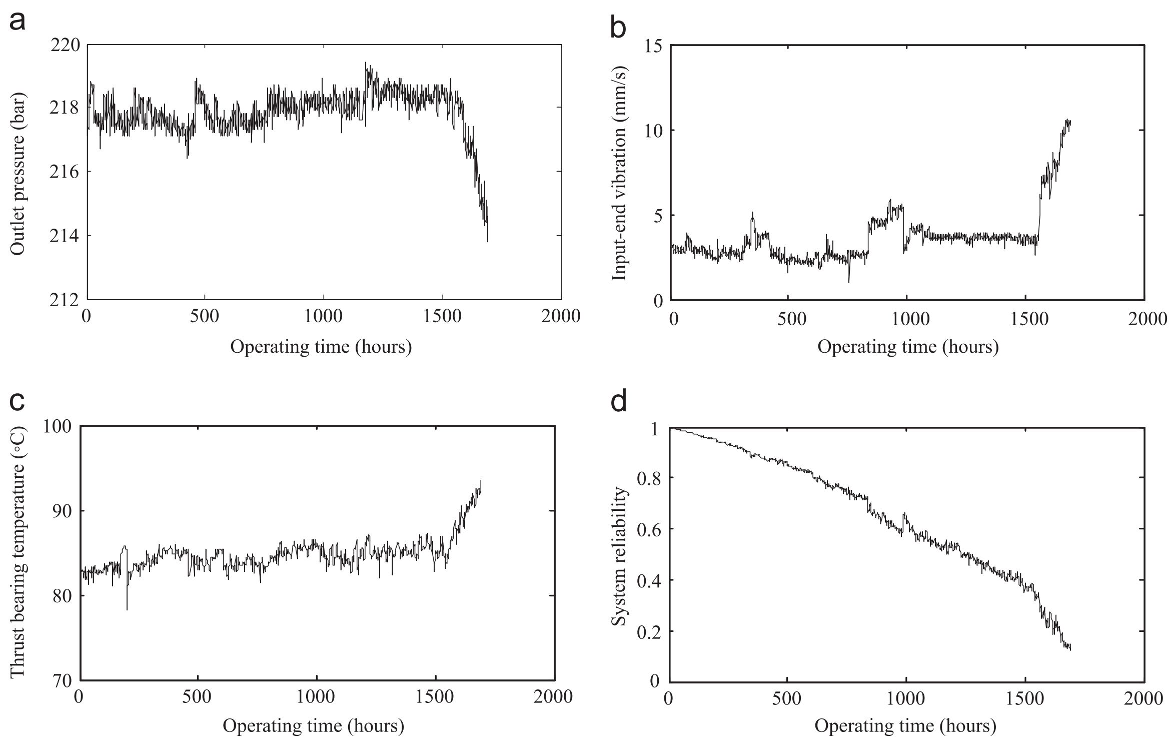

Fig. 5. Monitoring data and system reliability of a failure process: (a) Outlet pressure; (b) Input-end vibration; (c) Thrust bearing temperature; and (d) System reliability.

To verify the validity of the proposed model to predict failure, the Leave-One-Out Cross-Validation (LOOCV) method [14] is used. The reliability threshold R_0 is set at 0.2. Fig. 6 shows the predicted failure times in four failure processes, which are failure #2, #4, #13 and #14. Therein, the failures #2 and #4 belong to the failure mode of sealing ring wear, and the failures #13 and #14 belong to the failure mode of thrust bearing damage. While the sealing ring is worn, the outlet pressure would drop dramatically. It leads to the instability of the axial force provided by the thrust bearing. Therefore, fluctuations appeared in the input-end vibration and thrust bearing temperature. In the process of failures #2 and #4, the predicted failure time drops first and then fluctuates. To the failure mode of sealing ring wear, replacement is the only choice of maintenance. After the dropping of pressure, a right time is selected to shut down the pump and replace the sealing ring. From Fig. 6(a) and (b), it can be found that the predicted and actual failure times are very close while the failure is impending. The failure behaviour and maintenance strategy of thrust bearing damage are different with the sealing ring wear. After an initial stage of rapid degradation, the trend of monitoring data may be reversed in two cases. In the first case, the mechanical damage would be "healing" without manual interventions [15,16]. The failure #13 just goes through such a stage after 740 h, and Fig. 6(c) shows the change of predicted failure time influenced by the monitoring data. In another case, some temporary repair measures, such as shaft alignment and forced cooling, are implemented to restore the system performance. In the process of failure #14, the

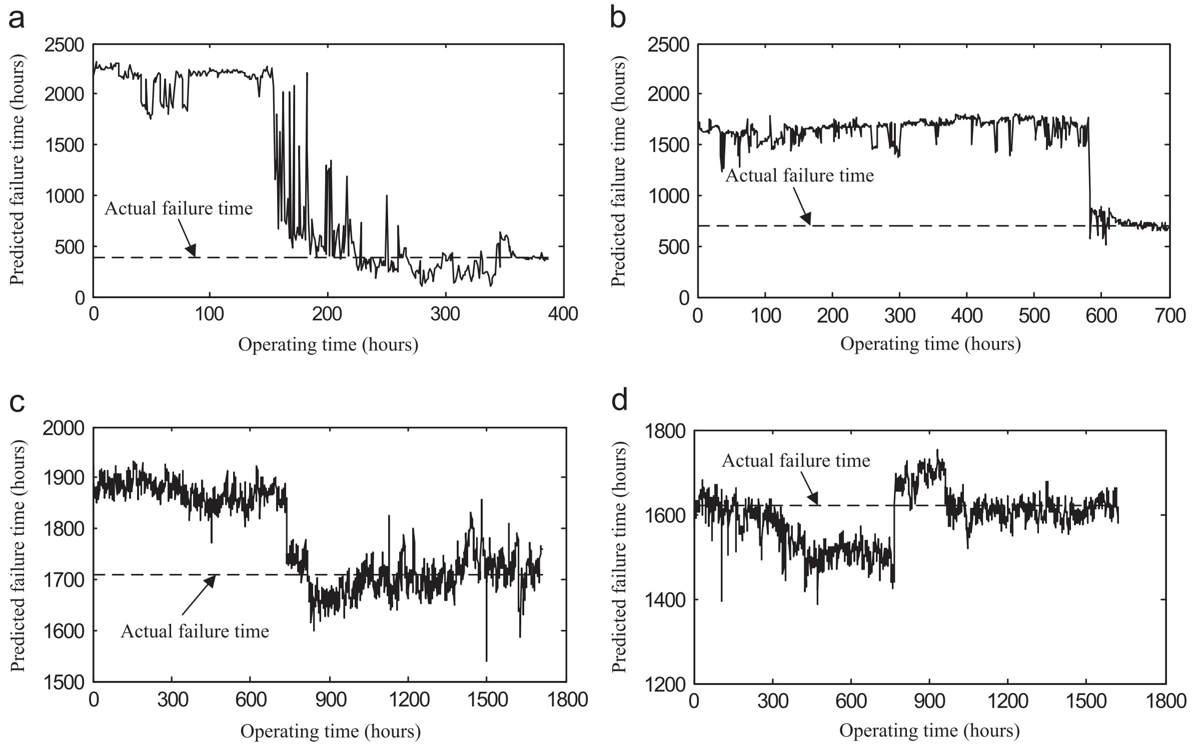

Fig. 6. Failure predictions for the descaling pump using the MWPHM: (a) Failure #2; (b) Failure #4; (c) Failure #13; and (d) Failure #14.

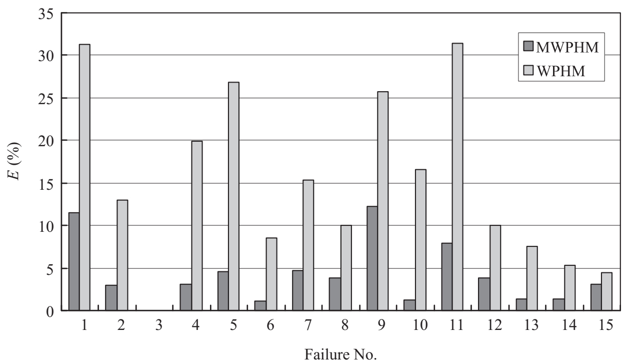

Fig. 7. Prediction errors of the MWPHM and the WPHM.

repair operation prolongs the system residual life. The curve, shown in Fig. 6(d), reflects the increasing of predicted failure time at 769 h. After the "healing" or repair, the system performance continues to decrease. In the second degradation stage, the predicted failure time has been closed to the actual failure time for a long time before failure. Results show that the failures of a system with different failure modes are successfully predicted.

The mean error between the predicted and the actual failure time over a period of time before failure is given as follows:

$$E = \frac{1}{s} \sum_{t=T_a-s+1}^{T_a} \left| \frac{T_t^{'}(R_0) - T_a}{T_a} \right| \times 100\%, \tag{21}$$

where $T_t^{'}(R_0)$ is the predicted failure time at time t, T_a is the actual failure time and s is the time period of error estimation. To compare the failure prediction accuracy of the MWPHM and the WPHM, the LOOCV method is applied to examine each failure listed in Table 2. To take the failure samples of short lifetime into account, the value of 24 h for s is used. E is calculated with the failure times predicted by the MWPHM and the WPHM respectively. The failure #3 is ignored because the actual failure time is less than s. The bar graph of E is shown in Fig. 7. It can be found that the prediction errors of the MWPHM are significantly less than those of the WPHM. Most of the predictions made by the proposed model are within 10% error of the actual failure time. In conclusion, the MWPHM provides the capability for estimating failure times of a mechanical system with multiple failure modes. The results of the proposed model are superior compared to the WPHM.

6. Conclusion and discussion

It is common for a mechanical system to have multiple failure modes. Due to the interaction and competition between the different failure modes, a method for determining a single failure mode is not suitable for analysing the overall system failure. In this paper, a mixture Weibull proportional hazard model is proposed to predict the failure time of a mechanical system with multiple failure modes. Compared with the traditional WPHM, the MWPHM has two significant features. First, the system failure density is obtained by proportionally mixing the failure density of the multiple failure modes. When mixed with the probability density, the MWPHM not only accounts for the contribution of different failure modes on the system failure, but it also provides more detailed information on the lifetime distribution. Second, the model parameters of the MWPHM are estimated using the lifetime and monitoring data of all failure modes. The influence of monitoring variables on different failure modes is coordinated by the maximum likelihood estimation for the mixed parameter family. Then, the monitoring data are input into the MWPHM to estimate the system reliability and predict system failure time. The simulated and experimental results demonstrate that our method can provide satisfactory failure distribution estimation and lifetime prediction.

However, the estimated failure time still depends on the choice of the defined reliability threshold. Since the system reliability at the failure time shows a clear trend as the number of maintenance operations increases, the reliability threshold should be adaptively adjusted according to the system maintenance experience. It is what our research work will focus on next.

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References

[1] A.K.S. Jardine, D. Lin, D. Banjevic, A review on machinery diagnostics and prognostics implementing condition-based maintenance, Mech. Syst. Signal Process. 20 (2006) 1483–1510.
[2] A.K.S. Jardine, P.M. Anderson, D.S. Mann, Application of the Weibull proportional hazards model to aircraft and marine engine failure data, Qual. Reliabil. Eng. Int. 3 (1987) 77–82.
[3] P.J Volk, M. Wnek, M. Zygmunt, Utilising statistical residual life estimates of bearing to quantify the influence of preventive maintenance actions, Mech. Syst. Signal Process. 18 (2004) 833–847.
[4] Y. Sun, L. Ma, J. Mathew, W. Wang, S. Zhang, Mechanical systems hazard estimation using condition monitoring, Mech. Syst. Signal Process. 20 (2006) 1189–1201.
[5] K.B. Goode, J. Moore, B.J. Roylance, Plant machinery working life prediction method utilizing reliability and condition-monitoring data, Proc. Inst. Mech. Eng. E: J. Process Mech. Eng. 214 (2000) 109–122.
[6] K.A.H. Kobbacy, B.B. Fawzi, D.F. Percy, H.E. Ascher, A full history proportional hazards model for preventive maintenance scheduling, Qual. Reliabil. Eng. Int. 13 (1997) 187–198.
[7] S.K. Ng, G.J. Mclachlan, K.K.W. Yau, A.H. Lee, Modelling the distribution of ischaemic stroke-specific survival time using an EM-based mixture approach with random effects adjustment, Stat. Med. 23 (2004) 2729–2744.
[8] A.H. Lee, Y. Zhao, K.K.W. Yau, S.K. NG, A computer graphical user interface for survival mixture modeling of recurrent infections, Comput. Biol. Med. 39 (2009) 301–307.
[9] F. Louzada-Neto, J. Mazucheli, J.A. Achcar, Mixture hazard models for lifetime data, Biometr. J. 44 (2002) 3–14.
[10] D.R. Cox, Regression models and life-tables (with discussion), J. R. Stat. Soc. Ser. B (Methodol.) 34 (1972) 187–220.
[11] C.E. Ebeling, An Introduction to Reliability and Maintainability Engineering, Waveland Press, Illinois, USA, 2005.
[12] J.A. Nelder, R. Mead, A simplex method for function minimization, Comput. J. 7 (1965) 308–313.
[13] J.C. Lagarias, J.A. Reeds, M.H. Wright, P.E. Wright, Convergence properties of the Nelder–Mead simplex method in low dimensions, SIAM J. Optimizations 9 (1998) 112–147.
[14] R. Kohavi, A study of cross-validation and bootstrap for accuracy estimation and model selection, in: Proceedings of the 14th International Conference on Artificial Intelligence (IJCAI), San Mateo, CA, Morgan Kaufmann, Los Altos, CA, 1995, pp. 1137–1143.
[15] T. Williams, X. Ribadeneira, S. Billington, T. Kurfess, Rolling element bearing diagnostics in run-to-failure lifetime testing, Mech. Syst. Signal Process. 15 (2001) 979–993.
[16] H. Qiu, J. Lee, J. Lin, G. Yu, Wavelet filter-based weak signature detection method and its application on rolling element bearing prognostics, J. Sound Vib. 289 (2006) 1066–1090.