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A Geometric Programming to Importance Sampling for Power System Reliability Evaluation

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***Abstract*—A novel Geometric Programming (GP) is presented in the first time by the optimization model of importance sampling parameters (ISP) in Variance Minimization (VM) for importance sampling (IS) of power system reliability evaluation. The key point of the proposed method is that the equality constraints of VM optimization model can be relaxed into inequalities because of its special structure, thus a new GP-VM convex optimization model can be built exactly to solve the difficulty of obtaining the optimal ISP. Numerical results of two test systems verify the effectiveness of the proposed method.**

***Index Terms*—Geometric programming (GP), importance sampling (IS), power system reliability, variance minimization (VM).**

I. Introduction

MONTE Carlo Simulation (MCS) is widely used in power system reliability evaluation, especially for large and complex power systems, due to its strong robustness to system scale and complexity. However, MCS involves a great amount of computation to deal with rare events. It needs huge numbers of samples to pick up the rare event with small probability. To address this issue, Importance Sampling (IS) has been introduced in MCS-based power system reliability evaluation to reduce the computational effort [1]–[3]. The essence of this approach is to utilize important sampling parameters (ISP) to emphasize those important values more frequently and reduce the coefficient of variation (COV). However, it is crucial to determine the optimal ISP, which can minimize the COV and significantly improve computation efficiency.

Mathematically, there are commonly two popular methods to obtain the optimal ISP: Cross-Entropy (CE) [1]–[3] and Variance Minimization (VM) [4], [5]. Comparing the two methods, it has been proved that VM results in smaller COV than CE, so that it presents better performance in terms of the convergence accuracy [5]. But VM demands to solve a complex optimization model, which is very difficult to handle. Hence, CE has been widely used in power system reliability evaluations, whereas VM has not been introduced. In particular, [5] suggested that if VM can be solved efficiently, it will be promising.

In this letter, we investigate that the parametric importance sampling density function in power system reliability evaluation follows a binomial distribution [1], [2]. Moreover, it is interesting to find that the VM based optimization model for power system reliability evaluation to obtain the optimal ISP has a special structure. Therefore, it can be set up as a Geometric Programming (GP), which is equal to a convex optimization model and can be very efficiently solved to its global optimality in polynomial time [6], even for large-scale problems. To our best knowledge, it is the first time that the GP is introduced to VM to obtain the optimal ISP for power system reliability evaluation. Therefore, we name it as "GP-VM" method.

II. Geometric Programming to Importance Sampling

A GP is a type of mathematical optimization problem. Its objective and constraint functions have the specific form as

$$\min_{x} \quad f_0(x) \tag{1-a}$$

$$s.t. \quad f_i(x) \le 1, \quad i = 1, 2, ..., m \tag{1-b}$$

$$g_i(x) = 1, \quad i = 1, 2, ..., p \tag{1-c}$$

where f_i are posynomials, g_i are monomials, and $\boldsymbol{x} = (x_1, \ldots, x_{\mathrm{n}})$ are n real non-negative variables. A monomial is a real value function in the form of $g(x) = cx_1^{a_1} x_2^{a_2} \ldots x_n^{a_n}$, $c \ge 0$ and $a_i \in R$. A posynomial refers to a sum of one or more monomials, i.e., a function in the form of $f(x) = \sum_{j=1}^{m} c_j x_1^{a_1} x_2^{a_2} \ldots x_n^{a_n}$, $c_k \ge 0$.

A GP model can be efficiently solved by the logarithmic transformation and general interior point method [6]. Generally, a GP solver can be thought of as a reliable black box that solves any problem put in GP form. In the following, we will make an effort to transform the VM based optimization model for IS into the standard GP model, yielding "GP-VM" method.

With respect to the LOLP index, IS based power system reliability evaluation can be briefly formulated as:

$$\mathrm{LOLP} = \frac{1}{N} \sum_{i=1}^{N} \mathrm{I}\{S(\boldsymbol{X}_i) < L\} \, \mathrm{W}(\boldsymbol{X}_i; n, \boldsymbol{u}, \boldsymbol{v}) \tag{2}$$

where $\mathrm{W}(\mathbf{X}_i;\, n, \boldsymbol{u}, \boldsymbol{v})$ is the known likelihood and represents a correction that must be introduced into sampling process to avoid any biased estimates; $\boldsymbol{u}$ is the original component forced outage rate vector; $\boldsymbol{v}$ is the new component forced outage rate vector; $\mathbf{X}_i$ is the i-th sampled system state; n is the total number of components; $\mathrm{S}(\mathbf{X}_i)$ is the performance function in i-th state; L represents the load level; N represents the number of samples; $\mathrm{I}\{\bullet\}$ is a 0–1 indicator function, which equals to 1 if the expression in $\mathrm{I}\{\bullet\}$ is true, and 0 otherwise.

Specifically, the expression of $\mathrm{W}(\mathbf{X}_i; n, \boldsymbol{u}, \boldsymbol{v})$ can be expressed as follows:

$$\mathrm{W}(X_i; n, \mathbf{u}, \mathbf{v}) = \frac{f(X_i; n, \mathbf{u})}{f(X_i; n, \mathbf{v})} = \frac{\prod_{j=1}^{n} u_j^{(1-X_{\mathrm{ij}})} (1-u_j)^{X_{\mathrm{ij}}}}{\prod_{j=1}^{n} v_j^{(1-X_{\mathrm{ij}})} (1-v_j)^{X_{\mathrm{ij}}}} \tag{3}$$

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where $f(\cdot;n,\boldsymbol{u})$ and $f(\cdot;n,\boldsymbol{v})$ are the original sampling and parametric importance sampling density function; $X_{\rm ij}$ denotes the j-th element in $\mathbf{X}_i$, which equals to 0 if the j-th component is at a failure state at i-th sample and 0 otherwise. For convenience, we definite a subset $\boldsymbol{Y} = \{\boldsymbol{Y}_1, \boldsymbol{Y}_2, \ldots, \boldsymbol{Y}_{\rm M}\}$ selected from $\boldsymbol{X} = \{\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_{\rm N}\}$, with each element satisfying $S(\boldsymbol{Y}_i) < L \cap \{\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots \boldsymbol{X}_i, \ldots, \boldsymbol{X}_N\}$ and M as the cardinality of subset $\boldsymbol{Y}$.

The VM method to obtain the optimal ISP $\boldsymbol{v}$ aims to solve the following optimization problem [6]:

$$\min_{v} \quad \frac{1}{N}\sum_{i=1}^{M} \frac{\prod_{j=1}^{n} u_j^{(1-Y_{\rm ij})}\,(1-u_j)^{Y_{\rm ij}}}{\prod_{j=1}^{n} v_j^{(1-Y_{\rm ij})}\,(1-v_j)^{Y_{\rm ij}}} \quad s.t.\, 0 \le v \le 1. \tag{4}$$

Let $c_i = \prod_{j=1}^{n} u_j^{(1-Y_{\rm ij})}\,(1-u_j)^{Y_{\rm ij}}$ for $i = 1, 2 \ldots, M$ be positive constant numbers and $t_j = 1\text{-}v_j$ be positive dummy variables. Then, (4) can be equivalently reformulated as follows:

$$\min_{v} \quad \frac{1}{N}\sum_{i=1}^{M}\left(c_i \prod_{j=1}^{n} v_j^{(Y_{\rm ij}-1)} t_j^{-Y_{\rm ij}}\right) \tag{5-a}$$

$$s.t. \quad 0 \le v \le 1,\ v_j + t_j = 1,\ j = 1, 2, \ldots, \text{n}. \tag{5-b}$$

However, it should be noted that the model (5) is not a GP model, since the equality constraints in (5-b) are posynomials, instead of monomials. Therefore, the model (5) is difficult to solve. To address this problem, the model (5) can be relaxed to (6) by changing the equality constraints in (5-b) into inequalities, which leads to a GP model as

$$\min_{v} \quad \frac{1}{N}\sum_{i=1}^{M}\left(c_i \prod_{j=1}^{n} v_j^{(Y_{\rm ij}-1)} t_j^{-Y_{\rm ij}}\right) \tag{6-a}$$

$$s.t. \quad 0 \le v \le 1, v_j + t_j \le 1,\ j = 1, 2, \ldots, n. \tag{6-b}$$

Fortunately, we can show that the relaxed model (6) is strictly equal to the model (5), since in model (6) there is no other constraints on the dummy variables t_j expect (6-b) and the objective (6-a) aims to minimize the objective function. Thus, t_j should be optimized as large as possible and its upper bound is $1\text{-}v_j$. In this term, the optimal solution must be located at $t_j = 1\text{-}v_j$, i.e., the inequalities must be binding. Consequently, model (6) is strictly equal to model (5) as well as model (4).

Finally, the GP model (6) can be solved by a general interior point method [6]. Consequently, the optimal ISP is achieved. Certainly, not only LOLP index, but other reliability evaluation indexes (e.g., EENS) also can be used for the proposed method.

III. Numerical Illustration

The proposed GP-VM and the traditional CE methods were compared on two test systems: (i) The standard IEEE RTS-79 system including 32 generators and 38 transmission lines [7]; (ii) A real-world 'XY' grid in the Northwest China including 64 generators and 1104 transmission lines. The computation is carried on a computer with an Intel® Core$^{\rm TM}$ i5 Duo Processor (2.30 GHz) and 4 GB RAM in C++ environment by MOSEK commercial solver.

Firstly, we adopt different pre-sampling sizes (PS) of MCS to obtain ISP by CE and GP-VM method, respectively. Then, the reliability evaluation runs until the COV of LOLP reaches a specified value (e.g., 1%). Next, the index of mean relative error (MRE) in 10 simulations is calculated to measure the accuracy. Notably, to compute MRE, the exact LOLP is termed as the value through crude MCS with 10^6 samples. In addition, we assume that the total computational time (TCT) includes both pre-sampling time (PCT) for ISP and main-sampling time (MCT) for evaluating indices. Here, PCT includes an additional time for computing optimal ISP by the GP optimization model.

TABLE I
Comparison of Results Between GP-VM and CE Methods

Test System	PS	Methods	Results			
			LOLP	MRE_{LOLP}	TCT(s)	SNs
IEEE RTS-79 (1%)	3000	CE	0.0936	2.197%	7373	22 798
		GP-VM	0.0942	1.567%	5377	19 027
	6000	CE	0.0943	1.463%	5970	19 866
		GP-VM	0.0947	1.045%	5401	18 416
XY Grid (1%)	3000	CE	0.0356	8.483%	11390	31 540
		GP-VM	0.0363	6.684%	8574	27 040
	6000	CE	0.0377	3.084%	8530	23 465
		GP-VM	0.0381	2.056%	5905	16 004

TABLE II
A Comparison of Detailed CT Between GP-VM and CE Methods

Test System	Methods	Detailed TCT (seconds)			
		PCT/3000	MCT/3000	PCT/6000	MCT/6000
IEEE RTS-79	CE	73.00	7300	147.21	5823
	GP-VM	73.12	5304	147.45	4894
XY Grid	CE	316.01	11 029	632.33	7998
	GP-VM	321.35	8253	643.72	5262

As shown in Table I, GP-VM attains a lower MRE, which suggests higher estimation accuracy in LOLP. As for computational time, it can be observed from Table II that GP-VM costs a little additional time than CE at pre-sampling, yet it can save much more time at main-sampling. Thus, the TCT of the proposed method in the whole evaluation process is much less than the CE method. Meanwhile, fewer sample numbers (SNs) is needed by the proposed method to reach the given same convergence criteria.

IV. Conclusion

The letter proposes a "GP-VM" method for power system reliability evaluation. Numerical results show that it obtains better convergence performance and higher estimation accuracy than CE method with even less simulation time. Further, the proposed method applied to the reliability evaluation in other research fields.

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