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# Remaining useful life estimation for mechanical systems based on similarity of phase space trajectory



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# ABSTRACT

When evolving from a normal state to failure, mechanical systems undergo a gradual degradation process. Due to the nonlinearity of damage accumulation, degradation data always exhibit a distinctive trend and random fluctuations. It makes the prediction of remaining useful life (RUL) very difficult and inaccurate. The phase space trajectory reconstructed from the time series of degradation data is capable of reliably elucidating the nonlinear degradation behavior. In this paper, a novel method based on the similarity of the phase space trajectory is proposed for estimating the RUL of mechanical systems. First, the reference degradation trajectories are built with historical degradation data using the phase space reconstruction. Second, the similarities between the current degradation trajectory and the reference degradation trajectories are measured with a normalized cross correlation indicator, which is determined solely by the trajectory shape and is not interfered with the scaling and shifting of the trajectory. Trajectory shape and degradation stage matching algorithms are combined to find the optimal segments in the reference degradation trajectories compared with the current degradation trajectory. Finally, the RULs corresponding to the optimal matching segments are subjected to weighted averaging to obtain the RUL of the current degradation process. The proposed method is evaluated utilizing both simulated data in stochastic degradation processes and experimental data measured on an actual pump. The results show that the predicted RULs are very close to the actual RUL.

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# 1. Introduction

Following increasing demands in the field of operational safety, asset availability and resource conservation, the area of prognostics has emerged as one of the key foundations for the maintenance scheme of modern industry. The main task of prognostics is to estimate the remaining useful life (RUL) of a mechanical system, which is defined as the period from the current service time until the component or system fails. It is important to predict the RUL of an asset as the incipient damage or performance degradation occurs because it provides valuable information toward decreasing future risk and loss due to failures or accidents. Over the past decade, RUL prediction has become a research topic of high interest, investigated in application fields (Heng, Zhang, Tan, & Mathew, 2009; Si, Wang, Hu, & Zhou, 2011; Sun, Zeng, Kang, & Pecht, 2012).

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E-mail addresses: zhangq@mail.xjtu.edu.cn (Q. Zhang), Peter.W.Tse@cityu.edu. hk (P.W.-T. Tse), wx72420@163.com (X. Wan), ghxu@mail.xjtu.edu.cn (G. Xu). Most failures in mechanical systems result from gradual degradation processes rather than sudden occurrences. Incipient damage is formed under the effects of repeated load and adverse conditions, such as wear and erosion, and then evolves into a distinct failure. Numerous prognostic approaches have been developed to model the degradation processes of mechanical systems and estimate the RUL. The physics-based and data-driven models are representative prognostic approaches, which have a wide range of applications.

Generally, physics-based models implement the mathematic formulas deduced from the physics of failures to predict the theoretical damage evolution, such as crack propagation and spall growth. Due to its convenience and accuracy, physics-based models are used as the basis of some expert systems for RUL prediction, which can be found in references (Jin, Matthews, Fan, & Liu, 2013; Kim, Song, & Park, 2009; Liu, Xuan, Si, & Tu, 2008; Zhao, Tian, & Zeng, 2013). However, the damage that modeled by physics-based models is specific and cannot be used for reference to other types of mechanical components. Moreover, it is hard to construct an



adequate physics-based model when the real-life system is complex.

Data-driven models are the more available solutions in many practical cases where degradation data are collected either continuously or periodically from operating systems. This type of models can be further classified as random coefficient models, artificial intelligence approaches and trend-based approaches. In random coefficient models, the degradation process is usually represented as a linear, polynomial, exponential, or any other functional form (Gebraeel, 2006). To characterize the complicated relationship between the hidden degradation behavior and the observed fluctuating data, stochastic processes, such as the Wiener process (Si, Wang, Hu, & Zhou, 2013; Son, Fouladirad, Barros, Levrat, & Iung, 2013) and Gamma process (Guida & Pulcini, 2013), are used to fit the distribution of the degradation path. For the random coefficient models, it is necessary to acquire prior degradation knowledge and abundant historical data to determine the model form and stochastic parameters. Therefore, some improved strategies, such as expectation maximization (Si et al., 2013) and Bayesian updating (Gebraeel, Lawley, Li, & Ryan, 2005), have been utilized to reduce the prediction errors caused by inappropriate parameters and to enhance the generalization ability of the models. Artificial intelligence is currently the most common foundational technique in the prognostics literature due to its flexibility in generating appropriate model. Huang et al. (2007) developed a set of feed-forward back propagation networks to model the exponential degradation process and estimate the bearing life. Other types of neural networks, such as the cerebellar model articulation controller neural networks (Lee & Kramer, 1993), recurrent neural networks (Tse & Atherton, 1999) and self-organizing map neural networks (Niu & Yang, 2010), have also been used to quantify the degradation level and predict failure. In addition, some prognostics approaches are developed based on artificial intelligence algorithms, such as support vector machine (Kim, Tan, Mathew, & Choi, 2012; Widodo & Yang, 2011), relevance vector machine (Hu & Tse, 2013) and hidden Markov model (Peng & Dong, 2011). Because the degradation characteristics are learned by hidden neural units or are mapped into a high dimensionality space, artificial intelligence approaches usually provide non-transparent solutions to failure prognosis, or rather it cannot be observed that how predict results are inferred. Trend-based approaches built degradation model utilizing the time series of experience data acquired from long-term degradation processes. The main difference with random coefficient models is that the degradation path is not predefined but completely determined by historical data. These approaches utilize advanced statistical techniques, such as sequential Monte Carlo method (Caesarendra, Niu, & Yang, 2010), state-space model (Sun, Zuo, Wang, & Pecht, 2014) and Bayesian hierarchical model (Zaidan, Harrison, Mills, & Fleming, 2015), to deal with the various degradation trends of mechanical systems, which work on variable operating conditions. For the above data-driven models, the bottleneck problem is that their accuracy is highly dependent on the quantity and quality of available degradation data.

Recently, condition monitoring is widely applied to detect the degradation process of critical mechanical system. It provides a favorable situation for data-driven models. However, due to the nonlinearity of mechanical damage accumulation, unstable operating conditions and accidental disturbances can significantly alter the associated degradation behavior. Therefore, the practical degradation data, which represent the time series indicating system performance, always exhibit a distinctive trend and random fluctuations. In most data-driven models, the time series of degradation data are directly engaged as the learning samples to model degradation evolution. When the available samples are insufficient, the distinctive trend and random fluctuations within the degradation data may produce unacceptable errors.

Nonlinear degradation behavior is the major challenge confronting the effective prediction of the RUL of mechanical systems. From the viewpoint of dynamical systems, the time series of degradation data are products of systems, which are undergoing degradation progresses. Although the degradation data present nonlinear behavior and possible chaos, the underlying data generating mechanisms can still be identified by phase space reconstruction technique. By virtue of the ability of revealing the nature of system state, phase space reconstruction has become a powerful tool for pattern recognition (Sharma & Pachori, 2015) and been wildly applied to differentiate the failed state from the normal state for mechanical systems (Aydin, Karakose, & Akin, 2014; Wang, Li, & Luo, 2007). In phase space reconstruction, the time series is rearranged into a phase space based on time delay embedding. The evolving state of a system over time traces a path, which is called the phase space trajectory, through the reconstructed phase space. The shape of the trajectory represents the system behavior that is compatible with a particular operating state. Because degradation leads to changes in the dynamics that are characteristic of the system state, the phase space trajectory is capable of elucidating the latent degradation behavior from the observed time series. In our research, the phase space trajectory, rather than the original degradation data, is used to analyze the degradation process.

In this article, we present a method for remaining useful life estimation based on the similarity of the phase space trajectory. The phase space reconstruction is adopted to build reference degradation trajectories from the time series of historical degradation data. The similarities between the current trajectory and the reference trajectories are robustly measured and used to estimate the RUL. The reminder of the paper is organized as follows. Section 2 describes the main principle of our method, which includes the phase space reconstruction (PSR) and normalized cross correlation (NCC). The methodologies of RUL estimation are also given in this section. Section 3 shows the results from the simulation verification. Next, a case study demonstrating the application on an actual pump is presented in Section 4. Finally, the conclusions are given in Section 5.

# 2. Methods and principles

#### 2.1. Phase space reconstruction

According to Takens' theorem (Takens, 1981), the underlying dynamics characteristic of a system can be obtained by reconstructing the phase space, preserving the topological properties of the original unknown attractor. To characterize the nonlinear feature of a scalar time series, time delay embedding is commonly used, allowing for the construction of a high-dimensional phase space in which the time series are unfolded. Suppose a time series is  $x = (x_1, x_2, ..., x_N)$ ; then, a point in the phase space is represented as a row vector:

$$X_{i} = [x_{i-(d-1)\tau}, x_{i-(d-2)\tau}, \dots, x_{i-\tau}, x_{i}],$$
(1)

where *N* is the number of points in the time series, *i* is the index of the row vector, ranging from  $1 + (d - 1)\tau$  to *N*, *d* is the embedding dimension, and  $\tau$  is the time delay. The sufficient condition for the topological equivalent of the reconstructed phase space is that *d* is greater than twice the box counting dimension of the original system.

Because *d* is the most critical parameter for PSR, many have discussed how to determine the minimum embedding dimension from a scalar time series. In this paper, we adopt Cao's algorithm (Cao, 1997), which is a practical and non-subjective method. Suppose the embedding dimension is chosen as *d*; then, the *i*th point in

this *d*-dimensional phase space is written as  $X_i(d)$ , which has the same form as Eq. (1).  $X_{n(i,d)}(d)$  is the nearest point to  $X_i(d)$ , that is,

$$\|X_{i}(d) - X_{n(i,d)}(d)\| = \min_{j=1+(d-1)\tau,\dots,N,j \neq i} \|X_{i}(d) - X_{j}(d)\|_{\infty},$$
(2)

where  $\|\cdot\|_{\infty}$  is the  $L_{\infty}$  norm in  $\mathbb{R}^d$  space, n(i, d) is the index of the nearest point and is dependent on i and d. If d is qualified as an embedding dimension according to the embedding theorem, then any two points which are close in the d-dimensional reconstructed phase space will continue to be close in the d + 1-dimensional space. For all points in the d- and d + 1-dimensional phase spaces, a(i, d), E(d) and  $E_1(d)$  are separately defined as:

$$a(i,d) = \frac{\|X_i(d+1) - X_{n(i,d+1)}(d+1)\|}{\|X_i(d) - X_{n(i,d)}(d)\|},$$
(3)

$$E(d) = \frac{1}{N - d\tau} \sum_{i=1 + (d-1)\tau}^{N - \tau} a(i, d),$$
(4)

$$E_1(d) = \frac{E(d+1)}{E(d)}.$$
 (5)

While the *d*- and *d* + 1-dimensional space are topologically equivalent,  $E_1(d)$  tends to a stable value, which stops changing with the increase of *d*. Therefore, the minimum *d* that stabilizes  $E_1(d)$  is the embedding dimension for which we search. However, in a practical experiment, it is difficult to judge whether  $E_1(d)$  has stopped changing due to the finite length of the observed samples. To solve this problem,  $E^*(d)$  and  $E_2(d)$  are proposed as supplemental criteria.

$$E^{*}(d) = \frac{1}{N - d\tau} \sum_{i=1}^{N - d\tau} |x_{i+d\tau} - x_{n(i,d)+d\tau}|,$$
(6)

$$E_2(d) = \frac{E^*(d+1)}{E^*(d)}.$$
(7)

If x is a stochastic time series,  $E_2(d)$  is equal to 1. Conversely,  $E_2(d)$  changes with d for a deterministic time series. By tracking the variation of both  $E_1(d)$  and  $E_2(d)$  with d, the suitable embedding dimension of the time series can be determined.

Chelidze and Cusumano (2004) applied PSR to develop a dynamical systems approach, called phase space warping (PSW), for failure prognosis. In the approach, damage processes are described by a hierarchical dynamical system consisting of a directly observable fast-time subsystem and a hidden slow-time subsystem. The transformation from signals in fast-time subsystem to damage indicators in slow-time subsystem is achieved by PSR. Fan, Hu, Hu, and Gu (2012) utilized PSW to track the damage evolution of bearings. In PSW, the reconstructed phase space in the undamaged state is set as a reference system, and the residual error between the current system and the reference system is used to estimate the damage. In contrast, our research considers the whole degradation processes, including initial damage, developing damage and distinct damage, as the references to estimate RUL. This is the main difference between PSW and our method.

#### 2.2. Normalized cross correlation

The normalized cross correlation is a popular and easily implemented measure for evaluating the similarity between points in trajectories or images (Tsai, Lin, & Chen, 2003). Compared with other similarity measures, such as the Euler distance and standard cross correlation, the NCC has significant advantages because it is invariant to linear transformation and is less sensitive to noise. In the applications of NCC for matching and tracking (Hii, Hann, Chase, & Van Houten, 2006; Nakhmani & Tannenbaum, 2013), the reference trajectory and the observed trajectory are compared on a point-by-point basis. This approach involves taking a given pattern in one trajectory and shifting a template containing the same pattern in another trajectory until the best comparison is found.

Suppose the reference trajectory is  $\mathbf{Z} = \{Z_1, Z_2, ..., Z_{l_1}\}$  and the observed trajectory is  $\mathbf{Y} = \{Y_1, Y_2, ..., Y_{l_2}\}$ ; then, where  $l_1 > l_2$  and  $\mathbf{Y}$  is similar to some parts of  $\mathbf{Z}$ , the similarity measure of NCC is defined as:

$$s_{\mathbf{YZ}}(i) = \frac{(\mathbf{Y} - \bar{\mathbf{Y}}) \times (\mathbf{Z}_i - \bar{\mathbf{Z}}_i)'}{\|\mathbf{Y} - \bar{\mathbf{Y}}\|_2 \|\mathbf{Z}_i - \bar{\mathbf{Z}}_i\|_2} \quad i = 1, 2, \dots, l_1 - l_2 + 1,$$
(8)

where *i* is the shifting index of the reference trajectory, and  $\mathbf{Z}_i = \{Z_i, Z_{i+1}, \ldots, Z_{i+l_2-1}\}$  has the same length as **Y**. Additionally,  $\bar{\mathbf{Y}}$  and  $\bar{\mathbf{Z}}_i$  are the mean vectors of **Y** and  $\mathbf{Z}_i$ , respectively, and  $\|\mathbf{\bullet}\|_2$  is the  $L_2$  norm. It is obvious that  $s_{\mathbf{YZ}}(i)$  is less than or equal to 1 for any **Y** and  $\mathbf{Z}_i$ . If and only if  $\mathbf{Y} = \mathbf{Z}_i$ , then  $s_{\mathbf{YZ}}(i) = 1$ . The invariance to a linear transformation can be proved as follows:

$$s_{(a\mathbf{Y}+b)\mathbf{Z}}(i) = \frac{(a\mathbf{Y}+b-a\bar{\mathbf{Y}}-b) \times (\mathbf{Z}_{i}-\bar{\mathbf{Z}}_{i})'}{\|a\mathbf{Y}+b-a\bar{\mathbf{Y}}-b\|_{2}\|\mathbf{Z}_{i}-\bar{\mathbf{Z}}_{i}\|_{2}}$$
$$= \frac{a(\mathbf{Y}-\bar{\mathbf{Y}}) \times (\mathbf{Z}_{i}-\bar{\mathbf{Z}}_{i})'}{a\|\mathbf{Y}-\bar{\mathbf{Y}}\|_{2}\|\mathbf{Z}_{i}-\bar{\mathbf{Z}}_{i}\|_{2}} = s_{\mathbf{Y}\mathbf{Z}}(i).$$
(9)

This means that the scaling and shifting of the observed trajectory will not influence the similarity measure with the reference trajectory. The computation process is repeated by traversing the reference trajectory. The maximum of  $s_{YZ}(i)$  determines the similarity of two trajectories and the most similar segment in the reference trajectory is also indicated.

# 2.3. RUL estimation method based on similarity of phase space trajectory

Although the degradation data exhibit a distinctive trend and random fluctuations during each degradation process, similar trajectories always exist in the phase space due to the comparable evolution of the dynamics that are characteristic of the system state. Hence, the similarities of the phase space trajectory between the historical degradation processes and the current degradation process are analyzed and applied to estimate the RUL. Our method includes three steps, including the construction of the reference degradation trajectory, similarity matching and RUL estimating. The whole scheme is given in Fig. 1.

During the reference degradation trajectory construction step, PSR is used to embed the time series of the historical degradation data into the high-dimensional phase space. Suppose that  $m{x}^{j}=\left(x_{1}^{j},x_{2}^{j},\ldots,x_{N_{j}}^{j}
ight)$  is the time series in the *j*th degradation process, where *j* is the index of the available degradation processes, and  $N_i$  is the number of points in the time series  $\mathbf{x}^i$ . The operating time at the corresponding sampling epoch is denoted as  $t^j = (t^j_1, t^j_2, \dots, t^j_{N_i})$ . Hence,  $t^j_{N_i}$  is the operational lifetime, i.e., the failure time, of the *i*th degradation process. The minimum embedding dimension  $d^{j}$  is determined by Cao's algorithm, as mentioned in Section 2.1. To simplify the comparisons of different degradation processes, the uniform embedding dimension d, which is equal to or greater than the maximum of all  $d^{i}$ , is adopted. The time delay  $\tau$  is chosen as 1, which is the best choice for discrete time series. In the reconstructed phase space, a reference degradation trajectory  $\mathbf{Z}_i$  is obtained by:

$$\mathbf{Z}_{j} = \begin{bmatrix} X_{d}^{j}, X_{1+d}^{j}, \dots, X_{N_{j}}^{j} \end{bmatrix}^{T},$$
(10)



Fig. 1. RUL estimation based on the similarity of the phase space trajectory.

where  $X_i^j = [x_{i-(d-1)}^j, x_{i-(d-2)}^j, \dots, x_{i-1}^j, x_i^j]$  is a point in the *d*-dimensional phase space. Accordingly, a set of reference degradation trajectories is formed with all available degradation data.

In the similarity matching step, a time series of the current degradation data, denoted as  $\mathbf{y} = (y_1, y_2, \dots, y_c)$ , is obtained and embedded into the same phase space. An incomplete degradation trajectory can be constructed as:

$$\mathbf{Y} = [\mathbf{Y}_d, \mathbf{Y}_{1+d}, \dots, \mathbf{Y}_c]^T, \tag{11}$$

where  $Y_i = [y_{i-(d-1)}, y_{i-(d-2)}, \dots, y_{i-1}, y_i]$ . The operating time that corresponds to each point in trajectory **Y** is denoted as  $t_i$ , where i ranges from d to c. A time-based sliding window with length l is used to select a trajectory segment  $\mathbf{Y}^k = [Y_{k-l+1}, \dots, Y_k]$  from **Y**. The similarities between  $\mathbf{Y}^k$  and all of the reference degradation trajectories are measured by NCC, as mentioned in Section 2.2. For the *i*th reference degradation trajectory, the similarity measure is calculated and written as  $s_{YZ}(t_k^j)$ , where  $t_k^j$  is the corresponding time of the compared segment in  $\mathbf{Z}_{j}$ . According to NCC's invariance to linear transformation,  $s_{YZ}(t_k^j)$  is determined solely by the trajectory shape, and the scaling and shifting of the compared trajectory do not interfere. The parameter *l* determines the length of the compared trajectory segments. For larger values of l, the noise causes less disturbance. However, in practice, *l* is limited by the available degradation data and the computation load. The finite value of *l* and the random fluctuations within the degradation data may result in the identification of the best match in irrelevant degradation stages. To avoid this situation, the similarity measure is modified as:

$$s_k^j(t) = s_{\mathbf{YZ}}(t_k^j) + \left(1 - \frac{\left|t_k - t_k^j\right|}{t_k}\right),\tag{12}$$

where  $t_k$  is the operating time corresponding to  $\mathbf{Y}^k$ . In this similarity metric, the former portion measures the shape similarity of the compared trajectory segments, while the latter portion measures the similarity of the degradation time. The epoch in which  $s_k^i(t)$  obtains a maximum value, is denoted as  $T_k^j$ . In the reference degradation trajectory  $\mathbf{Z}_j$ , the segment at  $T_k^j$  is the optimal matching result, which reflects the compromise between trajectory shape and degradation stage.

In the RUL estimating step, the RUL of the current degradation process is predicted by applying a weighted average of the RULs of the most similar trajectory segments in all reference degradation processes. The RUL of the most-similar segment in the *j*th reference trajectory is obtained by:

$$L_{k}^{j} = t_{N_{j}}^{j} - T_{k}^{j}.$$
 (13)

The weight is calculated as:

$$\omega_{k}^{j} = \frac{s_{k}^{j}(T_{k}^{j})}{\sum_{j=1}^{M} s_{k}^{j}(T_{k}^{j})},$$
(14)

where *M* is the number of reference degradation trajectories. Then, the RUL of the current degradation process is estimated as:

$$RUL_k = \sum_{i=1}^{M} \omega_k^j L_k^j.$$
(15)

By the introduction of phase space trajectory, dynamics of degradation evolution are utilized for prognostics. When degradation processes are subject to finite dynamics modes, the dependence of the proposed method on the quantity of degradation data will be greatly weakened. However, reference degradation trajectories should be enough to ensure that the necessary dynamics modes are included in the available degradation processes.

# 3. Simulation result

Exponential degradation model (Gebraeel et al., 2005) is a good representation for cumulative damage and has successfully captured the degradation process of the rolling element bearing (Gebraeel, 2006). Therefore, the model is used here to generate the run-to-failure data for simulation verification. The logarithmic form of the model is defined as:

$$L(t) = \theta' + \beta t + \varepsilon(t), \tag{16}$$

where L(t) is the log value of the degradation data at time t,  $\theta' \sim N(\mu_0, \sigma_0), \beta \sim N(\mu_1, \sigma_1^2)$ , and the noise  $\varepsilon(t) \sim N(0, \sigma^2 t)$ . Because  $\theta', \beta$  and  $\varepsilon(t)$  are normally distributed random variables, the model represents a wide range of stochastic degradation processes. For convenience, L(t) is commonly substituted for degradation data.

According to Eq. (16), the training data and verification data are generated with the parameters shown in Table 1. The sampling interval is set as 0.1. To evaluate the estimation accuracy of the proposed method, it is necessary to determine the actual life of each simulated degradation process. In Eq. (16),  $\theta'$  and  $\varepsilon(t)$  statistically define only the stationary component and noise, whereas  $\beta t$ determines the degradation rate and degree. Considering the dominant effect of  $\beta t$ , the actual life is set as the time period from the onset of degradation until  $\beta t$  first reaches a failure threshold. In this simulation study, the failure threshold is defined as 150. Fig. 2(a) and (c) depict the generated training data and verification data, respectively. For the ease of distinguishing each degradation process within the figures, undegraded stages with different lengths Q. Zhang et al./Expert Systems with Applications 42 (2015) 2353-2360

# Table 1 The simulation parameters for the training data and verification data.

Parameters	$\mu_0$	$\sigma_0^2$	$\mu_1$	$\sigma_1^2$	$\sigma^2$	Length of the undegraded stage	Initial value
Training data 1	2	1	2.2	0.3	0.2	60	60
Training data 2	1	0.5	2	0.3	1	80	50
Training data 3	2	0.5	1.7	0.5	0.4	60	40
Training data 4	1	1	1.5	0.6	1	80	30
Training data 5	2	1	1.9	0.5	0.5	90	20
Training data 6	1	1	2.1	0.5	1	100	10
Verification data	1	1	1.8	0.5	0.1	40	5



**Fig. 2.** The simulated degradation processes: (a) the training degradation data; (b)  $E_1(d)$  and  $E_2(d)$  of the simulated degradation data; (c) the verification degradation data; (d) the RUL estimated by the SbRLP; (f) estimation errors of the proposed method and the SbRLP.

are inserted in front of the degradation data series. Additionally, the initial values, as shown in Table 1, are added to the data series.

The time series of the training data are embedded into the phase space to construct the reference trajectories. To determine the embedding dimension,  $E_1(d)$  and  $E_2(d)$  are calculated separately. Fig. 2(b) shows the values of  $E_1(d)$  and  $E_2(d)$  calculated with the set identified as Training data 1. It can be seen that  $E_1(d)$  tends to a stationary value when *d* is larger than 8 and  $E_2(d)$  changes with *d*. The same conclusions were drawn for the other training data sets. Hence, the embedding dimension is set to 8. The verification data are embedded into the uniform phase space. A sliding window, with a length of 30 and a sliding step of 20, is used to select the verification trajectory segment after 200 operating times. By similarity matching with the reference trajectories, the RUL is estimated.

Fig. 2(d) shows the estimated results and the corresponding actual RUL. For comparison, a similarity-based residual life prediction (SbRLP) approach (You & Meng, 2013) is utilized. Because the solution A of SbRLP is applicable to the situation of limited samples, it is chosen to deal with the same simulation samples.



Fig. 3. The high-pressure water pump layout.



Fig. 4. The degradation data of the high-pressure water pump.

Fig. 2(e) depicts the estimated results of SbRLP. To get a view of this comparison, the absolute errors between the estimated RUL and the actual RUL are shown in Fig. 2(f). It is readily apparent that the estimation errors of our method are significantly less than those of the SbRLP. The result verifies that the estimation accuracy of the proposed method is satisfactory, even if data in each simulated degradation process exhibit a distinctive trend and random fluctuations.

# 4. Case study

To demonstrate the RUL estimation effect of the proposed method, experimental data were collected from the degradation processes of a high-pressure water pump. The pump, which is shown in Fig. 3, is used to remove the oxide scale formed on the surface of stainless steel during the process of steel rolling. Due to the nature of continuous work in a high-pressure environment, pump degradation develops rapidly, and breakdown occurs frequently. The outlet pressure, as a direct performance indicator, is monitored at intervals of 1 h. Fig. 4 depicts the outlet pressure during six complete degradation processes, which range from the beginning of operation to failure. Significant difference can be observed between the degradation trends and the operating lifetimes, which range from approximately 1350–1700 h.

Due to the limitation of available samples, the Leave-One-Out-Cross-Validation (LOOCV) method is used to verify the proposed method. The time series of the outlet pressure in a degradation process serves as the validation sample, and the time series of the outlet pressure in the remaining degradation processes serve as the training samples. This is repeated such that the pressure time series in each degradation process is used once as the validation sample. All samples are embedded into the same phase space, whose embedding dimension d is set as 6 according to Cao's algorithm, and the time delay  $\tau$  is set as 1. The reference degradation trajectories are constructed with the training samples. The trajectory segments of the validation sample after 800 h of operating time are selected using a sliding window with a length of 100 h and a sliding step of 20 h. Then, the RUL is estimated during the period of operation from 800 h to failure at intervals of 20 h. Fig. 5 shows the comparisons of the estimated RUL and the actual RUL.

Despite the significant differences between the time series of the outlet pressure, it can be observed from Fig. 5 that the RULs estimated by the proposed method are close to the actual RUL.



Fig. 5. The RUL estimation for the high-pressure water pump: (a) degradation process 1; (b) degradation process 2; (c) degradation process 3; (d) degradation process 4; (e) degradation process 5; (f) degradation process 6.

For all degradation processes, the influence of random fluctuations is effectively restrained. In particular, in degradation process 5, an abrupt change of the outlet pressure occurs at 1380 h, while the operating parameters are manually adjusted. Because the abrupt change results in the local distortion of the degradation trajectory, a small decrease of the estimated RUL can be seen in Fig. 5(e). For degradation process 6, the pump has the shortest operating life. Because the RULs of the most-similar trajectory segments in other degradation processes are longer than the actual RUL, the curve of the estimated RUL, as shown in Fig. 5(f), is higher than the actual RUL curve in the early stage of degradation. After approximately 1100 h of operating time, the estimated error is decreased, due to the increasing influence of trajectory shape similarity during the later stage of degradation. In contrast, the estimated RULs in the longest degradation period, i.e., degradation process 2, are slightly less than the actual RULs during early degradation and coincide well with the actual RULs when the failure is impending. This RUL curve is given in Fig. 5(b). In conclusion, the proposed method provides the capability of estimating the RUL when confronted with the adverse condition of degradation data, which includes random fluctuations, a distinctive trend and various lifetimes.

# 5. Conclusion and discussion

Due to the nonlinearity of mechanical damage accumulation, historical degradation data are incapable of providing a direct reference for the prediction of future failure. In this paper, a novel method based on the similarity of the phase space trajectory is proposed to estimate the RUL for an ongoing degradation process. The proposed method has two significant features. First, the nonlinear degradation behavior of mechanical system is represented by the phase space trajectory. Second, similarity matching of trajectory is not affected by the scaling and shifting of the compared trajectory. Then, the underlying degradation evolution, which generates degradation data with a distinctive trend and random fluctuations, is revealed and used to estimate the RUL. This method is successfully applied in simulated data in stochastic degradation processes. In spite of the limited samples, the proposed method is greatly superior in prediction accuracy to the compared approach, which implements with the similarity of original degradation data. In addition, the proposed method is evaluated by the actual data acquired from a high-pressure water pump. The results show that the predicted RULs are very close to the actual RUL.

There are still a number of challenges and practical problems to be further studied. We summarized them as follows: (1) In some applications, the mechanical system is not always allowed to run to failure. Truncated data, which are collected from interrupted degradation processes, cannot provide a clear reference for RUL estimation. (2) Degradation paths of mechanical systems may transform in the situation of manual intervention or changed operation condition. It is common in the accelerated degradation process. (3) No completed degradation processes can be recorded. To some critical and long-lifetime systems, such as nuclear coolant pump, a continuous and hands-off degradation is unacceptable. (4) Multiple degradation data are available from a mechanical system. But their behaviors in a degradation process are inconsistent.

To tackle these challenges, some possible solutions are suggested. The relationship between the degradation rate and the operation condition can be analyzed according to the evolution of the degradation trajectory. A phase space threshold, which determined by the normal state, would be established to identify the abnormal phase space trajectory. If a library of phase space trajectory, which includes trajectories of different monitoring parameters and in different working stages, can be constructed, data mining technique has potential as well. These are also where we intend to focus our research in the future.

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