A Fine-Resolution Frequency Estimator in the Odd-DFT Domain

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***Abstract*—Although many frequency estimation methods are available, few are designed for high-quality speech and audio processing systems, which typically use the modified discrete cosine transform (MDCT) as their analysis filter bank. In this letter, we propose a low complexity frequency estimator that is suitable for MDCT-based systems and that operates in the odd-DFT domain. Taking a complex exponential in noise as the input and deriving the analytical expression of its odd-DFT coefficient, we obtain an interpolated odd-DFT-based frequency estimator. Experiments show that the proposed estimator outperforms all other reported odd-DFT/MDCT domain estimators and has precision that is similar to that of representative DFT domain frequency estimators. The overhead for incorporating this estimator into a speech and audio processing system is small due to the simple odd-DFT to MDCT conversion. The corresponding magnitude and phase estimators are also proposed in this letter.**

***Index Terms*—Audio processing, frequency estimation, MDCT, odd-DFT.**

I. Introduction

FREQUENCY estimation is a fundamental signal processing problem that is relevant to a wide range of applications, such as communication, instrumentation, medical and audio. The odd frequency discrete Fourier transform (odd-DFT) domain method that we propose in this letter is designed for audio applications that use the modified discrete cosine transform (MDCT) filter bank [1], [2] as their basic analysis/synthesis framework, e.g., high-quality audio analysis, modification and coding. Low complexity of the estimator and low overhead for incorporating it into audio applications are our major concerns.

Although many low complexity estimation approaches exist [3]–[7], the majority of these methods require a high overhead to be incorporated into MDCT-analysis-based audio systems. This is because they require a separate signal analysis procedure, such as linear prediction, subspace decomposition, or discrete Fourier transform (DFT). But this separate module can not be re-utilized by the audio system.

To obtain the low overhead algorithms, two classes of estimators have been proposed over the past decade. One class operates with the MDCT coefficients provided by the audio system directly [8]. However, the magnitude of MDCT is known to be modulated by phase information, which greatly impacts the estimator precision, particularly when noise exists. Although efforts have been made to compensate such modulation [9], [10] or to obtain more accurate estimations through an iterative method [11], none of these methods are immune to the effect of the phase modulation. Investigations on the performances of these MDCT domain algorithms under noisy conditions show that their mean square errors (MSE) perform similarly and are all slightly far from the Cramer-Rao bound (CRB) (above 10 dB) [12]. The other class uses the odd-DFT coefficients to obtain the estimated value [13], [14] and then obtains the required MDCT from odd-DFT [15], [16]. In this way, the effect of phase modulation can be avoided. However, the existing odd-DFT domain estimators are based on an approximate fitting function; thus, their precisions are even worse than those of the MDCT domain methods.

The motivation of this letter is to obtain an accurate low complexity frequency estimator in the odd-DFT domain. The major contributions can be summarized as follows: (i) the analytical expression of the odd-DFT coefficients for a sinusoidal signal is derived; (ii) an interpolated odd-DFT-based frequency estimator is proposed. Experiments show that the proposed method has a similar MSE level as the DFT domain estimators, which is considerably less than the reported methods in the odd-DFT and MDCT domains. In addition, the proposed method is more applicable for MDCT-based applications than a DFT domain estimator since it allows the use of a window function required for perfect reconstruction and has lower complexity.

The rest of this letter is organized as follows: in Section II, the relationship between the odd-DFT and MDCT coefficients and the description of the odd-DFT domain frequency estimation are presented. The odd-DFT domain sinusoidal analysis and the proposed estimator are given in Section III Simulation results are presented in Section IV, and the conclusions are drawn in Section V.

II. Problem Description

A. Odd-DFT Analysis

The odd-DFT, which is also called ODFT, is a frequency shift version of common DFT with a half DFT bin. Using the twiddle factor notation $W_N = e^{-j\frac{2\pi}{N}}$, the odd-DFT coefficient of an N-point input signal $x(n)$ is given by

$$X_O(k) = \sum_{n=0}^{N-1} x(n) W_N^{(k+\frac{1}{2})n}, \tag{1}$$

where $k = 0, 1, \cdots, N-1$ is the bin index. In other words, the odd-DFT coefficients are the samples of the discrete-time Fourier transform (DTFT) taken directly between every two neighboring DFT sample bins. For a single tone complex ex-

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ponential signal with frequency $\omega = \frac{2\pi}{N}l$, its odd-DFT is equal to the DFT of its frequency shifting signal at $\omega' = \frac{2\pi}{N}(l - \frac{1}{2})$. Therefore, technically speaking, the odd-DFT domain is also suitable for estimating the signal's frequency as the DFT domain does. These DFT-domain estimators can directly be used in the ODFT domain with the output shifting a half bin.

The conversion from odd-DFT to MDCT is straightforward. Here, we use $Re(\cdot)$ and $Im(\cdot)$ to represent the real and imaginary parts of a complex value, respectively. For the odd-DFT coefficient of a real-valued $x(n)$, defining a phase factor $\theta(k) = \frac{2\pi}{N}(k + \frac{1}{2})n_0$ with $n_0 = \frac{1}{2} + \frac{N}{4}$ and taking the real part of its phase shifting version, we obtain

$$Re\left(X_O(k)e^{-j\theta(k)}\right) = \sum_{n=0}^{N-1} x(n)\cos\frac{2\pi}{N}(n + n_0)\left(k + \frac{1}{2}\right), \quad (2)$$

which is the same as the definition of the MDCT. Clearly, the MDCT coefficient $X_M(k)$ is the phase-modulated version of the odd-DFT coefficient magnitude $|X_O(k)|$, i.e., $X_M(k) = |X_O(k)|\cos[\angle X_O(k) - \theta(k)]$, and it can be computed from $X_O(k)$ as follows,

$$X_M(k) = Re(X_O(k))\cos\theta(k) + Im(X_O(k))\sin\theta(k). \quad (3)$$

Only two real multiplications and one real addition are required to compute one $X_M(k)$ value. The convenience of converting the odd-DFT to MDCT makes the odd-DFT domain estimator a candidate for MDCT-based applications [17].

B. Frequency Estimation in Odd-DFT Domain

We use a complex model in the following discussion. Although the audio signals are generally real, they can be viewed as consisting of a positive frequency component plus a negative (imaginary) component. Hence, using the complex model can separate the error caused by the estimator and the influence caused by the negative component. We consider a typical frequency estimation problem with the signal modeled as a single tone $s(n)$ polluted by the additive noise $w(n)$,

$$x(n) = s(n) + w(n), \quad n = 0, 1, \cdots, N-1, \quad (4)$$

and windowed by $h(n)$. For $s(n) = Ae^{j(\omega n + \phi)}$, the amplitude A, frequency ω and initial phase ϕ are deterministic but unknown constants. $w(n)$ is zero-mean white complex Gaussian noise with variance σ^2. The signal-to-noise ratio (SNR) is defined as A^2/σ^2. Define $\omega = \frac{2\pi}{N}l = \frac{2\pi}{N}(l_0 + \delta)$, where $l_0 (0, 1, \cdots, N-1)$ and $\delta (0 \le \delta < 1)$ are the integer and fractional parts of the digital frequency l in the odd-DFT bin scale, respectively. The objective is therefore to estimate the values of l_0 and δ using several odd-DFT coefficients of the windowed signal $x(n)h(n)$.

To make the algorithm applicable for MDCT-based audio applications, two aspects must be addressed.

- First, the windowing function $h(n)$ must satisfy the Princen-Bradley conditions [1], which is required by the MDCT to achieve perfect reconstruction. However, the windows that are commonly used in the existing DFT-domain estimators, such as rectangular, hamming or hanning, do not satisfy such conditions. Because the window function significantly affects the performance of an estimator [6], [18], these DFT domain estimators will inevitably under-perform if the window is altered. In this letter, we consider a typically used sine window $h(n) = \sin\frac{\pi}{N}(n + \frac{1}{2})$ and propose the corresponding estimator.
- Second, the algorithm should target good performance for real-valued inputs because the audio signals are in practice real valued. In fact, the transform domain estimation of a real signal is equivalent to that of a complex one polluted by its mirroring frequency, which will be illustrated in Section IV. Moreover, the odd-DFT of a real signal is symmetric, $X_O(k) = X_O(N - k - 1)$, and the MDCT takes the first $\frac{N}{2}$ values only. Therefore we constrain the bin index k to $[0, \frac{N}{2} - 1]$ in the following discussion.

III. Parameter Estimation Method

A. Sinusoidal Analysis in Odd-DFT Domain

The odd-DFT of a signal $x(n)$ windowed by $h(n)$ is defined as

$$X_O(k) = \sum_{n=0}^{N-1} x(n)h(n)W_N^{(k+\frac{1}{2})n}. \quad (5)$$

Consider a noise-free sinusoidal signal

$$x(n) = s(n) = Ae^{j(\frac{2\pi}{N}ln + \phi)}, \quad (6)$$

by substituting (6) into (5), we have

$$X_O(k) = Ae^{j\phi}\sum_{n=0}^{N-1} h(n)W_N^{(k-l+\frac{1}{2})n} = Ae^{j\phi}H_O(k - l), \quad (7)$$

where $H_O(\xi)$ represents the odd-DFT of the window $h(n)$, but ξ is not limited to being an integer or non-negative. With $\xi = k - l$, we consider $\xi \in [-\frac{N}{2}, \frac{N}{2}]$ here. It is clear that the odd-DFT coefficients are primarily determined by the window function.

For the sine window $h(n) = \sin\frac{\pi}{N}(n + \frac{1}{2})$,

$$\begin{aligned} H_O(\xi) &= \sum_{n=0}^{N-1} \sin\frac{\pi}{N}\left(n + \frac{1}{2}\right)W_N^{(\xi+\frac{1}{2})n} \quad (8) \\ &= \frac{1}{2}\sin(\pi\xi)\left[\frac{1}{\sin\frac{\pi\xi}{N}} - \frac{1}{\sin\frac{\pi(\xi+1)}{N}}\right]W_N^{\frac{N-1}{2}(\xi+\frac{1}{2})}. \end{aligned}$$

Note that the two poles at 0 and -1 are canceled out by the zeros at the same locations, the values of $H_O(\xi)$ can be obtained using L'Hospital's rule in this case. $H_O(\xi)$ has fast-fading side-lobes as illustrated in Fig. 1; significant values appear only when ξ is near 0. Via further approximation on (8) for ξ near 0 by

$$H_O(\xi) \approx \frac{N\sin(\pi\xi)}{2\pi\xi(\xi + 1)}W_N^{\frac{N-1}{2}(\xi+\frac{1}{2})}, \quad (9)$$

one can obtain an analytical expression of (7) given by

$$X_O(k) = \frac{AN\sin\pi l}{2\pi(k - l)(k - l + 1)}e^{j\psi}, \quad (10)$$

where

$$\psi = \frac{\pi}{N}\left(k + \frac{1}{2}\right) + \frac{N-1}{N}\pi l + \frac{\pi}{2} + \phi. \quad (11)$$

These results are the basis of the frequency estimator proposed in this letter.

B. Frequency Estimation

Now we present the estimation algorithm for the frequency $l = l_0 + \delta$. We limit our algorithm to non-iterative and single

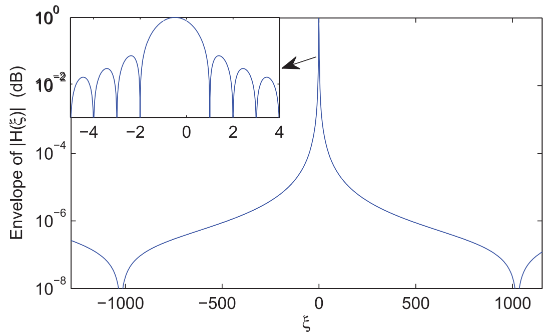

Fig. 1. Plot of $H_O(\xi)$, the odd-DFT magnitude of the sine window. The plot shows the envelope of the normalized values and the amplified curve of values near zero. Here, $N = 2048$.

frame based, and we use $\hat{\cdot}$ to denote an estimated value of a parameter. The estimations of l_0 and δ are according to the values of $|H_O(\xi)|$ (as plotted in Fig. 1) with an interpolated method.

1) The Integer Part: Recall that the odd-DFT is a frequency shift version of the DFT, the magnitude peak of the DFT at $k = l_0$ for $-0.5 < \delta < 0.5$ corresponds to that of the odd-DFT at $k = l_0$ for $0 < \delta < 1$. Thus l_0 is estimated as

$$\hat{l}_0 = \arg\max_k(|X_O(k)|). \tag{12}$$

For $\delta = 0$, a neighboring peak appears at $k_1 = l_0 - 1$; in this case, the estimation result is still $\hat{l}_0 = k$.

2) The Fractional Part: Because $\delta = 0$ has been addressed, here we use $X_O(\hat{l}_0)$ and its two immediate neighbors to obtain the estimated value when $0 < \delta < 1$. By denoting $X_i = X_O(\hat{l}_0 + i)$, $i = 0, \pm 1$, defining a ratio of the odd-DFT coefficients $\alpha_0 = \frac{|X_{-1}|}{|X_{+1}|}$, and substituting (10) into the ratio, we can obtain the relationship between α_0 and δ,

$$\alpha_0 = \frac{(1-\delta)(2-\delta)}{\delta(1+\delta)}. \tag{13}$$

Thus, a reasonable estimator of δ is given by

$$\hat{\delta} = \begin{cases} \frac{3+\alpha_0-\sqrt{\alpha_0^2+14\alpha_0+1}}{2(1-\alpha_0)} & \text{for } \alpha_0 \neq 1, \\ 0.5 & \text{for } \alpha_0 = 1. \end{cases} \tag{14}$$

Because the main lobe of the sine window is three-bin wide (as shown in Fig. 1), when δ is close to 0 or 1, either $|X_{-1}|$ or $|X_{+1}|$ is close to 0. Such a very small value is sensitive to noise and will decrease the precision of the estimator. Therefore, we define two other ratios, $\frac{|X_{-1}|}{|X_0|}$ and $\frac{|X_0|}{|X_{+1}|}$, and substitute (10) into the ratios again; consequently, two new estimators are obtained as

$$\hat{\delta} = \frac{|X_0| - |X_{-1}|}{|X_0| + |X_{-1}|}, \tag{15}$$

$$\hat{\delta} = \frac{2|X_{+1}|}{|X_0| + |X_{+1}|}. \tag{16}$$

Tests on these three estimators show that a combination of them enhances the precision of the estimation, which will be given in Section IV.

C. Magnitude and Phase Estimation

With the estimated frequency value, $\hat{l} = \hat{l}_0 + \hat{\delta}$, we can obtain the magnitude,

$$\hat{A} = \frac{|X_0|}{|H(-\hat{\delta})|} \approx \frac{2\pi\hat{\delta}(1-\hat{\delta})}{N\sin(\pi\hat{\delta})}|X_0|, \tag{17}$$

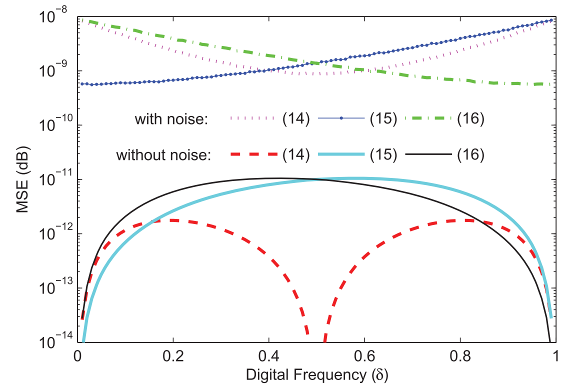

Fig. 2. MSE of the three frequency estimation equations (14), (15) and (16). The fractional part δ varies from 0 to 1 with a step of 0.01. A pure single complex exponential is used in the test *without noise*, and its noise polluted counterpart with SNR = 80 is used in the test *with noise*.

and the initial phase

$$\hat{\phi} = \angle X_0 - \frac{\pi}{2N} - \frac{N-1}{N}\pi\hat{\delta} + \frac{\pi}{2}. \tag{18}$$

For $\hat{\delta} = 0$, $\hat{A} = \frac{2}{N}|X_O(l)|$ because $|H(0)| = \frac{N}{2}$.

IV. Simulation Results and Discussion

In this section, we present the simulation results of the proposed estimator. According to the audio applications, we set the window length to $N = 2048$, sampling frequency to $f_s = 44.1$ kHz, and magnitude to $A = 1$. ϕ is generated randomly in the range $(-\pi, \pi)$ obeying a uniform distribution. All of the results are the averages of 10 000 independent runs.

A. The Combined Estimator and Its Complexity

In this test, the performances of the three equations (14), (15) and (16) in Section III-B for the frequency estimation of a single tone complex exponential signal are compared. The comparison was conducted under both noisy and noiseless conditions when the fractional part δ changes from 0 to 1. The MSE curves are given in Fig. 2. This plot shows that although the SNR is set to 80 dB for the noisy case, the noise greatly affects the precision when δ is close to 0 or 1, making the MSE curves deviate more from the noiseless ones. None of the equations outperform the others for most of the δ values.

To decrease the estimation error, the proposed estimator is a combination of these three equations,

$$\begin{cases} (15) & \text{for } \hat{\alpha}_0 > \alpha_0|_{\delta=0.5-\gamma/2}; \\ (14) & \text{for } \alpha_0|_{\delta=0.5+\gamma/2} \leq \hat{\alpha}_0 \leq \alpha_0|_{\delta=0.5-\gamma/2}; \\ (16) & \text{for } \hat{\alpha}_0 < \alpha_0|_{\delta=0.5+\gamma/2}; \end{cases}$$

where γ is the width centered at $\delta = 0.5$ and $\hat{\alpha}_0$ is the calculated ratio. We repeated this test with various SNRs and found that $\gamma = 0.2$ is a good choice. The computational complexity of the proposed estimator consists of a division, one or two comparison(s) and a calculation according to one of the formulas of (14)-(16). For low complexity implementation, set $\gamma = 0$ and use (15) for $|X_{-1}| > |X_{+1}|$ and (16) for the other case. The division and the square-root operation in (14) are avoided. In the following test, we present the results of these two combination settings.

B. Estimation for a Complex Signal

Simulations were performed to verify the frequency estimation precision of the proposed odd-DFT domain estimator (in

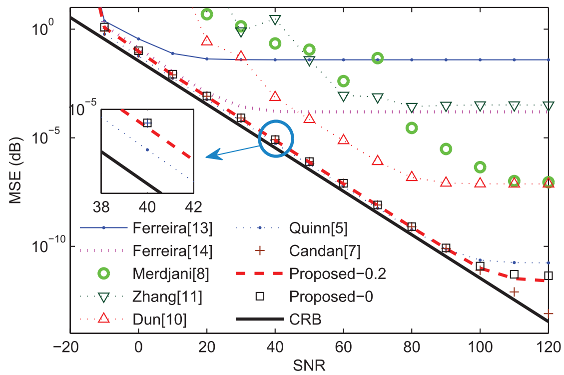

Fig. 3. Plot of the MSE vs. SNR of different frequency estimators for single tone complex exponential input. $N = 2048$, and δ is set randomly obeying a uniform distribution in the range of 0 to 1.

combined form) with a single complex exponential signal. For comparison, two odd-DFT domain estimators (Ferreira's single rule [13] and combined [14]), three MDCT domain estimators (Merdjani [8], Zhang [11] and Dun [10]), and two DFT domain estimators (Quinn [5] and Candan [7]) are taken as benchmarks. The single-frame-based envelope method without iteration is used for Zhang's estimator [11]. The fractional part δ is set to be uniformly distributed in the range (0,1). For all of the odd-DFT and MDCT domain estimators, the sine window is used. For the two DFT domain estimators [5], [7], we only consider the rectangular window case. This is because using the sine window greatly changes the signal spectrum and significantly degrades the performance of a DFT domain estimator.

Fig. 3 presents the MSE of these estimators as the SNR varies. The CRB [19] for frequency estimation is also plotted. The number following 'Proposed' in the legend is the value of γ. The three MDCT domain estimators are far from the CRB because of their inherent phase-modulation effect. The two Ferreira's odd-DFT domain methods exhibit considerable bias because of the approximate fitting function they use. The proposed odd-DFT domain estimator (with $\gamma = 0$ and $\gamma = 0.2$) and the two DFT domain estimators closely follow the CRB. For the proposed method, only when the SNR is greater than 100 dB does some slight difference appear between $\gamma = 0$ and $\gamma = 0.2$. Therefore the proposed odd-DFT domain estimator is among the best estimators and is the most suitable method for sine-window-based audio applications.

C. *Estimation for a Real Signal*

The influence of the negative (imaginary) frequency component of a real signal to the frequency estimation is presented in this part. Here, we use $s(n) = A\sin[\omega n + \phi]$ as the input, and ω varies from 0 to π. The MSE curves of different estimators at SNR $= 100$ dB are shown in Fig. 4. The single-frame-based envelope method with a maximum of 10 iterations is used for Zhang's estimator [11] here. Most of the curves in the plot are nearly even-symmetric with respect to $\frac{\pi}{2}$ because the 2π-period of a discrete signal's spectrum makes the influence of the negative frequency component increases when ω is close to 0 or π. However, two of the MDCT estimators [8], [11] exhibit large deviations because the additive noise considerably affects them when a bin value is close to 0. For the two Ferreira estimators [13], [14], the errors are nearly constant because their fitting function brings larger errors than the negative frequency does. All other estimators exhibit an obvious error decrease when ω

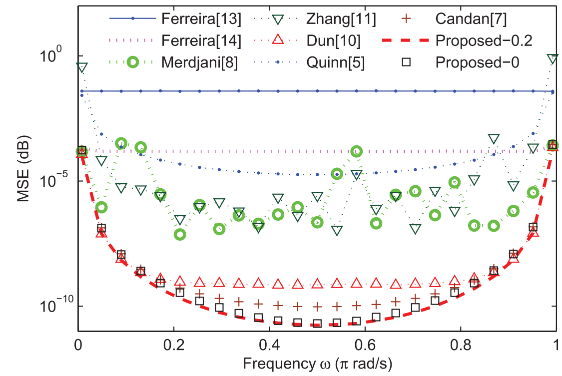

Fig. 4. MSE of the frequency estimators while ω varies from 0 to π. Here, we use a set of the l_0 values, and δ is set randomly, obeying a uniform distribution in the range of 0 to 1 for each l_0. SNR $= 100$ dB.

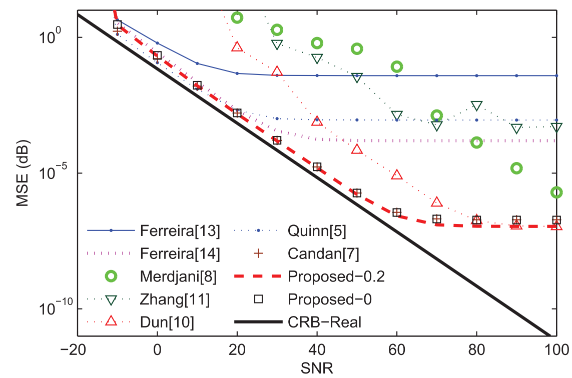

Fig. 5. Plot of the MSE vs. SNR of different frequency estimators for single tone real-valued input. $N = 2048$, $l_0 = 46$, and δ is set randomly, obeying uniform distribution in the range 0 to 1. Thus, $\omega \approx 0.045\pi$, corresponding to approximately 1 kHz with $f_s = 44.1$ kHz and $N = 2048$.

varies from 0 to $\frac{\pi}{2}$. The proposed odd-DFT domain estimator (for both γ values) shows smaller errors than the DFT-domain estimators [5], [7], this is because the sine window is used. Although the sine window is not a maximum side-lobe decay window that is used to obtain high imaginary component interference rejection estimator [20], it ensures better resistance to the imaginary frequency than the rectangular window does.

We also tested the variances at different SNRs for frequencies near $\omega = 0.045\pi$, corresponding to approximately 1 kHz with $f_s = 44.1$ kHz and $N = 2048$. The result is given in Fig. 5. The bound CRB-Real $\approx$ 2CRB [21] is used. As shown, the curves are similar to that of the complex signal input case shown in Fig. 3, but the two DFT-domain estimators [5], [7] exhibit either comparable or larger bias here. The proposed estimator is the smallest biased among the sine window-based non-iterative estimators.

V. Conclusions

This letter presents an interpolated odd-DFT-based sinusoidal frequency estimator with fine resolution. The estimator is based on the derived analytical expression of the odd-DFT coefficient for a sine windowed sinusoidal signal. The proposed frequency estimator outperforms other odd-DFT and MDCT domain methods. Its fine resolution, which is similar to a modern interpolated DFT estimator, together with its low complexity ensures that it is a good choice for MDCT-based audio applications. The magnitude and phase estimators are also provided in this letter. Further, the method can also be generalized to other window cases through numerical simulations.

References

[1] J. Princen and A. Bradley, "Analysis/synthesis filter bank design based on time domain aliasing cancellation," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-34, no. 5, pp. 1153–1161, 1986.

[2] H. S. Malvar, "Lapped transforms for efficient transform/subband coding," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 38, no. 6, pp. 969–978, 1990.

[3] H. So, K. W. Chan, Y. Chan, and K. Ho, "Linear prediction approach for efficient frequency estimation of multiple real sinusoids: Algorithms and analyses," *IEEE Trans. Signal Process*, vol. 53, no. 7, pp. 2290–2305, Jul. 2005.

[4] H. C. So, F. Chan, and W. Sun, "Subspace approach for fast and accurate single-tone frequency estimation," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 827–831, Feb. 2011.

[5] B. Quinn, "Estimating frequency by interpolation using fourier coefficients," *IEEE Trans. Signal Process*, vol. 42, no. 5, pp. 1264–1268, May 1994.

[6] E. Jacobsen and P. Kootsookos, "Fast, accurate frequency estimators [DSP tips tricks]," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 123–125, May 2007.

[7] C. Candan, "Analysis and further improvement of fine resolution frequency estimation method from three DFT samples," *IEEE Signal Process. Lett.*, vol. 20, no. 9, pp. 913–916, Sep. 2013.

[8] S. Merdjani and L. Daudet, "Direct estimation of frequency from MDCT-encoded files," in *Proc. 6th Int. Conf. Digital Audio Effects (DAFx-03) Conf.*, Citeseer, Sep. 2003, pp. 1–4.

[9] M. Zhu, W. Zheng, D. Li, and M. Zhang, "An accurate low complexity algorithm for frequency estimation in MDCT domain," *IEEE Trans. Consumer Electron.*, vol. 54, no. 3, pp. 1022–1028, 2008.

[10] Y. Dun and G. Liu, "An improved MDCT domain frequency estimation method," in *Proc. IEEE Int. Conf. Signal and Information Process. (ChinaSIP2014)*, Jul. 2014, pp. 120–123.

[11] S. Zhang, W. Dou, and H. Yang, "MDCT sinusoidal analysis for audio signals analysis and processing," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 21, no. 7, pp. 1403–1414, Jul. 2013.

[12] Y. Dun and G. Liu, "Frequency estimation performance of several MDCT domain algorithms," in *Proc. Int. Conf. Internet Multimedia Computing and Service*, 2014, pp. 399–402, ACM.

[13] A. Ferreira, "Accurate estimation in the ODFT domain of the frequency, phase and magnitude of stationary sinusoids," in *2001 IEEE Workshop on Applications of Signal Process. to Audio and Acoustics*, 2001, pp. 47–50, IEEE.

[14] A. Ferreira and D. Sinha, "Accurate and robust frequency estimation in the ODFT domain," in *2005 IEEE Workshop on Applications of Signal Process. to Audio and Acoustics*, Oct. 2005, pp. 203–206.

[15] A. Ferreira, "An odd-DFT based approach to time-scale expansion of audio signals," *IEEE Trans. Speech Audio Process.*, vol. 7, no. 4, pp. 441–453, Jul. 1999.

[16] R. Sousa, A. Ferreira, and P. Alku, "The harmonic and noise information of the glottal pulses in speech," *Biomed. Signal Process. Contr.*, vol. 10, no. 0, pp. 137–143, 2014 [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1746809413001791

[17] M. Fabiani, "Frequency, phase and amplitude estimation of overlapping partials in monaural musical signals," in *13th Int. Conf. Digital Audio Effects, DAFx 2010*, Sep. 2010, pp. 1–8.

[18] S. Schuster, S. Scheiblhofer, and A. Stelzer, "The influence of windowing on bias and variance of DFT-based frequency and phase estimation," *IEEE Trans. Instrum. Meas.*, vol. 58, no. 6, pp. 1975–1990, Jun. 2009.

[19] D. Rife and R. Boorstyn, "Single tone parameter estimation from discrete-time observations," *IEEE Trans. Inf. Theory*, vol. 20, no. 5, pp. 591–598, Sep. 1974.

[20] D. Belega, D. Petri, and D. Dallet, "Frequency estimation of a sinusoidal signal via a three-point interpolated DFT method with high image component interference rejection capability," *Digital Signal Process.*, vol. 24, pp. 162–169, 2014 [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1051200413002194

[21] M. Macleod, "Fast nearly ML estimation of the parameters of real or complex single tones or resolved multiple tones," *IEEE Trans. Signal Process.*, vol. 46, no. 1, pp. 141–148, Jan. 1998.